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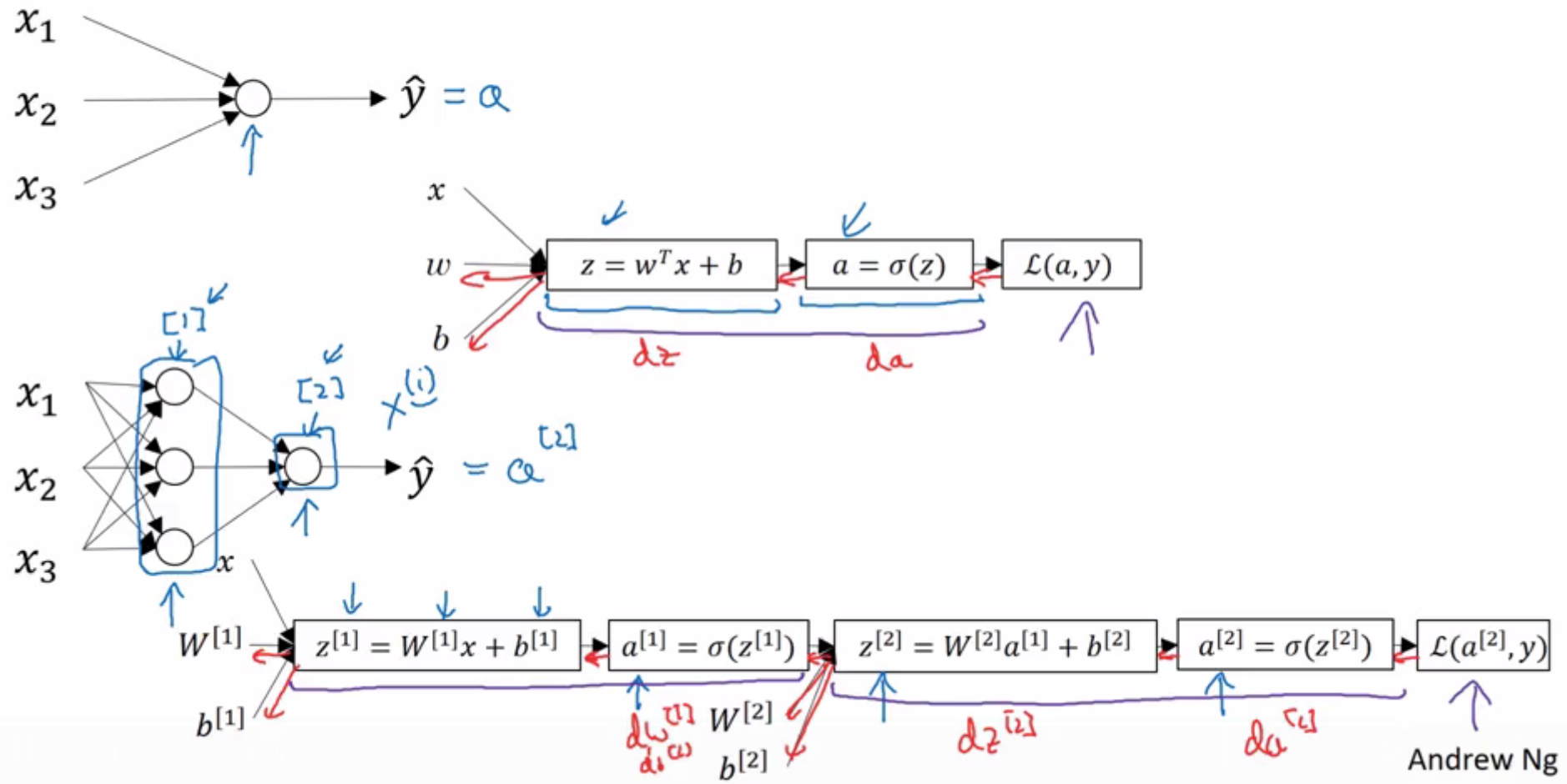
One hidden layer  
Neural Network

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# Neural Networks Overview

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# What is a Neural Network?





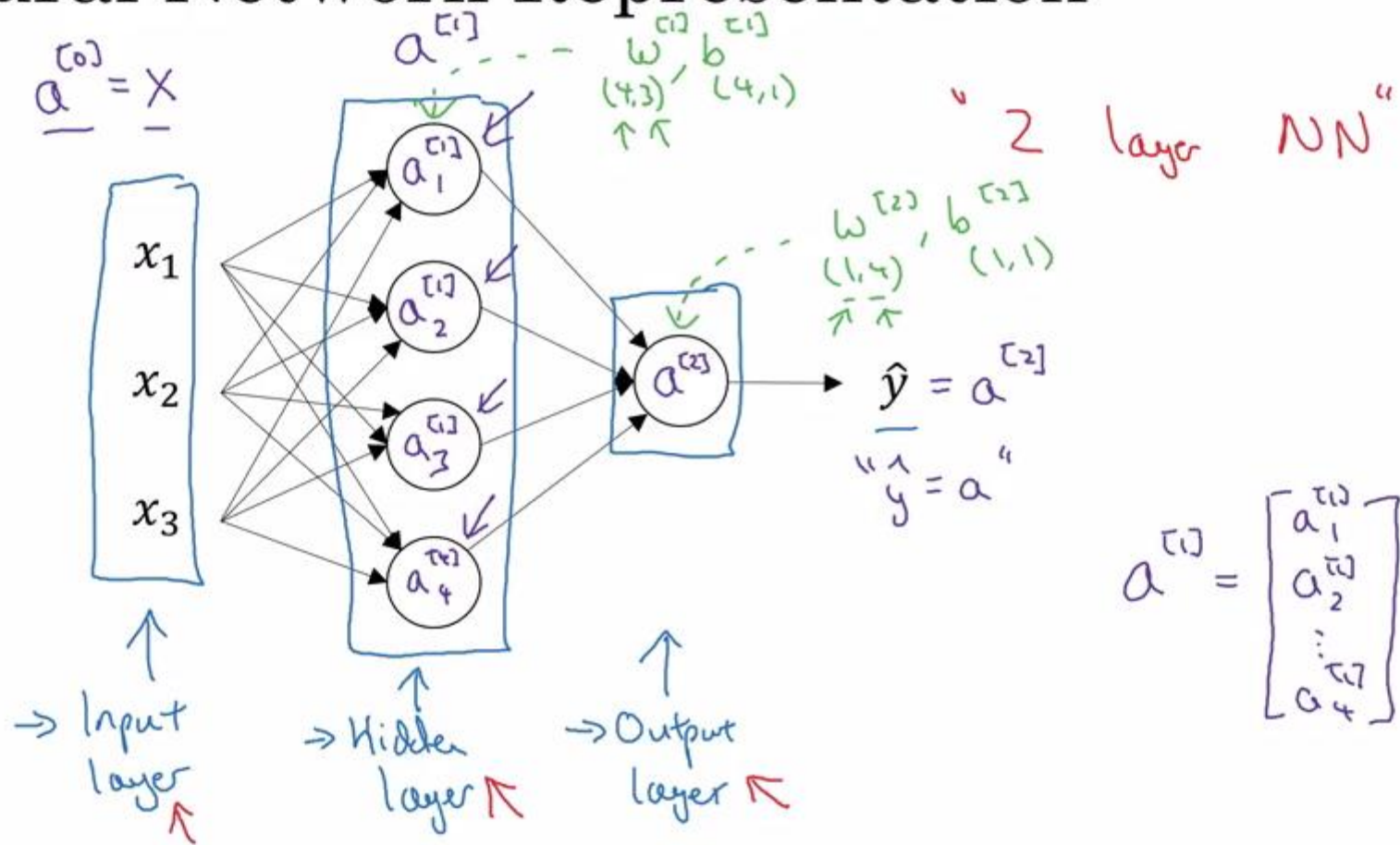
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One hidden layer  
Neural Network

