Indian Institute of Technology Bombay

Department of Aerospace Engineering

AE706 – Computational Fluid Dynamics

Assignment 5 Report

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Transformation Derivatives:

$$x = 4\xi$$

$$\therefore \frac{\partial \xi}{\partial x} = 0.5 \qquad \frac{\partial^2 \xi}{\partial x^2} = 0$$

$$\frac{\partial \xi}{\partial y} = 0 \qquad \frac{\partial^2 \xi}{\partial y^2} = 0$$

$$y = \eta + (1-\eta)\left[\frac{1 - \cos((x-1)\pi)}{10}\right]$$

$$\Rightarrow \eta = \frac{10y - 1 + \cos((x-1)\pi)}{9 + \cos((x-1)\pi)}$$

$$\frac{\partial \eta}{\partial x} = \frac{10\pi . \sin((x-1)\pi)(y-1)}{(9 + \cos((x-1)\pi))^2}$$

$$\frac{\partial^2 \eta}{\partial x^2} = \frac{10\pi^2 (y-1)\left[9\cos^2((x-1)\pi) + 9\cos((x-1)\pi) + 2\sin^2((x-1)\pi)\right]}{(9 + \cos((x-1)\pi))^3}$$

$$\frac{\partial \eta}{\partial y} = \frac{10}{9 + \cos((x-1)\pi)}$$

$$\frac{\partial^2 \eta}{\partial y^2} = 0$$

Laplace equation:

Laplace Equation :

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

converts to :

$$\frac{\partial^2 \psi}{\partial \xi^2}\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2\right] + \frac{\partial^2 \psi}{\partial \eta^2}\left[\left(\frac{\partial \eta}{\partial x}\right)^2 + \left(\frac{\partial \eta}{\partial y}\right)^2\right]$$

$$+ 2\frac{\partial^2 \psi}{\partial \xi \partial \eta}\left[\left(\frac{\partial \eta}{\partial x}\right)\left(\frac{\partial \xi}{\partial y}\right) + \left(\frac{\partial \eta}{\partial y}\right)\left(\frac{\partial \xi}{\partial x}\right)\right]$$

$$+ \frac{\partial \psi}{\partial \xi}\left[\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right] + \frac{\partial \psi}{\partial \eta}\left[\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right] = 0$$

Let $\Rightarrow$ $\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2 = a$

$$\left(\frac{\partial \eta}{\partial x}\right)^2 + \left(\frac{\partial \eta}{\partial y}\right)^2 = b$$

$$\left(\frac{\partial \eta}{\partial x}\right)\left(\frac{\partial \xi}{\partial y}\right) + \left(\frac{\partial \eta}{\partial y}\right)\left(\frac{\partial \xi}{\partial x}\right) = c$$

$$\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} = d$$

$$\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} = e$$

Finite Difference Equation:

$$\frac{\partial^2 \psi}{\partial \xi^2} = \frac{\psi_{\xi+1,\eta} - 2\psi_{\xi,\eta} + \psi_{\xi-1,\eta}}{(\Delta\xi)^2}$$

$$\frac{\partial^2 \psi}{\partial \eta^2} = \frac{\psi_{\xi,\eta+1} - 2\psi_{\xi,\eta} + \psi_{\xi,\eta-1}}{(\Delta\eta)^2}$$

$$\frac{\partial^2 \psi}{\partial \xi \partial \eta} = \frac{\psi_{\xi+1,\eta+1} - \psi_{\xi+1,\eta-1} - \psi_{\xi-1,\eta+1} + \psi_{\xi-1,\eta-1}}{4\Delta\xi . \Delta\eta}$$

$$\frac{\partial \psi}{\partial \xi} = \frac{\psi_{\xi+1,\eta} - \psi_{\xi-1,\eta}}{2\Delta\xi}$$

$$\frac{\partial \psi}{\partial \eta} = \frac{\psi_{\xi,\eta+1} - \psi_{\xi,\eta-1}}{2\Delta\eta}$$

