Question : Given an infnite collection A_n , $n=1,2,3,\ldots$ of intervals of the real line, their intersection is defned to be $\bigcap_{n=1}^{\infty} A_n = \{x \mid (\forall n \in \mathbb{N})[x \in A_n]\}$.
Give an example of a family of intervals A_n, n = 1, 2, . . ., such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n = \{\}$

(Idea:) Define a family of intervals A such that $A_n = [n, \infty)$

Answer:

Let A be family of sets, such that $A_n = \{x \in \mathbb{R} \mid n \leq x\}$ where $n \geq 1$

Statement 1: $(\forall n \geq 1)[A_{n+1} \subset A_n]$
proof:

for any positive integer n and real number m, it is obvious that if $n+1 \leq m$ then $n \leq m$
From the definition of family A, we can rephrase above statement as

$$(\forall m, n \in \mathbb{N})[m \in A_{n+1} \Rightarrow m \in A_n]$$

which is equivalent to saying that $A_{n+1} \subset A_n$

$\therefore$ Every set is a superset of it's succesor.

Statement 2: $\bigcap_{n=1}^{\infty} A_n = \{\}$
proof (by contradiction) :

Negation statement: there exists real number $p > 0$ such that for any natural number n, $p \in A_n$.
[Note: I am not checking any negative reals since my selected sets only contain reals above one]

Let $m \in \mathbb{N}$ such that $p < m$.
$p < m$
$\neg(m \leq p)$
$p \notin A_m$ $\because$ definition of family A

$\therefore$ There is no real number that is common to all sets in family A

$\therefore$ Intersection of all sets in family A is an empty set.

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