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Neural Network  
Representation

# Neural Network Representation





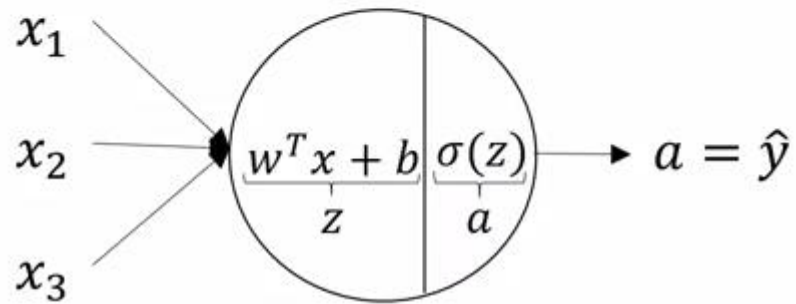
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One hidden layer  
Neural Network



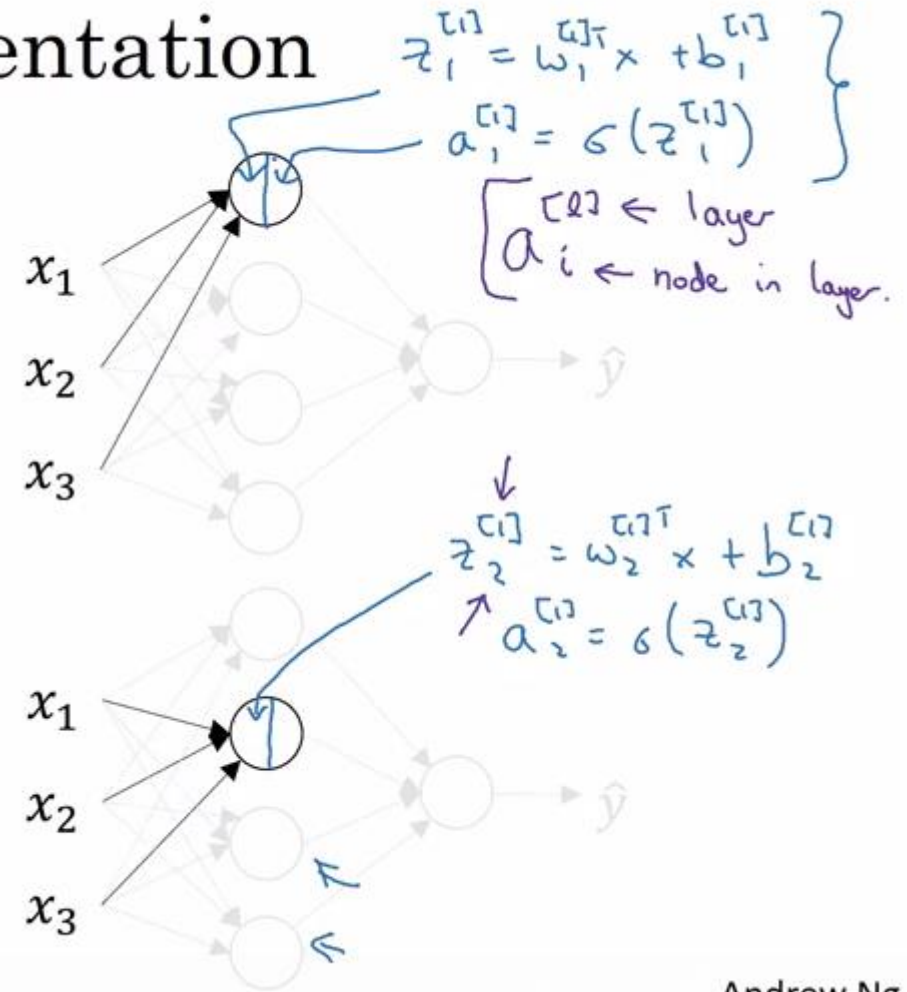
Computing a  
Neural Network's  
Output

# Neural Network Representation



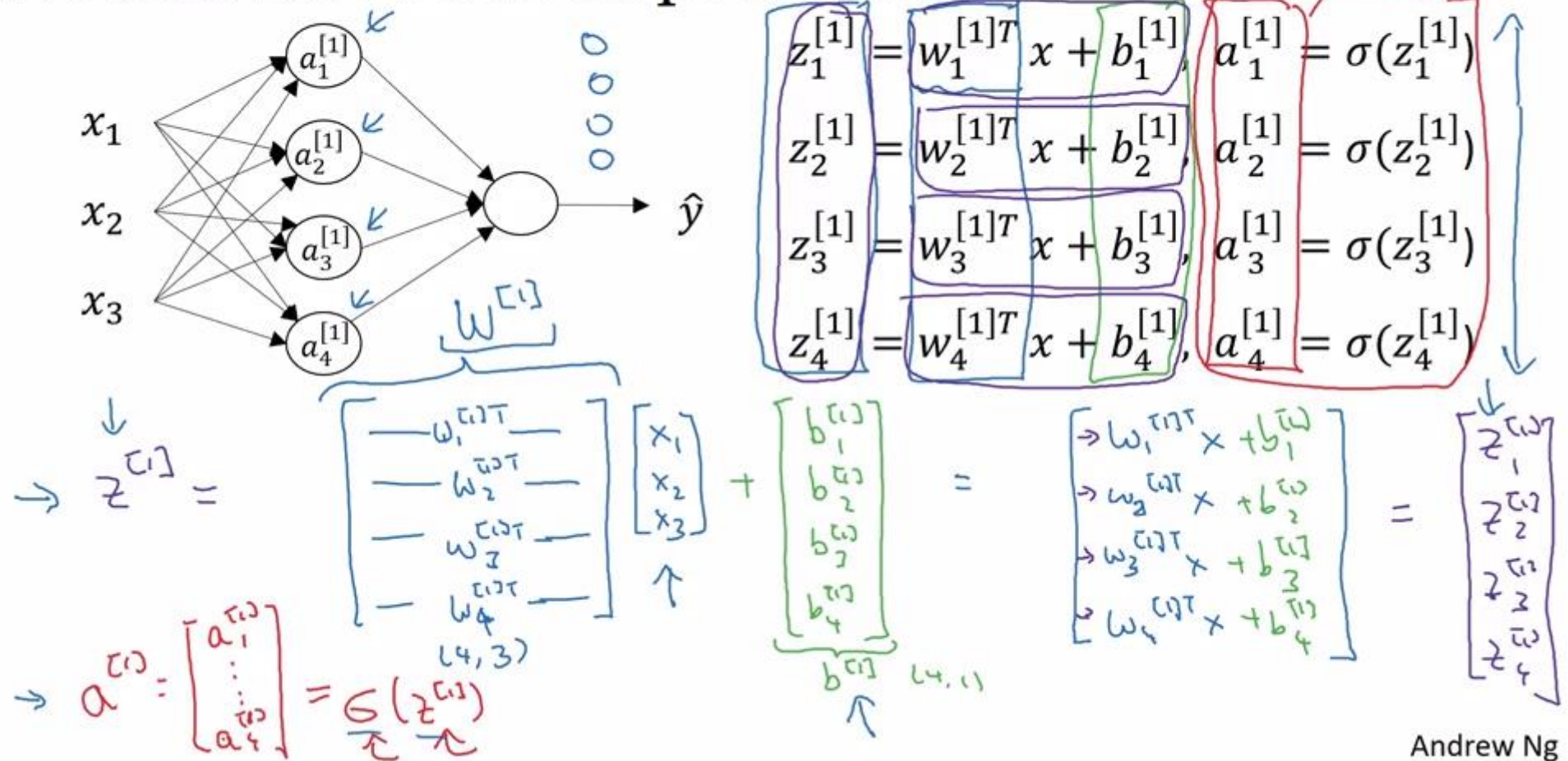
$$z = w^T x + b$$

$$a = \sigma(z)$$

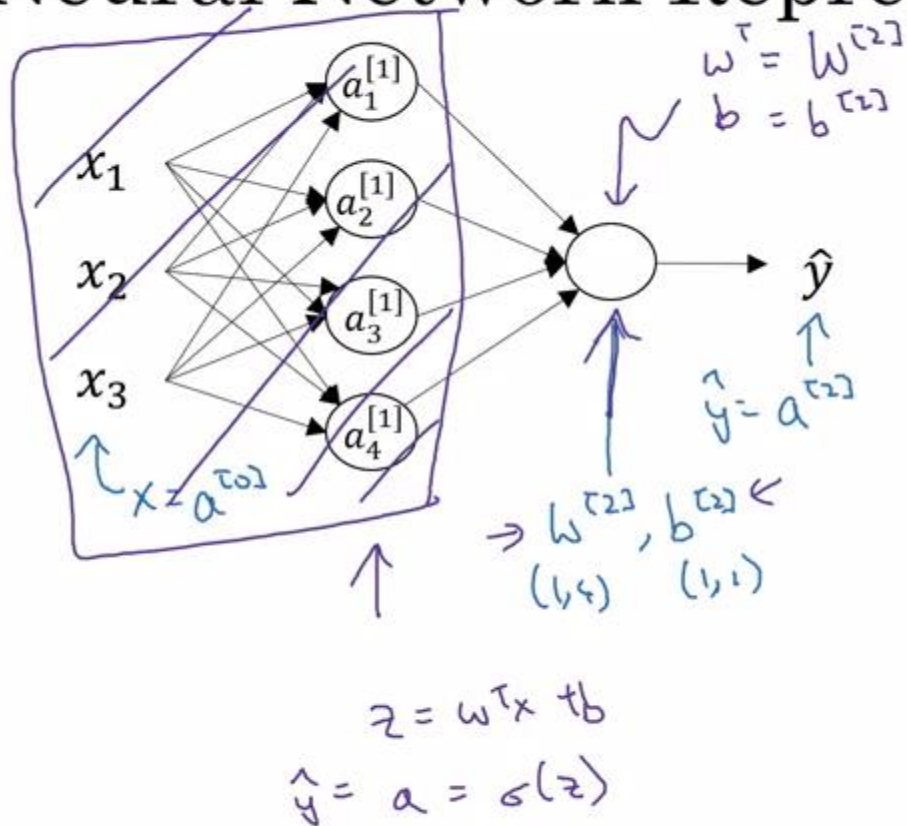




# Neural Network Representation



# Neural Network Representation learning



Given input  $x$ :

$$\begin{aligned} \rightarrow z^{[1]} &= W^{[1]} a^{[0]} + b^{[1]} \\ &\quad \begin{matrix} (4,1) & (4,3) & (3,1) & (4,1) \end{matrix} \\ \rightarrow a^{[1]} &= \sigma(z^{[1]}) \\ &\quad \begin{matrix} (4,1) & (4,1) \end{matrix} \\ \rightarrow z^{[2]} &= W^{[2]} a^{[1]} + b^{[2]} \\ &\quad \begin{matrix} (1,1) & (1,4) & (4,1) & (1,1) \end{matrix} \\ \rightarrow a^{[2]} &= \sigma(z^{[2]}) \\ &\quad \begin{matrix} (1,1) & (1,1) \end{matrix} \end{aligned}$$





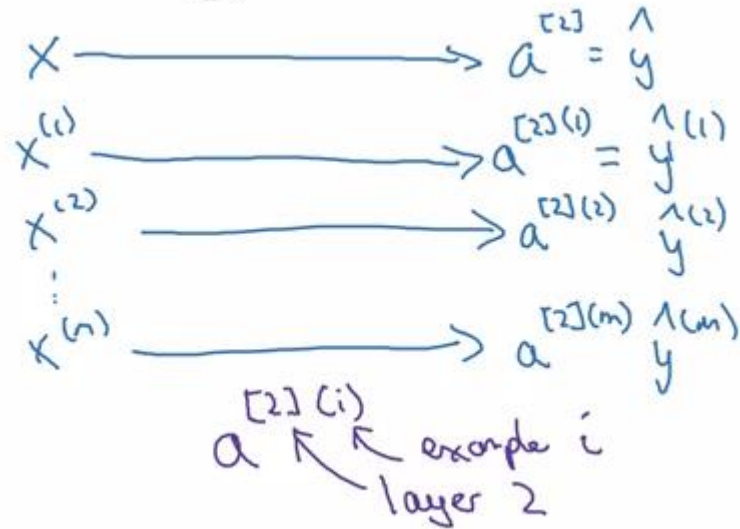
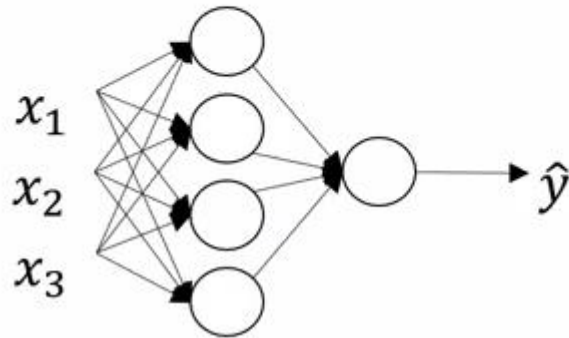
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# One hidden layer Neural Network

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Vectorizing across  
multiple examples

# Vectorizing across multiple examples



$$\left\{ \begin{array}{l} z^{[1]} = W^{[1]}x + b^{[1]} \\ a^{[1]} = \sigma(z^{[1]}) \\ z^{[2]} = W^{[2]}a^{[1]} + b^{[2]} \\ a^{[2]} = \sigma(z^{[2]}) \end{array} \right\} \leftarrow$$

→ for  $i = 1$  to  $m$ ,

$$\begin{aligned} z^{[1](i)} &= W^{[1]}x^{(i)} + b^{[1]} \\ a^{[1](i)} &= \sigma(z^{[1](i)}) \\ z^{[2](i)} &= W^{[2]}a^{[1](i)} + b^{[2]} \\ a^{[2](i)} &= \sigma(z^{[2](i)}) \end{aligned}$$