Putting ~~these~~ back into the eq^n

$$a. \frac{\psi_{\xi+1,\eta} - 2\psi_{\xi,\eta} + \psi_{\xi-1,\eta}}{(\Delta\xi)^2} +$$

$$b. \frac{\psi_{\xi,\eta+1} - 2\psi_{\xi,\eta} + \psi_{\xi,\eta-1}}{(\Delta\eta)^2} +$$

$$2c. \frac{\psi_{\xi+1,\eta+1} - \psi_{\xi+1,\eta-1} - \psi_{\xi-1,\eta+1} + \psi_{\xi-1,\eta-1}}{4\Delta\xi . \Delta\eta}$$

$$+ d. \frac{\psi_{\xi+1,\eta} - \psi_{\xi-1,\eta}}{2\Delta\xi} + e \frac{\psi_{\xi,\eta+1} - \psi_{\xi,\eta-1}}{2\Delta\eta}$$

$$= 0$$

$$\left[\frac{2a}{(\Delta\xi)^2} + \frac{2b}{(\Delta\eta)^2}\right]\psi_{\xi,\eta} = a.\frac{\psi_{\xi+1,\eta} + \psi_{\xi-1,\eta}}{(\Delta\xi)^2} + b.\frac{\psi_{\xi,\eta+1} + \psi_{\xi,\eta-1}}{(\Delta\eta)^2} +$$

$$c.\frac{\psi_{\xi+1,\eta+1} - \psi_{\xi+1,\eta-1} - \psi_{\xi-1,\eta+1} + \psi_{\xi-1,\eta-1}}{2(\Delta\xi)(\Delta\eta)}$$

$$+ d.\frac{\psi_{\xi+1,\eta} - \psi_{\xi-1,\eta}}{2(\Delta\xi)} + e\frac{\psi_{\xi,\eta+1} - \psi_{\xi,\eta-1}}{2(\Delta\eta)}$$

For Point Gauss Siedel we will use :

$$\psi_{\xi+1,\eta} \rightarrow \psi^{i}_{\xi+1,\eta}, \quad \psi_{\xi-1,\eta} \rightarrow \psi^{i+1}_{\xi-1,\eta}$$

$$\psi_{\xi,\eta+1} \rightarrow \psi^{i}_{\xi,\eta+1}, \quad \psi_{\xi,\eta-1} \rightarrow \psi^{i+1}_{\xi,\eta-1}$$

$$\psi_{\xi+1,\eta+1} \rightarrow \psi^{i}_{\xi+1,\eta+1}, \quad \psi_{\xi+1,\eta-1} \rightarrow \psi^{i+1}_{\xi+1,\eta-1}$$

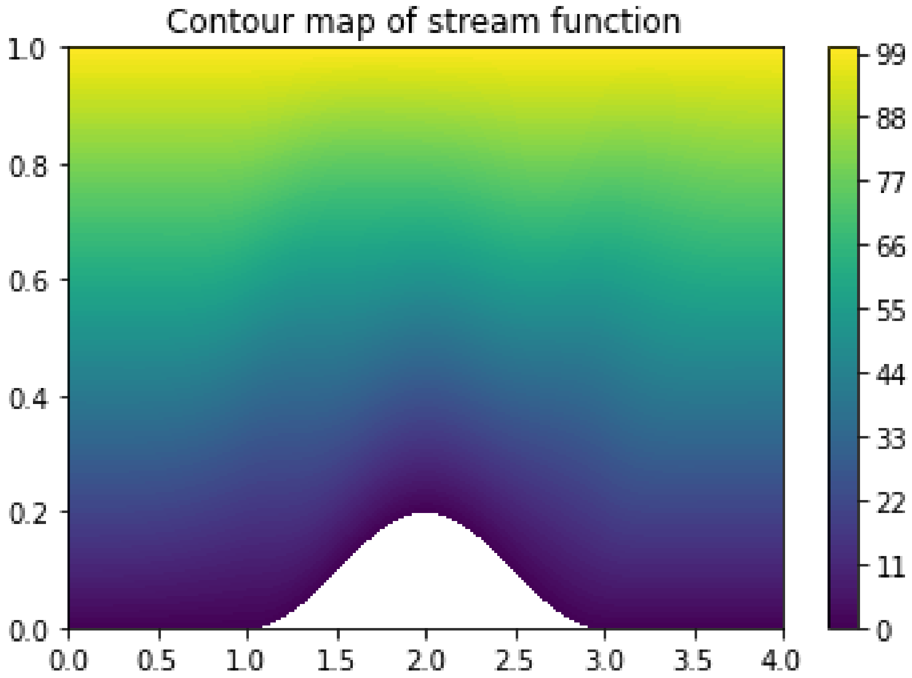
$$\psi_{\xi-1,\eta+1} \rightarrow \psi^{i}_{\xi-1,\eta+1}, \quad \psi_{\xi-1,\eta-1} \rightarrow \psi^{i+1}_{\xi-1,\eta-1}$$

