

[Note from the writer: It is my first time making documents and I do 'nt know TeX yet. Forgive any ugly symbols or odd formatting and provide feedback. I am looking to get better at using Libreoffice to make documents]

Say whether the following is true or false and support your answer by a proof.

$$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})[3m + 5n = 12]$$

Answer:

The given statement is false

proof (via contradiction):

Assume given statement is true.

Let  $m, n$  be 2 naturals such that  $3m + 5n = 12$

$$3m + 5n = 12 \dots\dots (1)$$

$$5n = 12 - 3m$$

$$5n = 3(4 - m)$$

$$3|5n$$

$$3|n \text{ (}\because 5 \text{ is a prime number)}$$

let  $k$  such that  $n = 3k$

put  $n = 3k$  in (1)

$$3m + 5(3k) = 12$$

$$3m + 15k = 12$$

$$3(m + 5k) = 12$$

$$m + 5k = 4 \dots\dots \text{False, because } m + 5k \text{ cannot be less than 6 (remember that we are working with natural numbers only)}$$

$\therefore$  There exists no naturals  $m, n$  such that  $3m + 5n = 12$ .