# Vectorizing across multiple examples

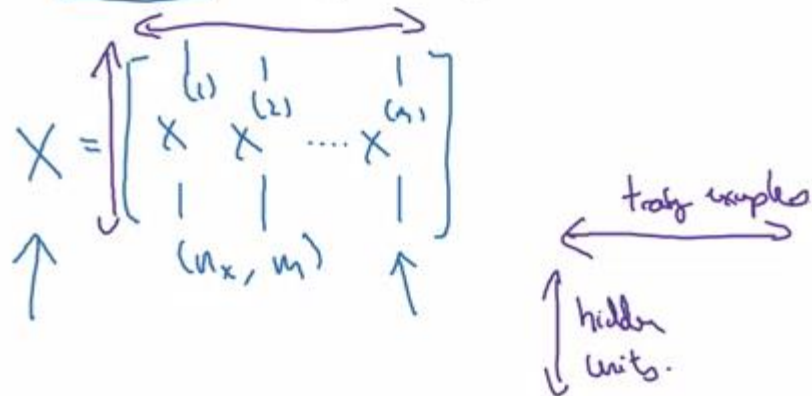
for  $i = 1$  to  $m$ :

$$z^{[1]}(i) = W^{[1]}x^{(i)} + b^{[1]}$$

$$a^{[1]}(i) = \sigma(z^{[1]}(i))$$

$$z^{[2]}(i) = W^{[2]}a^{[1]}(i) + b^{[2]}$$

$$a^{[2]}(i) = \sigma(z^{[2]}(i))$$

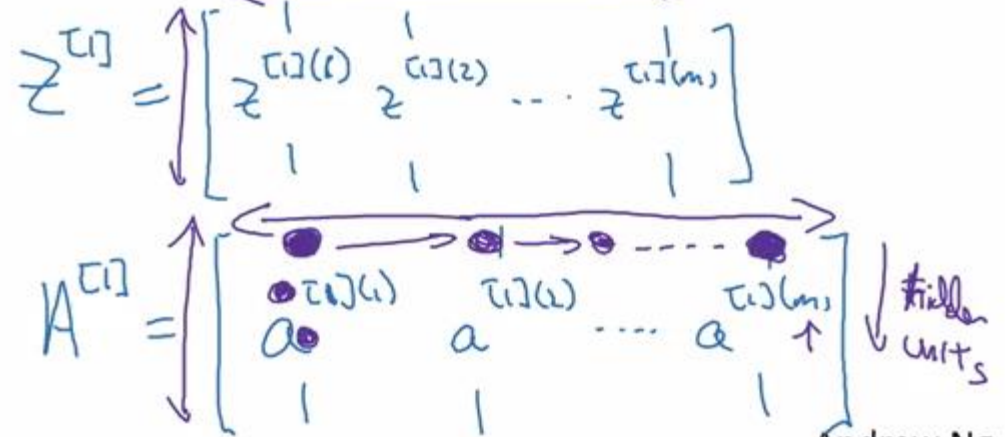


$$z^{[1]} = W^{[1]}X + b^{[1]}$$

$$\rightarrow A^{[1]} = \sigma(z^{[1]})$$

$$\rightarrow z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$\rightarrow A^{[2]} = \sigma(z^{[2]})$$





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# One hidden layer Neural Network

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Explanation  
for vectorized  
implementation

# Justification for vectorized implementation

$$z^{[1](1)} = \omega^{[1]} x^{(1)} + \cancel{b^{[1]}} \quad , \quad z^{[1](2)} = \omega^{[1]} x^{(2)} + \cancel{b^{[1]}} \quad , \quad z^{[1](3)} = \omega^{[1]} x^{(3)} + \cancel{b^{[1]}}$$

↑ 0      ↑ 0      ↑ 0

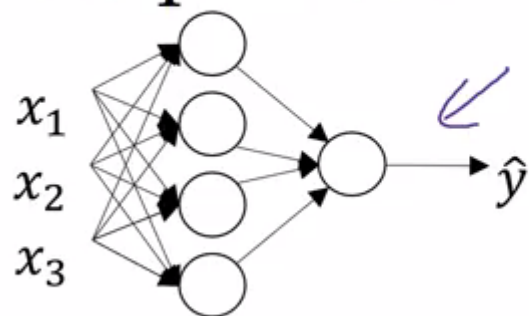
$$\omega^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \quad \omega^{[1]} x^{(1)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} \quad \omega^{[1]} x^{(2)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} \quad \omega^{[1]} x^{(3)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix}$$

$$\omega^{[1]} \begin{bmatrix} | & | & | & \dots \\ x^{(1)} & x^{(2)} & x^{(3)} & \dots \\ | & | & | & \dots \end{bmatrix} = \begin{bmatrix} \bullet & \bullet & \bullet & \dots \\ \bullet & \bullet & \bullet & \dots \\ \bullet & \bullet & \bullet & \dots \\ \bullet & \bullet & \bullet & \dots \end{bmatrix} = \begin{bmatrix} | & | & | & \dots \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \dots \\ | & | & | & \dots \end{bmatrix} = z^{[1]}$$

$\cancel{z^{[1]} = \omega^{[1]} X + b^{[1]}}$

$\omega^{[1]} X = z^{[1]}$

# Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & \dots & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & \dots & | \end{bmatrix}$$

A blue arrow points to the matrix  $X$ .

$$\underline{A^{[1]}} = \begin{bmatrix} | & | & \dots & | \\ a^{[1](1)} & a^{[1](2)} & \dots & a^{[1](m)} \\ | & | & \dots & | \end{bmatrix}$$

A blue arrow points to the matrix  $A^{[1]}$ .

for  $i = 1$  to  $m$

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$

$$\rightarrow a^{[1](i)} = \sigma(z^{[1](i)})$$

$$\rightarrow z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$\rightarrow a^{[2](i)} = \sigma(z^{[2](i)})$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

$$x = a^{[0]} \quad x^{(i)} = a^{[0](i)}$$

$$W^{[1]}A^{[0]} + b^{[1]}$$





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# One hidden layer Neural Network

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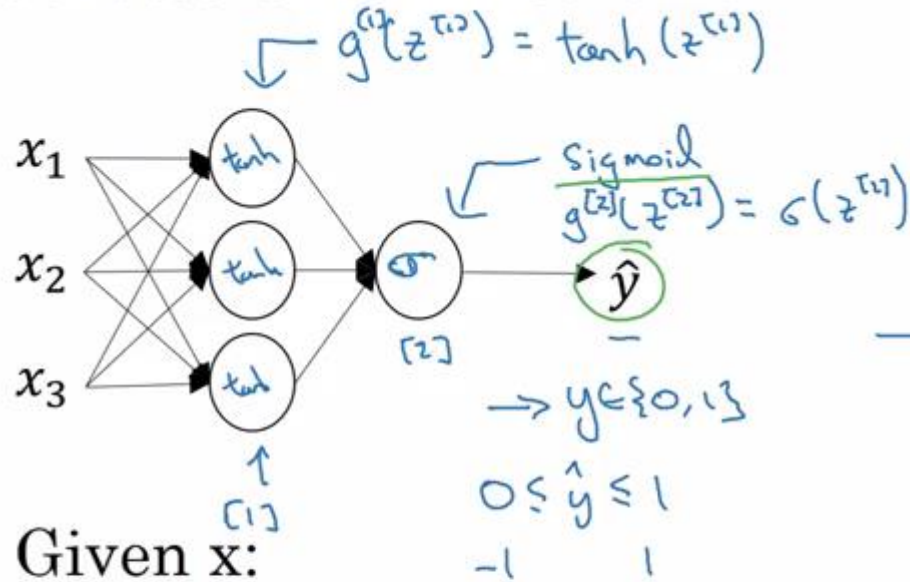
## Activation functions