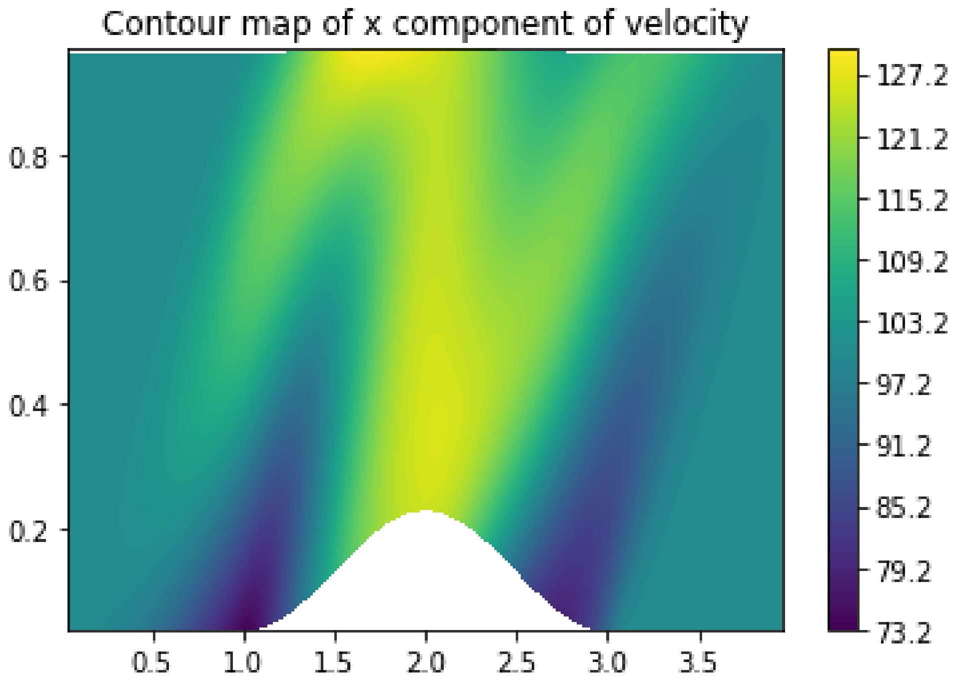
$$\therefore \left[\frac{2a}{(\Delta\xi)^2} + \frac{2b}{(\Delta\eta)^2}\right]\psi^{i+1}_{\xi,\eta} = a.\frac{\psi^{i}_{\xi+1,\eta} + \psi^{i+1}_{\xi-1,\eta}}{(\Delta\xi)^2} + b\frac{\psi^{i}_{\xi,\eta+1} + \psi^{i+1}_{\xi,\eta-1}}{(\Delta\eta)^2} +$$

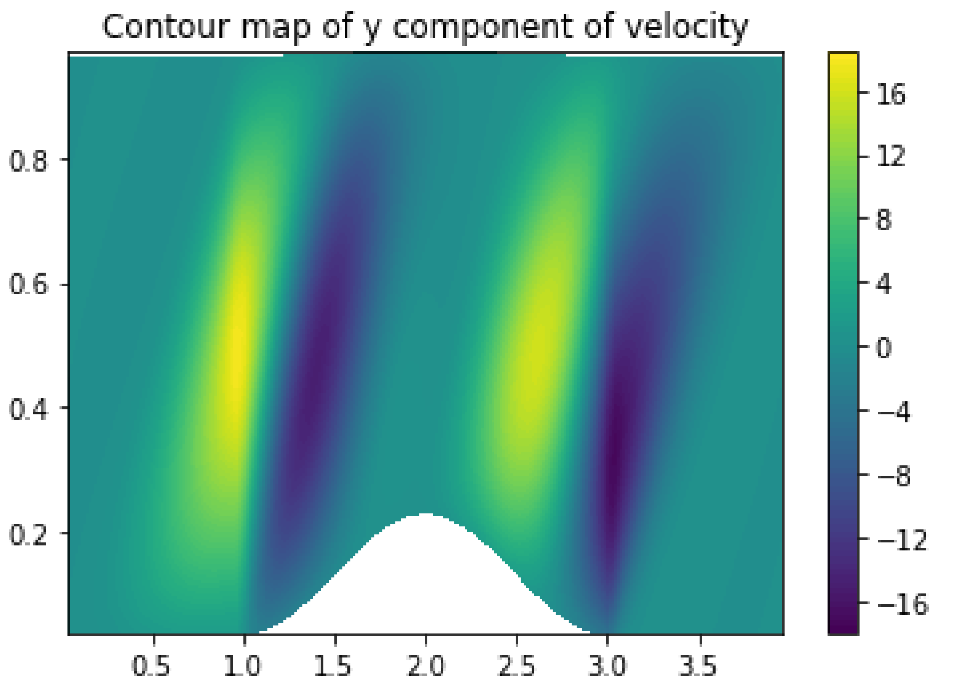
$$c.\frac{\psi^{i}_{\xi+1,\eta+1} - \psi^{i+1}_{\xi+1,\eta-1} - \psi^{i}_{\xi-1,\eta+1} + \psi^{i+1}_{\xi-1,\eta-1}}{2(\Delta\xi)(\Delta\eta)}$$

