Dependence Notation

name	symbol	alt.	summ.	example
true dependence	S1 δ^t S2	δ, δ^f	W$\rightarrow$R	`S1: x=10;` `S2: y=x+2;`
anti-dependence	S1 δ^a S2	δ^{-1}	R$\rightarrow$W	`S1: y=x+2;` `S2: x=10;`
output dependence	S1 δ^o S2		W$\rightarrow$W	`S1: x=10;` `S2: x=20;`

notation:

R: Read

$\rightarrow$: then

W: Write

Iteration Vector

```
for (i1 = 0; i1 < N1; i1++) {
  for (i2 = 0; i2 < N2; i2++) {
    ...
      for (in = 0; in < Nn; in++) {
        ...
      }
    ...
  }
}
```

- iteration vector $\vec{i} := (\texttt{i1}, \texttt{i2}, ..., \texttt{in})$ indicates progression through loops
  - e.g. $\vec{i} := (2, 5, 4)$ means there are 3 nested loops:
    - the first (outermost) loop is in its 2nd iteration
    - the second (middle) loop is in its 5th iteration
    - the third (innermost) loop is in its 4th iteration

Dependence Vectors I

- *distance vector*: $d(\vec{i}, \vec{j}) = \vec{j} - \vec{i}$
- e.g. $\mathrm{S}(\vec{i} = (\mathtt{i1} - 2, \mathtt{i2} - 5, \mathtt{i3} - 4))\ \delta_1^t\ \mathrm{S}(\vec{j} = (\mathtt{i1}, \mathtt{i2}, \mathtt{i3}))$ in the code below has $d(\vec{i}, \vec{j}) = (2, 5, 4)$

```
for (i1 = 2; i1 < N1; i1++) {
  for (i2 = 5; i2 < N2; i2++) {
    for (i3 = 4; i3 < N3; i3++) {
S:      A[i1][i2][i3] = f(A[i1-2][i2-5][i3-4]);
    }
  }
}
```

Dependence Vectors II

- *direction vector*: $D(\vec{i},\vec{j})_k = \begin{cases} \text{``}<\text{''}, & d(\vec{i},\vec{j})_k > 0 \\ \text{``}=\text{''}, & d(\vec{i},\vec{j})_k = 0 \\ \text{``}>\text{''}, & d(\vec{i},\vec{j})_k < 0 \end{cases}$
  - e.g. $d(\vec{i},\vec{j}) = (2, 0, -4) \implies D(\vec{i},\vec{j}) = (\text{``}<\text{''}, \text{``}=\text{''}, \text{``}>\text{''})$
- the *level* is the index of the leftmost "$<$" or "$>$" in $D(\vec{i},\vec{j})$
  - equivalently, the index of the leftmost nonzero in $d(\vec{i},\vec{j})$
  - corresponds to outermost loop involved in dependency
  - e.g. $d(\vec{i},\vec{j}) = (0, 0, 4, 0)$ has level 3
  - e.g. $D(\vec{i},\vec{j}) = (\text{``}=\text{''}, \text{``}=\text{''}, \dots)$ has level ∞
  - denote with $\delta^{\cdots}_{\text{level}}$