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# Activation functions



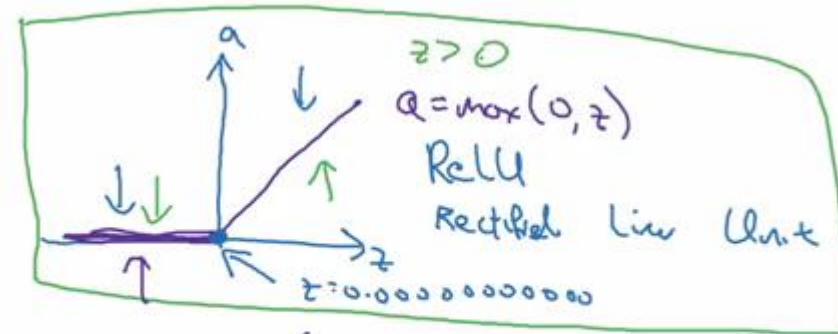
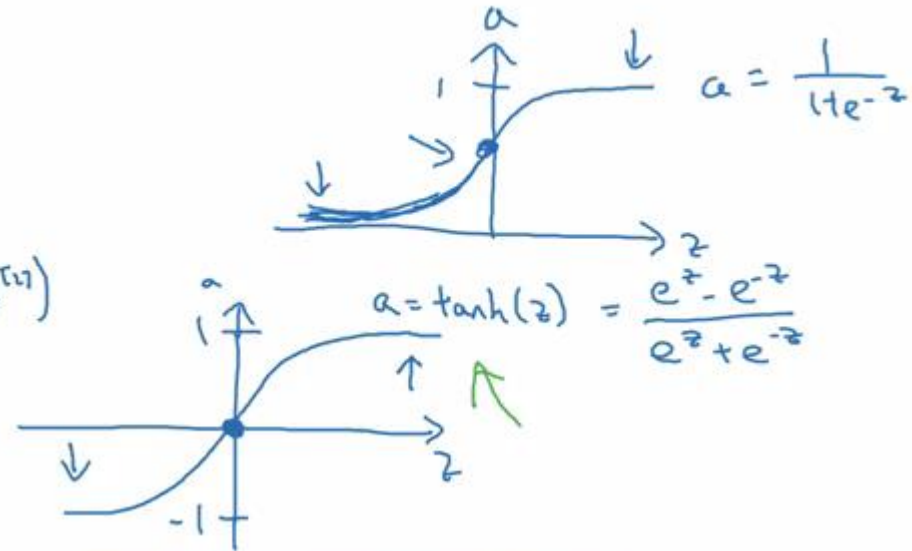
Given  $x$ :

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

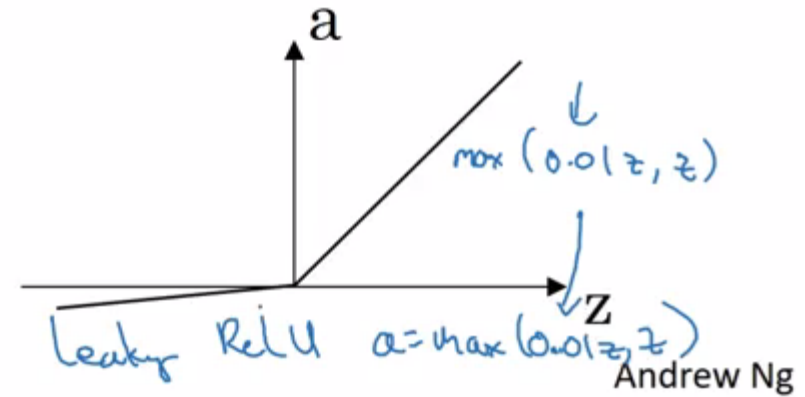
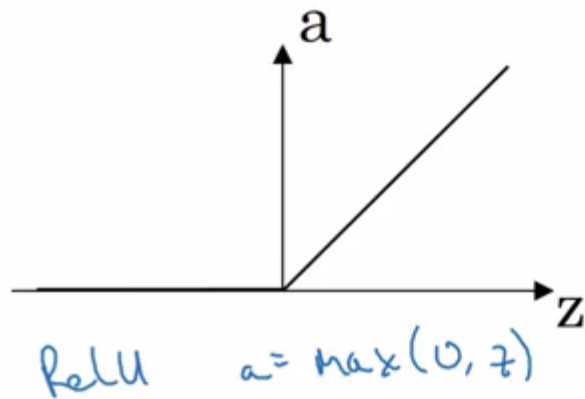
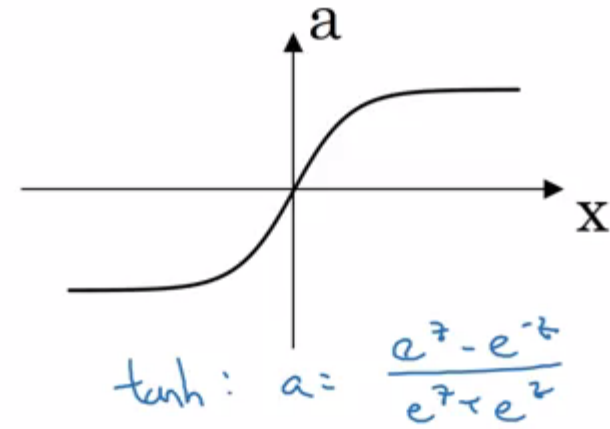
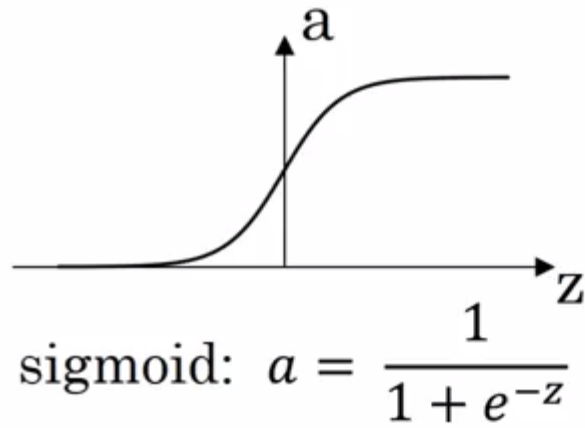
$$\rightarrow a^{[1]} = \sigma(z^{[1]}) \quad g^{(1)}(z^{(1)})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$\rightarrow a^{[2]} = \sigma(z^{[2]}) \quad g^{(2)}(z^{(2)})$$



# Pros and cons of activation functions





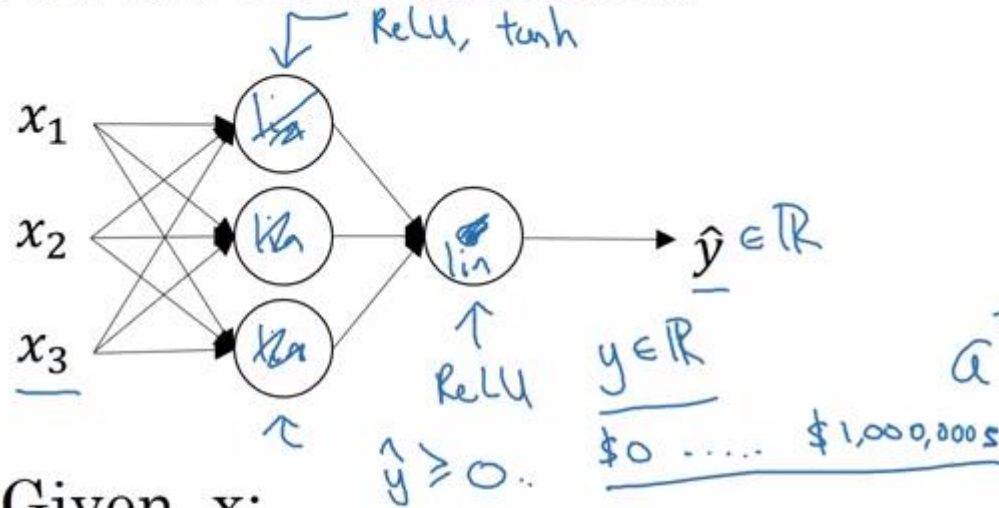
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One hidden layer  
Neural Network

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Why do you  
need non-linear  
activation functions?