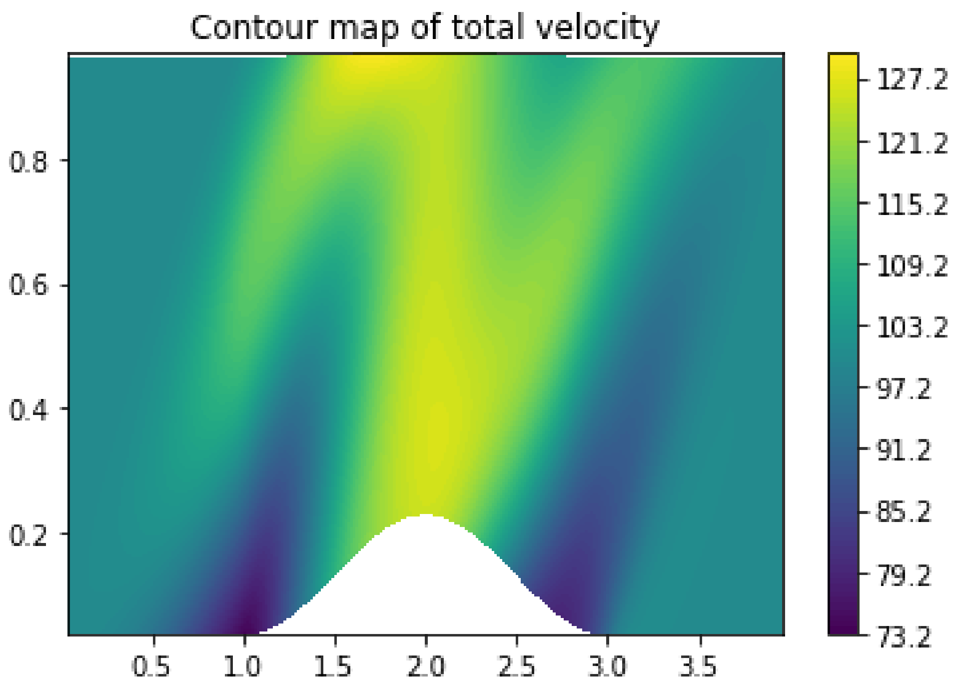
$$+ d.\frac{\psi^{i}_{\xi+1,\eta} - \psi^{i+1}_{\xi-1,\eta}}{2(\Delta\xi)} + e\frac{\psi^{i}_{\xi,\eta+1} - \psi^{i+1}_{\xi,\eta-1}}{2(\Delta\eta)}$$

