A4Q3

ϕ_1

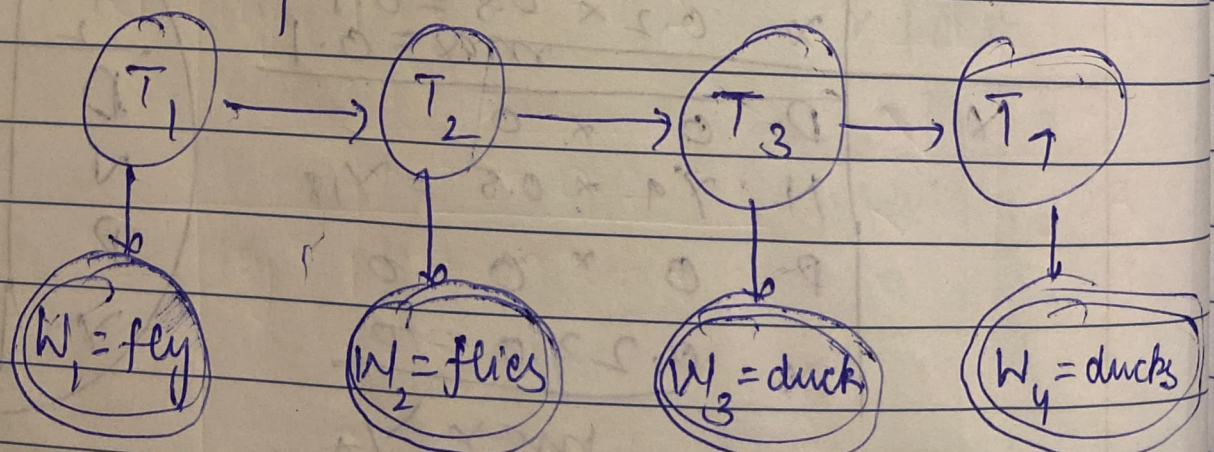
T_1	$P(T_1)$
N	0.8
Y	0.2

ϕ_2

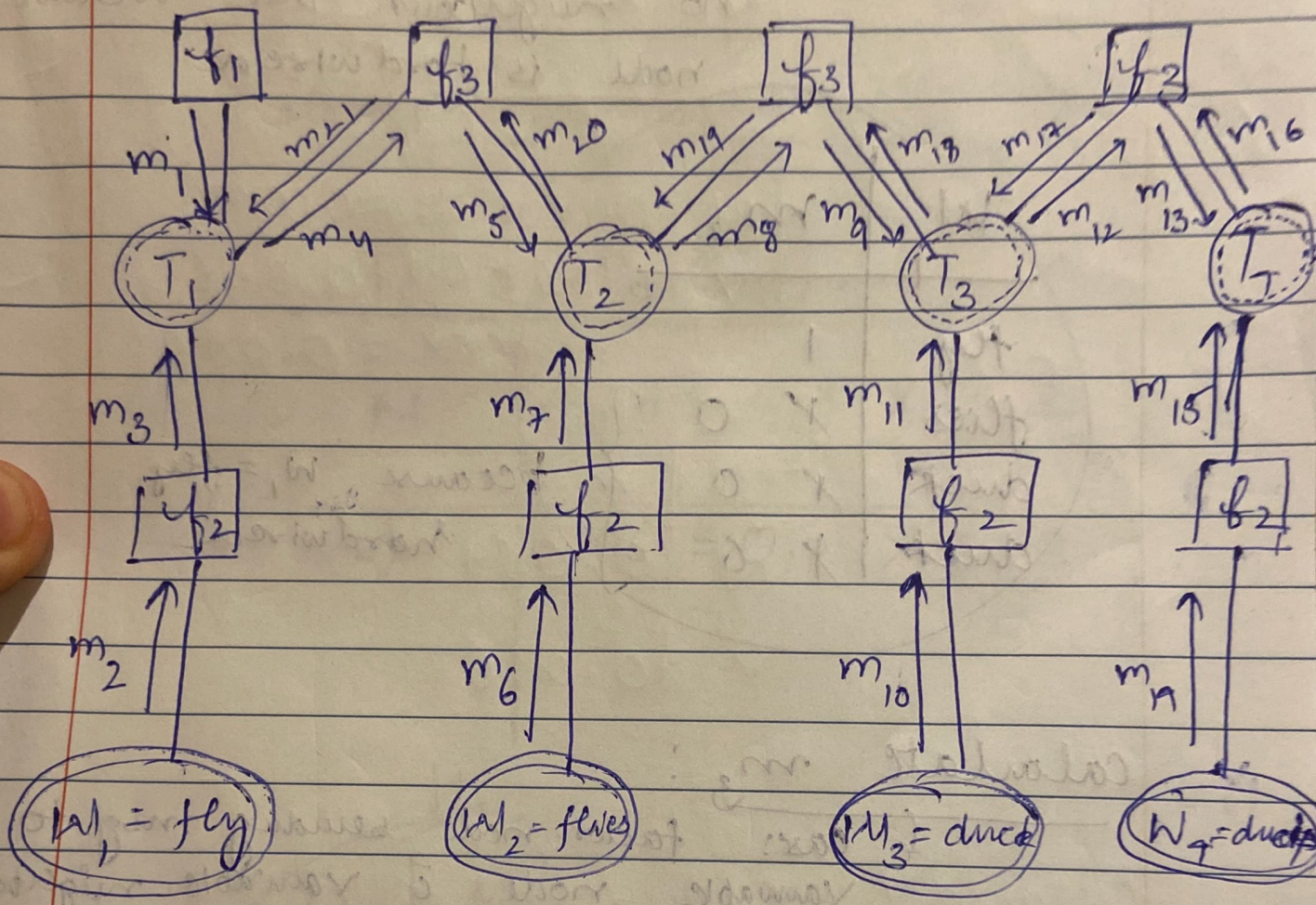
T_i	W_i	$P(W_i \mid T_i)$
N	duck	0.5
N	ducks	0.2
N	fly	0.2
N	flies	0.1
Y	duck	0.1
Y	ducks	0.2
Y	fly	0.3
Y	flies	0.4

