Credentials

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Code to calculate the Bayes Theorem

In [1]:
```
def bayesTheorem(ba,a,b):
    res=(ba*a)/b
    print("The Posterior Probability [ P(A|B) ] is : ",res)

# Input

ba=float(input("Enter the value of P(B|A) : "))
a=float(input("Enter the value of P(A) : "))
b=float(input("Enter the value of P(B) : "))

bayesTheorem(ba,a,b)
```

```
Enter the value of P(B|A) : 0.07
Enter the value of P(A) : 0.10
Enter the value of P(B) : 0.05
The Posterior Probability [ P(A|B) ] is :  0.14
```

Code to calculate Eigen Value and Eigen Vector of a Matrix

In [2]:
```
import numpy as np
```

In [3]:
```
# 3 x 3 Matrix

matrix=[[1,2,3],[4,5,6],[7,8,9]]

array=np.array(matrix)
x,y=np.linalg.eig(array)
```

In [4]:
```
# to print eigen values

print("Eigen Values of the given Matrix is :\n",x)
```

```
Eigen Values of the given Matrix is :
 [ 1.61168440e+01 -1.11684397e+00 -1.30367773e-15]
```

In [5]:
```
# to print eigen vectors

print("Eigen Vectors of the given Matrix is :\n",y)
```

```
Eigen Vectors of the given Matrix is :
 [[-0.23197069 -0.78583024  0.40824829]
 [-0.52532209 -0.08675134 -0.81649658]
 [-0.8186735   0.61232756  0.40824829]]
```

Code to calculate Determinant of a Matrix

In [6]:
```
matrix=[[1,2,3],[4,5,6],[7,2,9]]
array=np.array(matrix)

det = np.linalg.det(array)

# to print the determinant
print("Determinant of the given Matrix is : ",det)
```

```
Determinant of the given Matrix is :  -36.0
```

Code to calculate the Inverse of a Matrix

In [7]:
```
matrix=[[1,2,3],[4,5,6],[7,2,9]]
array=np.array(matrix)

det = np.linalg.det(array)

if det==0:
    print("Inverse do not exist for the given matrix.")
else:
    print(np.linalg.inv(array))
```

```
[[-0.91666667  0.33333333  0.08333333]
 [-0.16666667  0.33333333 -0.16666667]
 [ 0.75       -0.33333333  0.08333333]]
```

Assignment
(Week 4)

1. Covid 19 tests are common nowdays, but some results of tests are not true. let's assume, a diagonstic test has 99% accuracy and 60% of all people have Covid 19. If a patient tests positive, what is the probability that they actually have the disease?

⇒ Let $P(A)$: Probability that a person has Covid 19
$P(B)$: Probability that a person do not have Covid 19
$P(T|A)$: Probability of testing positive given that person has Covid 19
$P(T|B)$: Probability of testing +ve given that person do not have covid 19
$P(A|T)$: Probability that a person has Covid 19 given that they tested positive.
(we need to find this)

Given,
$$P(A) = 0.60$$
$$P(T|A) = 0.99$$
$$P(T|B) = 0.01$$
$$P(B) = 1 - P(A) = 1 - 0.60$$
$$= 0.40$$

Using Bayes' Theorem :—

$$P(A|T) = \frac{P(T|A).P(A)}{P(T)}$$

Now,
$$P(T) = P(T|A)\,P(A) + P(T|B).\,P(B)$$

$P(T) = (0.99 * 0.60) + (0.01 \times 0.40)$

$P(T) = 0.594 + 0.004$

$P(T) = 0.598$

Using Bayes Theorem Formula :

$$P(A|T) = \frac{P(T|A) . P(A)}{P(T)}$$

$$= \frac{(0.99)(0.60)}{0.598} = \frac{0.594}{0.598}$$

$$= 0.9933$$

So, the probability that a person actually has covid 19 given that tested positive is 99.33 %.

