

Statement: Prove that every odd natural number is of one of the forms  $4n + 1$  or  $4n + 3$ , where  $n$  is an integer.

(Idea:)

proof:

For an arbitrary integer  $n$ , let integer  $k$  such that  $n = 2k + 1$

case 1:  $k$  is even

$$\text{let } k = 2m$$

$$n = 2(2m) + 1$$

$$n = 4m + 1 \quad \dots\dots\dots(1)$$

$n$  can be of form  $4m + 1$

Case 2:  $k$  is odd

$$\text{let } k = 2m + 1$$

$$n = 2(2m + 1) + 1$$

$$n = 4m + 3 \quad \dots\dots\dots(2)$$

$n$  can be of form  $4m + 3$

$\therefore$  Every odd integer can be of form  $4n + 1$  or  $4n + 3$