Statement: Prove that for any natural number n ,$P(n)$ holds true

where $P(n) := \sum_{k=1}^{n} 2^k = 2^{(n+1)} - 2$

(Idea:) Proof via Induction

proof:

Take n=1

$$\sum_{k=1}^{1} 2^k = 2^{(1+1)} - 2$$

$$2 = 4 - 2$$

$$2 = 2$$

Left hand side = Right hand side,

$\therefore$ $P(1)$ holds True

Step 2: For all n, assume $\sum_{k=1}^{n} 2^k = 2^{(n+1)} - 2$

add $2^{(n+1)}$ on both sides

$$2^{(n+1)} + \sum_{k=1}^{n} 2^k = 2^{(n+1)} + 2^{(n+1)} - 2$$

$$\sum_{k=1}^{n+1} 2^k = 2 \cdot 2^{(n+1)} - 2$$

$$\sum_{k=1}^{n+1} 2^k = 2^{(n+2)} - 2$$

$$\sum_{k=1}^{n+1} 2^k = 2^{((n+1)+1)} - 2 \Leftrightarrow P(k+1)$$

$\therefore$ $\forall\, n : P(n) \Rightarrow P(n+1)$ holds true

$\therefore$Proof by Induction.