# Activation function



Given  $x$ :

$$\begin{aligned} \rightarrow z^{[1]} &= W^{[1]}x + b^{[1]} \\ \rightarrow a^{[1]} &= \underline{g^{[1]}(z^{[1]})} \quad z^{[1]} \\ \rightarrow z^{[2]} &= W^{[2]}a^{[1]} + b^{[2]} \\ \rightarrow a^{[2]} &= \underline{g^{[2]}(z^{[2]})} \quad z^{[2]} \end{aligned}$$

$g(z) = z$   
"linear activation function"

$$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = W^{[2]}(W^{[1]}x + b^{[1]}) + b^{[2]}$$

$$= \underbrace{(W^{[2]}W^{[1]})}_{w'}x + \underbrace{(W^{[2]}b^{[1]} + b^{[2]})}_{b'}$$

$$= \underline{w'x + b'}$$

$$g(z) = z$$



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One hidden layer  
Neural Network

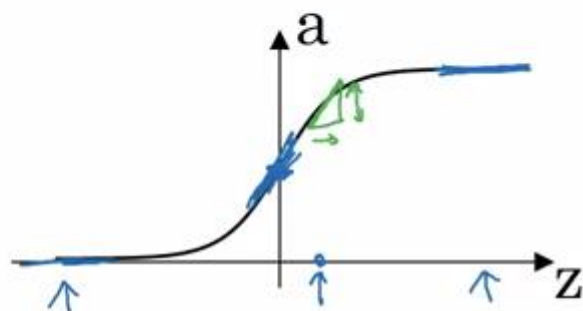
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Derivatives of  
activation functions

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# Sigmoid activation function



$$g(z) = \frac{1}{1 + e^{-z}}$$

$$a = g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z$$

$$= \frac{1}{1 + e^{-z}} \left( 1 - \frac{1}{1 + e^{-z}} \right)$$

$$= g(z) (1 - g(z)) \leftarrow$$

$$= \boxed{a(1-a)} \quad \left| \begin{array}{l} g'(z) = a(1-a) \\ \uparrow \\ a \end{array} \right.$$

$$z = 10, \quad g(z) \approx 1$$

$$\frac{d}{dz} g(z) \approx 1(1-1) \approx 0$$

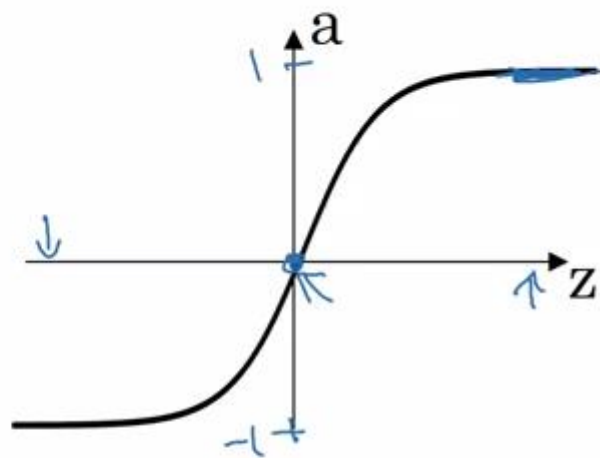
$$z = -10, \quad g(z) \approx 0$$

$$\frac{d}{dz} g(z) \approx 0(1-0) \approx 0$$

$$z = 0, \quad g(z) = \frac{1}{2}$$

$$\frac{d}{dz} g(z) = \frac{1}{2} \left( 1 - \frac{1}{2} \right) = \frac{1}{4}$$

# Tanh activation function



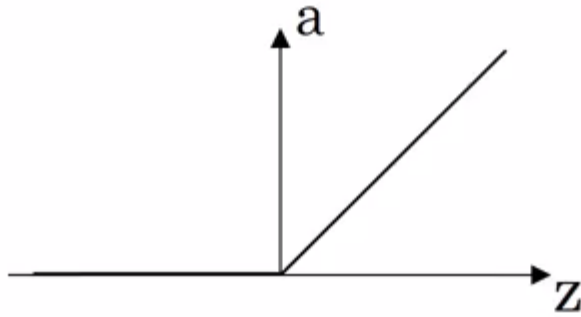
$$g(z) = \tanh(z) \\ = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z \\ = \underline{1 - (\tanh(z))^2} \leftarrow$$

$$a = g(z), \quad g'(z) = 1 - a^2$$

$$\left| \begin{array}{ll} z=10 & \tanh(z) \approx 1 \\ & g'(z) \approx 0 \\ z=-10 & \tanh(z) \approx -1 \\ & g'(z) \approx 0 \\ z=0 & \tanh(z) = 0 \\ & g'(z) = 1 \end{array} \right.$$

# ReLU and Leaky ReLU

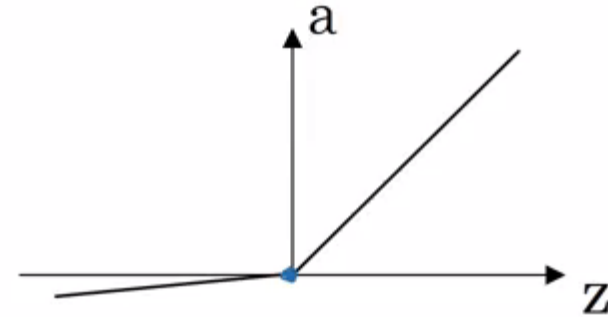


ReLU

$$g(z) = \max(0, z)$$

$$\rightarrow g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \text{undefined} & \text{if } z = 0 \end{cases}$$

$z = 0.0000000000$



Leaky ReLU

$$g(z) = \max(0.01z, z)$$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$



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One hidden layer  
Neural Network

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Gradient descent for  
neural networks

# Gradient descent for neural networks

Parameters:  $W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}$   
 $(n^{[1]}, n^{[0]})$   $(n^{[1]}, 1)$   $(n^{[2]}, n^{[1]})$   $(n^{[2]}, 1)$

$$n_x = n^{[0]}, \quad n^{[1]}, \quad \underline{n^{[2]} = 1}$$

$$\text{Cost function: } J(W^{[1]}, b^{[1]}, \underline{W^{[2]}}, \underline{b^{[2]}}) = \frac{1}{n} \sum_{i=1}^n \ell(\hat{y}, y)$$

$\uparrow$   $\uparrow$   $\uparrow$   
 $a^{[2]}$

Gradient descent:

