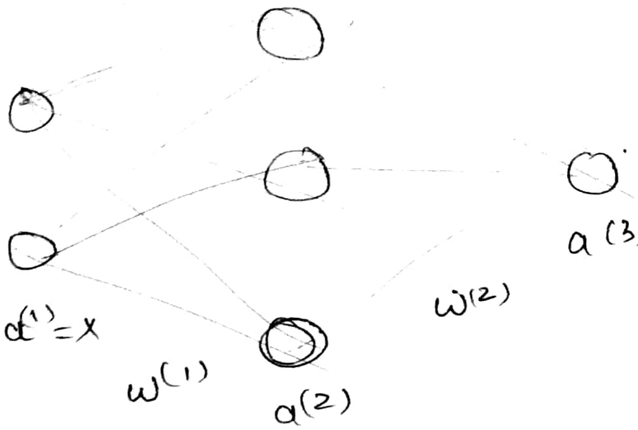
Neural Network Equations :- Unsupervised Learning



$a^{(3)} = z^{(3)}$ [No sigmoid applied at output layer]

J (cost function) $= \frac{1}{2} \sum (y - \hat{y})^2$ [squared mean error]

$$\frac{\partial J}{\partial w^{(2)}} = \sum \frac{\partial}{\partial w^{(2)}} \frac{1}{2} \times (y - \hat{y})^2 = \frac{2}{2} (y - \hat{y}) (-1) \frac{\partial \hat{y}}{\partial w^{(2)}}$$

$$= (-1)(y - \hat{y}) \frac{\partial \hat{y}}{\partial w^{(2)}} = (-1)(y - \hat{y}) \frac{\partial a^{(3)}}{\partial w^{(2)}} \quad \left[\hat{y} = a^{(3)} = z^{(3)} \right]$$

$$z^{(3)} = a^{(2)} w^{(2)}$$

$$\frac{\partial a^{(3)}}{\partial w^{(2)}} = \frac{\partial z^{(3)}}{\partial w^{(2)}} = a^{(2)}.$$

$$\boxed{\frac{\partial J}{\partial w^{(2)}} = (-1)(y - \hat{y})\, a^{(2)}}$$

Now finding $\frac{\partial J}{\partial w^{(1)}} = (-1)(y - \hat{y}) \frac{\partial a^{(3)}}{\partial w^{(1)}}$

$$\frac{\partial a^{(3)}}{\partial w^{(1)}} = \frac{\partial (a^{(2)} w^{(2)})}{\partial w^{(1)}} = \frac{\partial (a^{(2)} w^{(2)})}{\partial a^{(2)}} \times \frac{\partial a^{(2)}}{\partial w^{(1)}}$$

$$= w^{(2)} \times \frac{\partial a^{(2)}}{\partial w^{(1)}}$$

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$$\frac{\partial a^{(2)}}{\partial w^{(1)}} \qquad a^{(2)} = \sigma\left(z^{(2)}\right)$$

$$\frac{\partial a^{(2)}}{\partial w^{(1)}} = \frac{\partial a^{(2)}}{\partial z^{(2)}} \times \frac{\partial z^{(2)}}{\partial w^{(1)}}$$

$$\frac{\partial a^{(2)}}{\partial w^{(1)}} = \frac{\partial \cdot \sigma\left(z^{(2)}\right)}{\partial z^{(2)}} \times \frac{\partial z^{(2)}}{\partial w^{(1)}}$$

$$= \sigma'\left(z^{(2)}\right) \times \frac{\partial\, a^{(1)} \times w^{(1)}}{\partial w^{(1)}}$$

$$= \sigma'\left(z^{(2)}\right) \times a^{(1)}$$

$$= \sigma'\left(z^{(2)}\right) \times X$$

So

$$\boxed{\frac{\partial J}{\partial w^{(1)}} = (-1)\cdot(y-\hat{y})\cdot w^{(2)} \cdot \sigma'\left(z^{(2)}\right) \cdot X}$$

$$\boxed{\frac{\partial J}{\partial w^{(2)}} = (-1)\,(y-\hat{y})\cdot a^{(2)}}$$