2. Find the eigen value and eigen vector for the given matrix.

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$\Rightarrow$ $AX = \lambda I$

$A - \lambda I = 0$

$$\Rightarrow \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix}$$

$$\Rightarrow (2-\lambda)(3-\lambda) - 2$$
$$\Rightarrow 6 - 2\lambda - 3\lambda + \lambda^2 - 2$$
$$\Rightarrow \lambda^2 - 5\lambda + 4 = 0$$
$$\Rightarrow \lambda^2 - 4\lambda - \lambda + 4 = 0$$
$$\Rightarrow \lambda(\lambda-4) - 1(\lambda-4) = 0$$
$$\Rightarrow (\lambda-1)(\lambda-4) = 0$$

$$\lambda = \underline{\underline{1, 4}}$$

Now, for $\underline{\lambda = 4}$

$$\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\Rightarrow -2x + 2y = 0$$
$$\Rightarrow \cancel{-2}x = \cancel{-2}y$$

$$\begin{matrix} x \\ y \end{matrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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For $\lambda = 1$

$$\begin{bmatrix} 2-1 & 2 \\ 1 & 3-1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\Rightarrow 1x + 2y = 0$

$\Rightarrow 1x = -2y$

$$\begin{matrix} x \\ y \end{matrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Thus, the two eigen vectors are $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

3. Calculate determinant of a 3X3 matrix and find the inverse of the matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 2 & 9 \end{bmatrix}$$

$\Rightarrow$ Determinant of $A = 1\begin{vmatrix} 4 & 6 \\ 2 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 2 \end{vmatrix}$

$\Rightarrow (45-12) - 2(36-42) + 3(8-35)$

$\Rightarrow 33 + 12 + (24-105)$

$\Rightarrow 33 + 12 - 81$

$\Rightarrow \underline{\underline{-36}}$

As the determinant exist, so they will be inverse existing.
Now, let find the inverse of the 3X3 matrix.

lets find the minors of the above matrix:—

$$M_{11} = \begin{vmatrix} 5 & 2 \\ 6 & 9 \end{vmatrix} = 45-12 = 33$$

$$M_{12} = \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} = 36-42 = -6$$

$$M_{13} = \begin{vmatrix} 4 & 5 \\ 7 & 2 \end{vmatrix} = 8-35 = -27$$

$$M_{21} = \begin{vmatrix} 2 & 3 \\ 2 & 9 \end{vmatrix} = 18-6 = 12$$

$$M_{22} = \begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = 9-21 = -12$$

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 2 \end{vmatrix} = 2-14 = -12$$

$$M_{31} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = 12-15 = -3$$

$$M_{32} = \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = 6-12 = -6$$

$$M_{33} = \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = 5-8 = -3$$

Now, the positions are $= \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

Similarly, the cofactors are $= \begin{bmatrix} 33 & 6 & -27 \\ -12 & -12 & 12 \\ -3 & 6 & -3 \end{bmatrix}$

$$\text{Adjoint } A = (\text{cofactor})^T$$

$$= \begin{bmatrix} 33 & -12 & -3 \\ 6 & -12 & 6 \\ -27 & 12 & -3 \end{bmatrix}$$

Now,

$$A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{-36} \begin{bmatrix} 33 & -12 & -3 \\ 6 & -12 & 6 \\ -27 & 12 & -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -11/12 & 1/3 & 1/12 \\ -1/6 & 1/3 & -1/6 \\ 3/4 & -1/3 & 1/12 \end{bmatrix}$$

4. Describe the properties and applications of the Normal Distribution and calculate probabilities using its properties.

⇒

Properties of Normal Distribution :-

(i) The mean, median and mode are all equal and located at the center of the distribution.

(ii) The normal distribution is symmetric about its mean. This means the left and the right halves of the distribution are mirror images.

(iii) The graph is bell shaped, with the highest point at that mean.

(iv) The shape and location of the normal distribution are determined by its mean and standard deviation.

Applications of Normal Distribution :-

(i) It is used to analyze data and interpret pyscological tests and medical research.

(ii) It is applied in finance to model asset returns and price movements.

(iii) Used in quality control processes to detect variations and improve manifacturing.

(iv) It is used in various statistical methods like hypothesis testing and confidence intervals.

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Q. Suppose the height of adult men are normally distributed with mean of 70 inches and a standard deviation of 3 inches. Find the probability that the randomly selected man is taller than 74 inches.

$$\Rightarrow \quad z = \frac{74-70}{3} = \frac{4}{3} = \underline{1.33}$$

$P(z)$ from z table is $\underline{0.9082}$

$$P(x > 74) = 1 - P(z < 1.33)$$
$$= 1 - 0.9082$$
$$= \underline{0.0918}$$

So, the probability that a randomly selected man is taller than 74 inches is $\underline{9.18\%}$