ϕ_3

T_i	T_{i+1}	$P(T_{i+1} \mid T_i)$
N	N	0.7
N	Y	0.6
Y	N	0.9
Y	Y	0.1



(1) Create factor Graph



(2) Calculate m_1

↳ case: factor node sending msg to variable node with no neighbours

T_1	m_1
N	0.8
Y	0.2

f_1

T_1	$P(T_1)$
N	0.8
Y	0.2

Calculate m_2:

↳ case: variable node sending msg to factor node with no neighbour but variable node is hard wired

W_1	m_2
fly	1
flies	0
duck	0
duck	0

because W_1 = fly hardwired

calculate m_3:

↳ case: factor node sending msg to variable node & variable neighbours

T_1	M_3	
	W_1	$m_2 * f_2 =$
N	fly	$1 * 0.2 = 0.2$
		$0 = 0$
		$0 = 0$
	max	$= 0.2$
Y	fly	$1 * 0.3 = 0.3$
		$0 = 0$
		$0 = 0$
		$0 = 0$
	max	$= 0.3$

T_1	M_3
N	0.2
Y	0.3

Calculate m_4:

In case when variable node is sending msg to factor node & have variable neighbour

⇓

then do point wise multiplication of $m_1 \cdot m_3$

T_1	$M_4 = m_1 \cdot m_3$
1	$0.16 = 0.8 \times 0.2$
2	$0.06 = 0.2 \times 0.3$

⇓

T_1	M_4
1	0.16
2	0.06

Calculate m_5:

T_2	m_5
N	T_1 $m_4 * f_3 =$
	N $0.16 * 0.4 = 0.064$
	Y $0.06 * 0.9 = 0.054$
	max = 0.064
Y	T_1 $m_4 * f_3 =$
	N $0.16 * 0.6 = 0.096$
	Y $0.06 * 0.1 = 0.006$
	max = 0.096

T_2	m_5
N	0.064
Y	0.096

Calculate m_6:

lvl_2	m_6
fly	✗ 0
flies	1
duck	✗ 0
ducks	✗ 0

Hard-coding
lvl_2 = flies

∴

Calculate m_7:

T_2	m_7
N	~~flies~~ $m_6 * f_2 =$
	flies 1 * 0.1 = 0.1
	max = 0.1
Y	flies 1 * 0.4 = 0.4
	max = 0.4

⇓

T_2	m_7
N	0.1
Y	0.4

Calculate m_8:

