Statement: For any integer n, the number $n^2 + n + 1$ is odd.

(Idea:)

proof:

let n be an arbitary integer

case 1: n is even

let $n=2k$

$n^2+n+1=(2k)^2+(2k)+1$

$n^2+n+1=4k^2+2k+1$

$n^2+n+1=2(2k^2+k)+1$

which is of form 2m+1 ie odd.

Case 2:n is odd

let $n=2k+1$

$n^2+n+1=(2k+1)^2+(2k+1)+1$

$n^2+n+1=4k^2+4k+1+2k+1+1$

$n^2+n+1=4k^2+6k+3$

$n^2+n+1=2(2k^2+3k+1)+1$

which is of form 2m+1 ie, odd

In either case, it is odd

$\therefore$ For any natural number n, n^2+n+1 is an od number