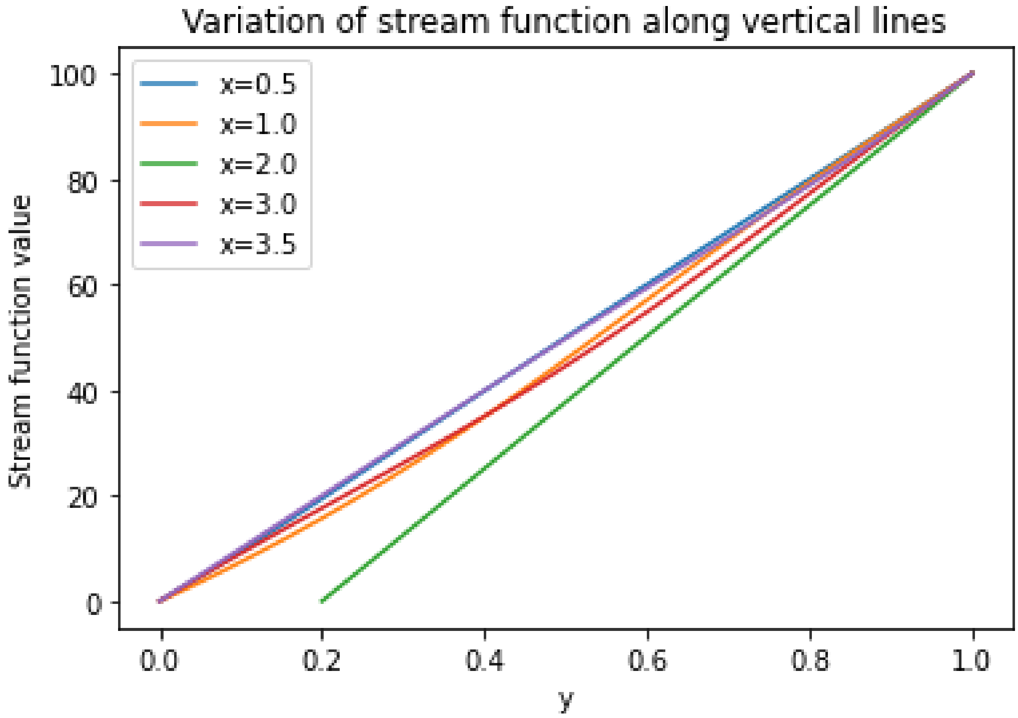
RESULTS:

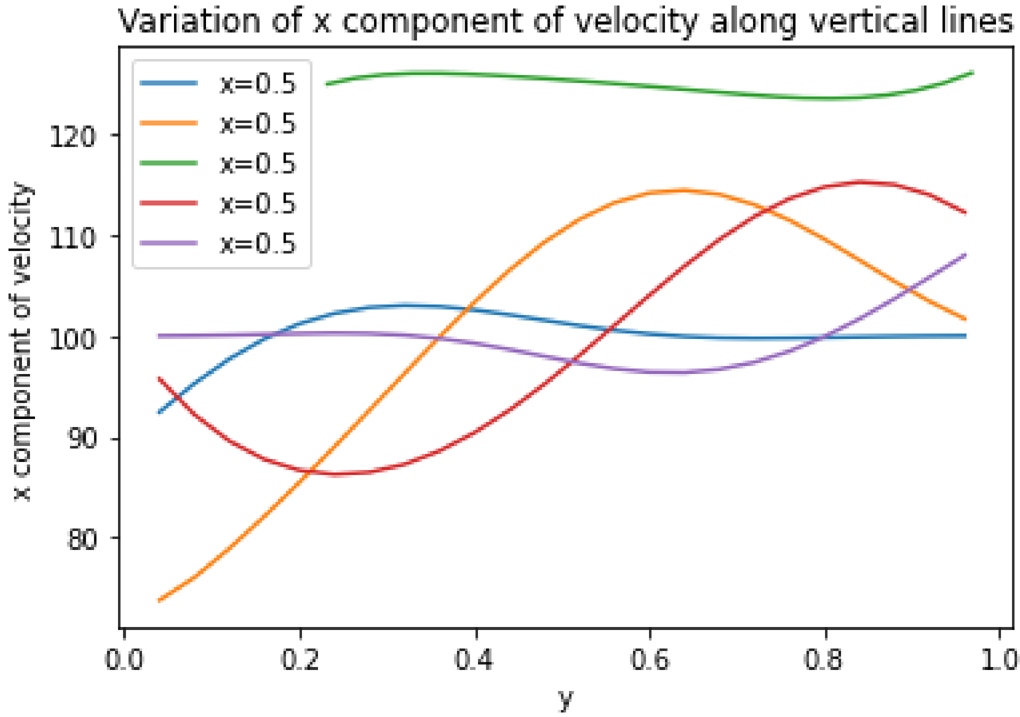

Contour map of stream function
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
99
88
77
66
55
44
33
22
11
0

