Assignment
Transcendental and Polynomial Equations & Matrix Operations

1) Find a real root of the equation $f(x) = x^3 - x - 1 = 0$, using Bisection method.

2) Find a root of $f(x) = xe^x - 1 = 0$, using Bisection method, correct to three decimal places.

3) Find a real root of the equation $f(x)=x^3 -2x-5=0$ by method of Regula- Falsi.

4) Determine the root of the equation $\cos x - x\,e^x = 0$ by the method of Regula- Falsi. https://www.youtube.com/watch?v=41lE1PN1Ap4

5) Find a real root of $2x - \log_{10} x = 7$ by the iteration method Ex 8

6) Find a root of $x \sin x + \cos x = 0$, using Newton – Raphson method Ex 10 https://www.youtube.com/watch?v=YSl37OYMLFw

7) Find the smallest positive root of $x - e^{-x} = 0$, using Newton – Raphson method Ex 11

8) Using Newton-Raphson method (a) Find square root of a number (b) Find a reciprocal of a number .

9) Find a real root for $x \tan x + 1 = 0$ using Newton–Raphson method

10) Find the value of $\sqrt{7}$ to 20 decimal places. Use initial approximation as $x_0 = 3$

11) Use Gaussian elimination to solve the following system.

$$
\begin{aligned}
0.143x_1 + 0.357x_2 + 2.01x_3 &= -5.173 \\
-1.31x_1 + 0.911x_2 + 1.99x_3 &= -5.458 \\
11.2x_1 - 4.30x_2 - 0.605x_3 &= 4.415
\end{aligned}
$$

After *each* intermediate calculation, round the result to three significant digits.

12) The **Hilbert matrix** of size $n \times n$ is the $n \times n$ symmetric matrix $H_n = [a_{ij}]$, where $a_{ij} = 1/(i + j - 1)$. As n increases, the Hilbert matrix becomes more and more ill-conditioned. Use Gaussian elimination to solve the following system of linear equations, rounding to two significant digits after each intermediate calculation. Compare this solution with the exact solution ($x_1 = 3$, $x_2 = -24$, and $x_3 = 30$).

$$
\begin{aligned}
x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{3}x_3 &= 1 \\
\tfrac{1}{2}x_1 + \tfrac{1}{3}x_2 + \tfrac{1}{4}x_3 &= 1 \\
\tfrac{1}{3}x_1 + \tfrac{1}{4}x_2 + \tfrac{1}{5}x_3 &= 1
\end{aligned}
$$

13) Use Gauss-Seidel and Jacobi Methods and check the solution. Initial approximation is

$(x_1, x_2, \ldots, x_n) = (0, 0, \ldots, 0)$

$$
\begin{aligned}
4x_1 + 2x_2 - 2x_3 &= 0 \\
x_1 - 3x_2 - x_3 &= 7 \\
3x_1 - x_2 + 4x_3 &= 5
\end{aligned}
$$

14) Write a program for the following Matrix equation set and solve

[1.0, -1.0, 0.0, 1.0, 2.0, -2.0, 0.0, 1.0]

$$
\begin{aligned}
4x_1 + x_2 - x_3 &= 3 \\
x_1 + 6x_2 - 2x_3 + x_4 - x_5 &= -6 \\
x_2 + 5x_3 - x_5 + x_6 &= -5 \\
2x_2 + 5x_4 - x_5 - x_7 - x_8 &= 0 \\
-x_3 - x_4 + 6x_5 - x_6 - x_8 &= 12 \\
-x_3 - x_5 + 5x_6 &= -12 \\
-x_4 + 4x_7 - x_8 &= -2 \\
-x_4 - x_5 - x_7 + 5x_8 &= 2
\end{aligned}
$$

-1

15) Apply the power method with scaling, starting with $\mathbf{x}_0$ $(1, 1, 1)$and observe the results of the first four iterations.

. $A = \begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ $\qquad$ $A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

16)

. Obtain the LU factorization of the matrix

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$$

Use this factorization to solve the system with $\boldsymbol{b} = (4, 4, 6)^T$.

. Show that the following matrix cannot be written in the LU factorization form: $\begin{bmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{bmatrix}$

. Show that the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

is invertible but has no LU factorization. Do a suitable interchange of rows and/or columns to get an invertible matrix, which has LU factorization.

17)

$$A = \begin{bmatrix} 0.7825 & 0.8154 & -0.1897 \\ -0.3676 & 2.2462 & -0.0573 \\ -0.1838 & 0.1231 & 1.9714 \end{bmatrix}$$

has eigenvalues $\lambda_1 = 2$, $\lambda = 2$ and $\lambda_3 = 1$. Does the power method converge for the above matrix? Justify your answer. Perform 5 iterations starting from the initial guess $\boldsymbol{x}^{(0)} = (1, 3, 6)$ to verify your answer.

18)

. The matrix

$$A = \begin{bmatrix} 5.4 & 0 & 0 \\ -113.0233 & -0.5388 & -0.6461 \\ -46.0567 & -6.4358 & -0.9612 \end{bmatrix}$$

has eigenvalues $\lambda_1 = 5.4$, $\lambda = 1.3$ and $\lambda_3 = -2.8$ with corresponding eigen vectors $\boldsymbol{v}_1 = (0.2, -4.1, 2.7)^T$, $\boldsymbol{v}_2 = (0, 1.3, -3.7)^T$ and $\boldsymbol{v}_3 = (0, 2.6, 9.1)^T$. To which eigenvalue and the corresponding eigen vector does the power method converge if we start with the initial guess $\boldsymbol{x}^{(0)} = (0, 1, 1)$? Justify your answer.

Use Gerschgorin's theorem to the following matrix and determine the intervals in which the eigenvalues lie.

$$A = \begin{bmatrix} 0.5 & 0 & 0.2 \\ 0 & 3.15 & -1 \\ 0.57 & 0 & -7.43 \end{bmatrix}$$

Can power method be used for this matrix? Justify your answer. Use Power method to compute the eigenvalue which is largest in the absolute value and the corresponding eigenvector each of the above matrix.