8. Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

Proof:
Assume $\lim_{n \to \infty} a_n = L$ and let M be an arbitary positive real number.
From definition of limit, we have $(\forall\, \varepsilon > 0)(\exists n \in \mathbb{N})(\forall\, m \geq n)[|a_m - L| \leq \varepsilon]$.
Since $\frac{\varepsilon}{M}$ is also a positive real, we can find another natural n_0 such that $(\forall\, m \geq n_0)\left[|a_m - L| \leq \frac{\varepsilon}{M}\right]$.
We will use this to prove limit for sequence $\{Ma_n\}_{n=1}^{\infty}$ below

For an arbitary ε>0 and for every $m \geq n_0$ show that $|Ma_m - ML| \leq \varepsilon$

$|Ma_m - ML| \leq \varepsilon$

$M|a_m - L| \leq \varepsilon$ ($\because M > 0$)

$\left[|a_m - L| \leq \frac{\varepsilon}{M}\right]$ which is true.

Hence sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to ML as $n \to \infty$.

$\therefore$ if $\lim_{n \to \infty} a_n = L$ then $\lim_{n \to \infty} Ma_n = ML$.