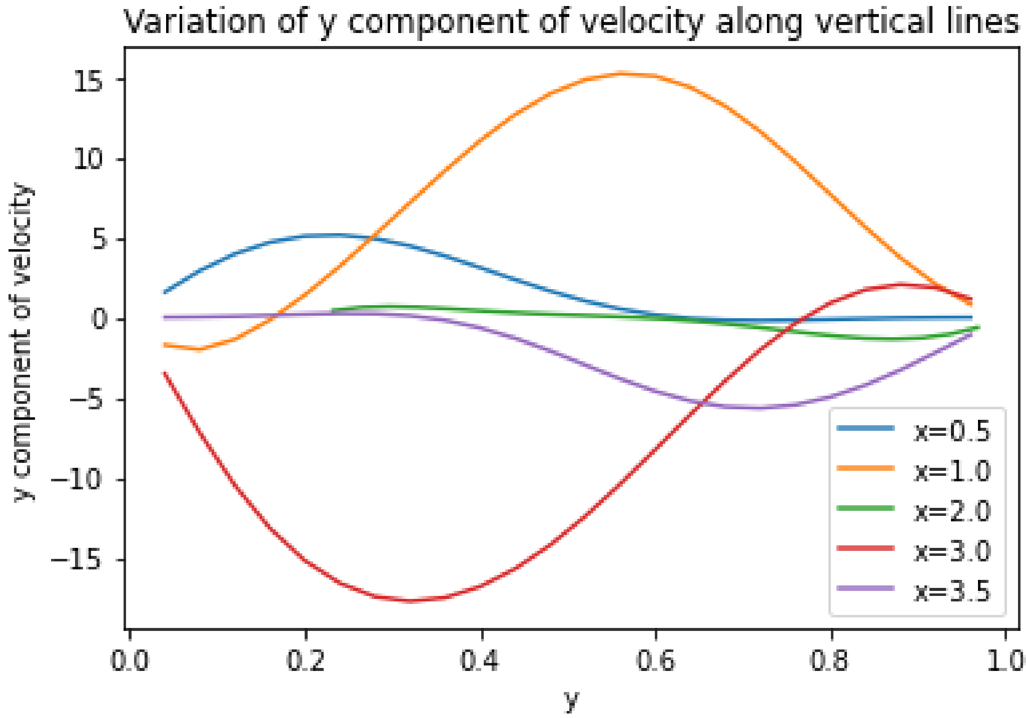

Contour map of x component of velocity
0.8
0.6
0.4
0.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
127.2
121.2
115.2
109.2
103.2
97.2
91.2
85.2
79.2
73.2

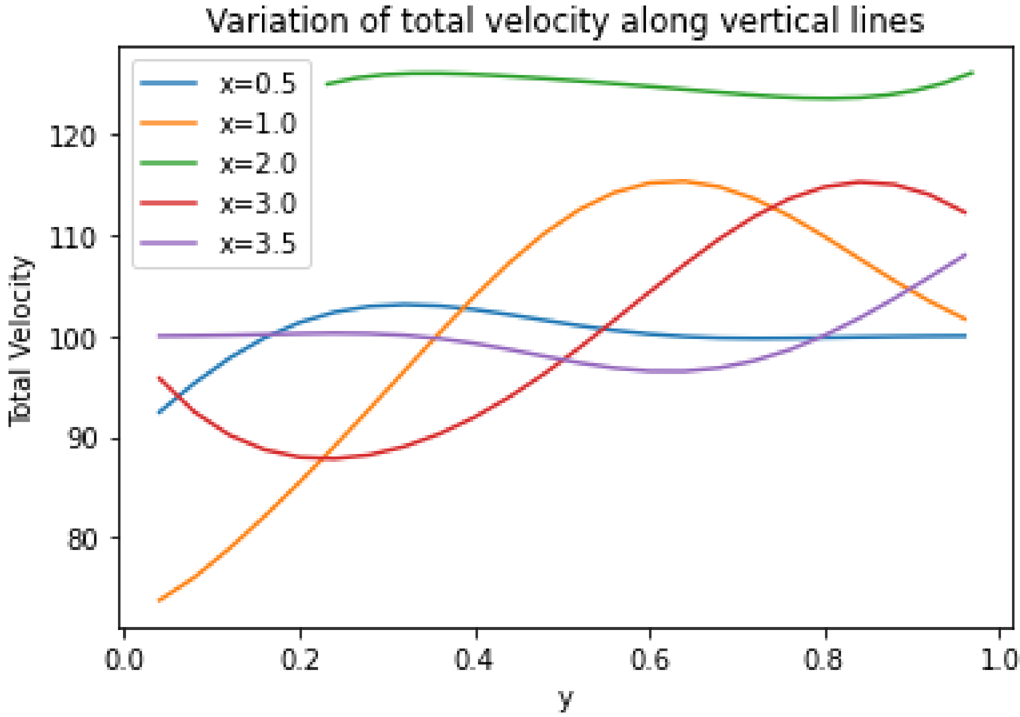

Contour map of y component of velocity
0.8
0.6
0.4
0.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
16
12
8
4
0
−4
−8
−12
−16