→ Repeat {

→ Compute predicts  $(\hat{y}^{(i)}, i=1, \dots, n)$

$$\underline{\frac{dJ}{dW^{[1]}}} = \frac{\partial J}{\partial W^{[1]}}, \quad \underline{\frac{dJ}{db^{[1]}}} = \frac{\partial J}{\partial b^{[1]}}, \dots$$

$$W^{[1]} := W^{[1]} - \alpha \frac{dJ}{dW^{[1]}}$$

$$b^{[1]} := b^{[1]} - \alpha \frac{dJ}{db^{[1]}}$$

# Formulas for computing derivatives

Forward propagation:

$$z^{[1]} = w^{[1]}x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]}) \leftarrow$$

$$z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$$

Back propagation:

$$dz^{[2]} = A^{[2]} - Y \leftarrow$$

$$dw^{[2]} = \frac{1}{n} dz^{[2]} A^{[1]T}$$

$$db^{[2]} = \frac{1}{n} \text{np.sum}(dz^{[2]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$$dz^{[1]} = \underbrace{w^{[2]T} dz^{[2]}}_{(n^{[1]}, m)} \star \underbrace{g^{[1]'}(z^{[1]})}_{\text{element-wise product}} \quad (n^{[1]}, m)$$

$$dw^{[1]} = \frac{1}{n} dz^{[1]} x^T$$

$$db^{[1]} = \frac{1}{n} \text{np.sum}(dz^{[1]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$(n^{[1]}, 1)$        $(n^{[1]}, 1)$        $\uparrow$  reshape

$$Y = [y^{(1)} \ y^{(2)} \ \dots \ y^{(m)}]$$

$$(n^{[1]}) \leftarrow$$

$$\downarrow (n^{[2]}, 1) \leftarrow$$





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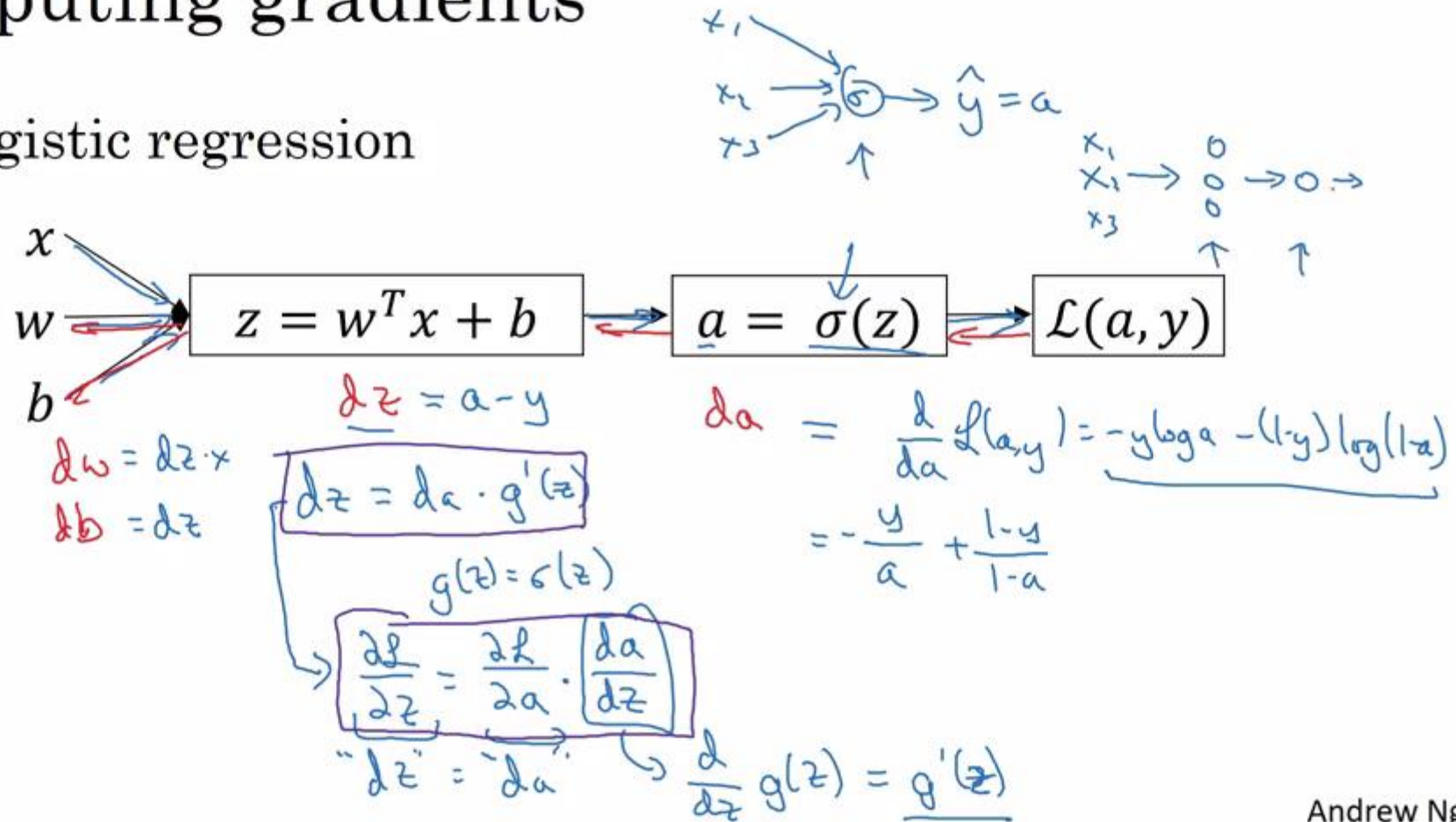
One hidden layer  
Neural Network

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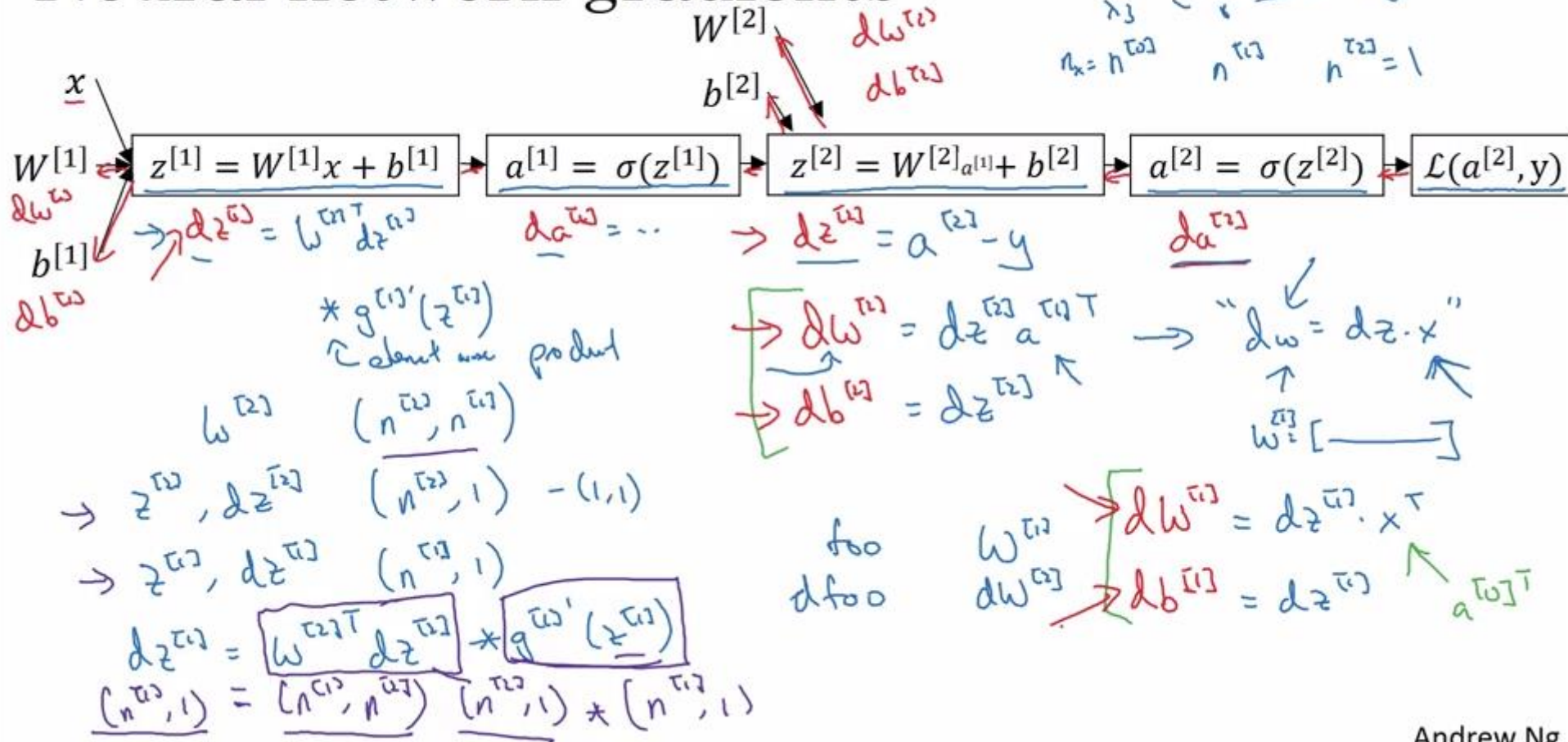
Backpropagation  
intuition (Optional)