T_2	$m_8 = m_5 . m_7$
N	0.0064 = 0.064 × 0.1
Y	0.0384 = 0.096 × 0.4

Calculate m_9

T_3	m_9
N	T_2 $m_8 * f_3$ =
	N $0.0064 \times 0.4 = 0.00256$
	Y $0.0384 \times 0.9 = 0.03456$ max
Y	T_2 $m_8 * f_3$ =
	N $0.0064 \times 0.6 = 0.00384$
	Y $0.0384 \times 0.1 = 0.00384$

tie ∴ choose Nover

max = 0.00384

T_3	m_9
N	0.03456
Y	0.00384

Calculate m_{10}

wl_3	m_{10}
fly	Y 0
flies	Y 0
duck	1
ducks	Y 0

Calculate m_{11}

T_3	m_{11}
N	$w_3 = m_{10} \times f_2$
	duck 1 $\times$ 0.5 = 0.8
	max = 0.5
Y	duck 1 $\times$ 0.1 = 0.1
	max = 0.1

=

T_3	m_{11}
N	0.5
Y	0.1

Calculate m_{12}

T_3	$m_{12} = m_{11} \times m_9$
N	0.01728 = 0.5 $\times$ 0.03456
Y	0.000384 = 0.1 $\times$ 0.00384

Calculate m_{13}

T_4	m_{13}
N	T_3 : $m_{12} * f_3$
	N $0.01728 \times 0.4 = 0.006912$
	V $0.000384 \times 0.9 = 0.000348$
	max $= 0.006912$
V	N $0.01728 \times 0.6 = 0.010368$
	V $0.000384 \times 0.1 = 0.0000384$
	max = 0.010368

T_4	m_{13}
N	0.006912
V	0.010368

Calculate m_{14}

W_4	m_{14}
fly	$\times$ 0
flies	$\times$ 0
duck	$\times$ 0
ducks	1

calculate m_{15}

Ty	m_{15}
N	$w_y \quad m_{1y} \times f_2$
	ducks $1 \times 0.2 = 0.2$
	⋮ ⋮ ⋮
	$max = 0.2$
V	ducks $1 \times 0.2 = 0.2$
	⋮ ⋮ ⋮
	$max = 0.2$

Ty	m_{15}
N	0.2
V	0.2

calculate m_{16}

T_1	m_{16}
N	0
V	0.2

T_4	$m_{13} \times m_{15}$
N	0.006912×0.2
	$= 0.0013824$
V	0.010368×0.2
	$= 0.0020736$

max V

$T_4^* = V$

calculate m_{17}

T_3	m_{17}
N	$m_{16} \cdot f_3$
	N $0 \times 0.4 = 0$
	V $0.2 \times 0.6 = 0.12$
	max 0.12
V	N $0.2 \times 0.6 = 0$
	V $0.2 \times 0.1 = 0.02$
	max 0.12

max

T_3	m_{17}
N	0.18
Y	0.02

T_3 = ?

T_3	$m_9 \times m_{11} \times m_{17}$
N	$0.03456 \times 0.5 \times 0.12 = 0.002074$
Y	$0.00384 \times 0.1 \times 0.02 = 0.00000768$

$= 0.0020736$

$max = 0.002073 6$ N

$\therefore T_3^* = N$ handwired

calculate m_{18}

T_3	$m_{18} = m_{17} \cdot m_{11}$
N	$0.06 = 0.12 \times 0.5$
Y	$0.002 = 0.02 \times 0.1$

T_3	m_{18}
N	0.06
Y	0

because hand wired

Calculate m_{19}

T_2	$m_{19} = m_{18} \cdot f_3$ for $T_3 = N$
N	$0.024 = 0.06 \times 0.4$
Y	$0.054 = 0.06 \times 0.9$

$T_2 = ?$

T_2	$m_{19} \times m_7 \times m_8$
N	$0.0264 \times 0.1 \times 0.064 = 0.001536$
V	$0.054 \times 0.4 \times 0.096 = 0.0020736$

$T_2^* = V$ max

Hand wired

calculate m_{20}

T_2	$m_{20} = m_7 \cdot m_{19}$
N	$0.0024 = 0.1 \times 0.024$
V	$0.0216 = 0.4 \times 0.054$

T_2	m_{20}
N	0
V	0.0216

because hand wired

calaulat m_{21}

T_1	$m_{21} = m_{20} \cdot t_3$ for $T_2 = V$
N	$0.01296 = 0.0216 \times 0.6$
V	$0.00216 = 0.0216 \times 0.1$

T_1	$m_1 \cdot m_3 \cdot m_2$
N	$0.8 \times 0.2 \times 0.01296 =$
x	$0.2 \times 0.3 \times 0.00216 =$

$N = 0.0020736$

$x = 0.0001296$

$\max$

$$\boxed{T_1^* = N}$$

(3) $$\boxed{T_1^* = M \quad T_2^* = V \quad T_3^* = N \quad T_4^* = Y}$$