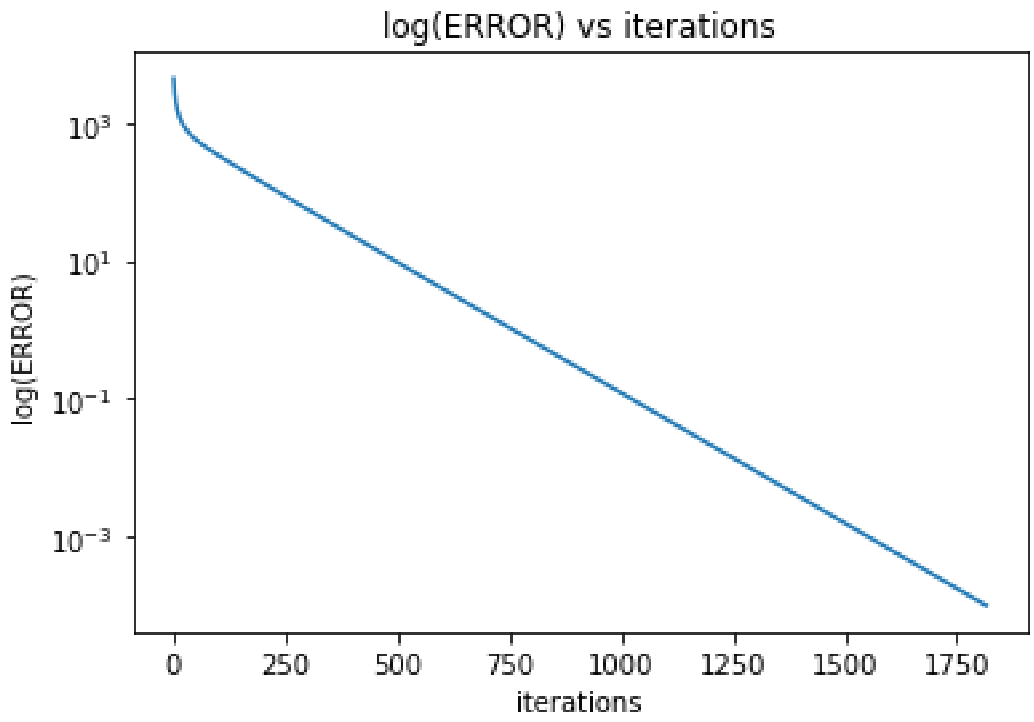

Contour map of total velocity
0.8
0.6
0.4
0.2
0.5
1.0
1.5
2.0
2.5
3.0
3.5
127.2
121.2
115.2
109.2
103.2
97.2
91.2
85.2
79.2
73.2


Variation of stream function along vertical lines
Stream function value
y
x=0.5
x=1.0
x=2.0
x=3.0
x=3.5
0
20
40
60
80
100
0.0
0.2
0.4
0.6
0.8
1.0


Variation of x component of velocity along vertical lines
x component of velocity
y
x=0.5
x=0.5
x=0.5
x=0.5
x=0.5
80
90
100
110
120
0.0
0.2
0.4
0.6
0.8
1.0


Variation of y component of velocity along vertical lines
y component of velocity
y
x=0.5
x=1.0
x=2.0
x=3.0
x=3.5
15
10
5
0
−5
−10
−15
0.0
0.2
0.4
0.6
0.8
1.0


Variation of total velocity along vertical lines
Total Velocity
y
x=0.5
x=1.0
x=2.0
x=3.0
x=3.5
120
110
100
90
80
0.0
0.2
0.4
0.6
0.8
1.0



OBSERVATIONS:

The Stream contour plot indicates a gradual increase in the stream function value from the base of the venturi to the top of the venturi.

The velocity values we can see that the velocity increases as the flow reaches on top of the bump and then decreases.

The Error plot also confines to the second order error.