# Computing gradients

## Logistic regression



# Neural network gradients



# Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} a^{[1]T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]'}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = W^{[1]} x + b^{[1]}$$
$$a^{[1]} = g^{[1]}(z^{[1]})$$
$$z^{[2]} = \begin{bmatrix} z^{[2](1)} & z^{[2](2)} & \dots & z^{[2](n)} \end{bmatrix}$$
$$z^{[2]} = W^{[2]} X + b^{[2]}$$
$$A^{[2]} = g^{[2]}(z^{[2]})$$

# Summary of gradient descent

$$\underline{dz}^{[2]} = \underline{a}^{[2]} - \underline{y}$$

$$dW^{[2]} = dz^{[2]} a^{[1]T}$$

$$db^{[2]} = dz^{[2]}$$

$$\underset{(n^{[1]}, 1)}{dz^{[1]}} = W^{[2]T} dz^{[2]} * g^{[1]'}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

$$\underline{dZ}^{[2]} = \underline{A}^{[2]} - \underline{Y}$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$$

$$db^{[2]} = \frac{1}{m} \text{np.sum}(dZ^{[2]}, \text{axis} = 1, \text{keepdims} = \text{True})$$

$$\underset{(n^{[2]}, m)}{dZ^{[1]}} = \underbrace{W^{[2]T} dz^{[2]}}_{(n^{[2]}, m)} * \underbrace{g^{[1]'}(Z^{[1]})}_{(n^{[2]}, m)}$$

↙ element-wise product

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} \text{np.sum}(dZ^{[1]}, \text{axis} = 1, \text{keepdims} = \text{True})$$

$$J(\dots) = \frac{1}{m} \sum_{i=1}^m \mathcal{L}(\hat{y}_i, y_i)$$



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# One hidden layer Neural Network

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## Random Initialization

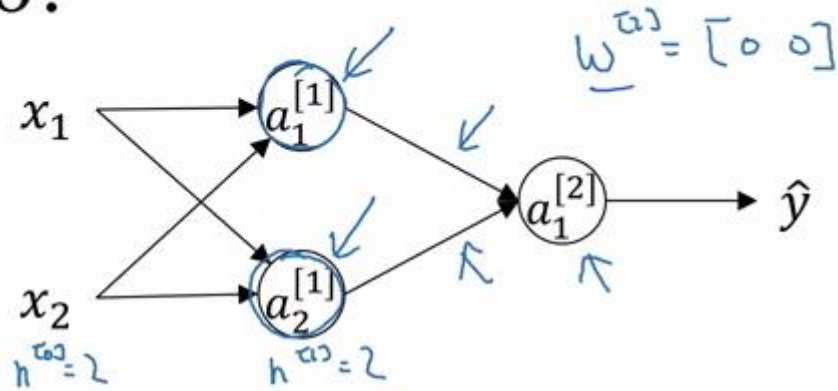


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# What happens if you initialize weights to zero?



$$w^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

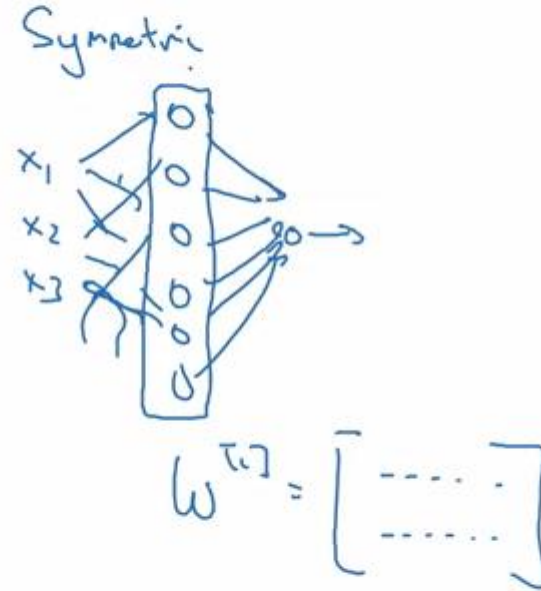
$$b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a_1^{[1]} = a_2^{[1]}$$

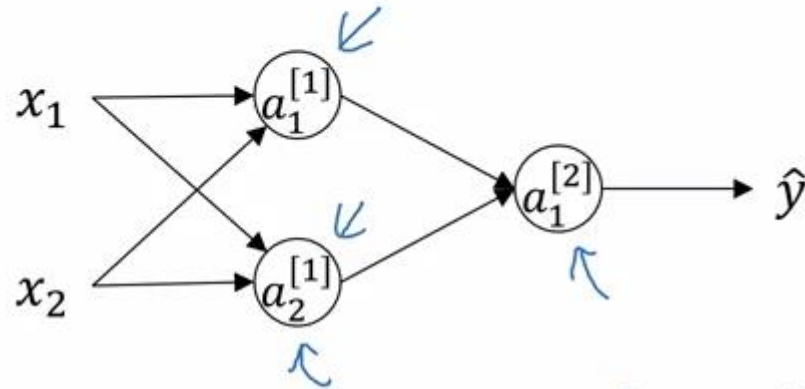
$$dz_1^{[1]} = dz_2^{[1]}$$

$$dW = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$$

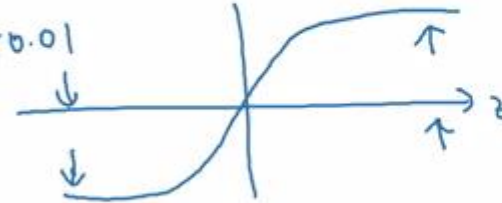
$$W^{[1]} = W^{[1]} - 2dW$$



# Random initialization



$$\begin{aligned} \rightarrow w^{[1]} &= \text{np.random.randn}(2,2) * \frac{0.01}{100?} \\ b^{[1]} &= \text{np.zeros}(2,1) \\ w^{[2]} &= \text{np.random.randn}(1,2) * 0.01 \\ b^{[2]} &= 0 \end{aligned}$$



$$\begin{aligned} \downarrow \\ z^{[1]} &= w^{[1]}x + b^{[1]} \\ a^{[1]} &= g^{[1]}(z^{[1]}) \end{aligned}$$