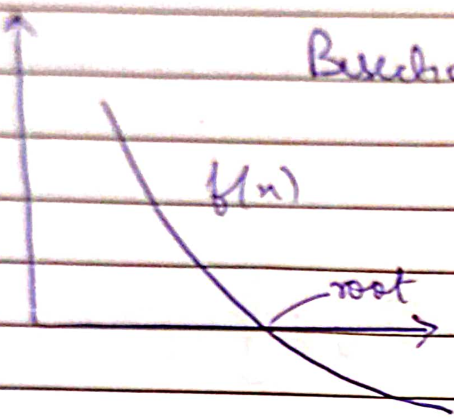
Assignment - 1

Roll Num: IMT2020045

Q1

Anil
Date
Page

Q1



Bisection method first lets find b/w which two numbers root lies

$f(x) = x^3 - x - 1 = 0$

$f(1) = 1^3 - 1 - 1 = -1 \quad < 0$

$f(2) = 2^3 - 2 - 1 = 5 \quad > 0$

$\therefore$ A root of $f(x)$ lies in 1 and 2

$$x_1 = \frac{(1+2)}{2} = \frac{3}{2} = 1.5$$

$f(x_1) = f(1.5) = (1.5)^3 - 1.5 - 1 = 0.875 \quad > 0$

root lies in 1 and 1.5

$$x_2 = \frac{1 + 1.5}{2} = 1.25$$

$f(x_2) = f(1.25) = (1.25)^3 - 1.25 - 1 = -0.29 < 0$

hence root lies in 1.25 and 1.5

$$x_3 = \frac{1.25 + 1.5}{2} = 1.375$$

Similarly, we get $x_4 = 1.3125$

$x_5 = 1.34375$

$x_6 = 1.328125$ etc.

Ans

Q2 $f(x) = xe^x - 1 = 0$ Bisection method

Q2

first lets find out b/w which two pts the root lies

$f(0) = 0 \cdot e^0 - 1 = -1 < 0$

$f(1) = 1 \cdot e^1 - 1 = 1.7183 > 0$

Hence, a root lies in 0 and 1

$x_1 = \frac{0+1}{2} = 0.5$

$f(0.5) = 0.5\, e^{0.5} - 1 = -0.1756 < 0$

Hence the root lies b/w 0.5 and 1

$x_2 = \frac{0.5+1}{2} = 0.75$

Proceed, like this we get

$x_3 = 0.625$

$x_4 = 0.5625$

$x_5 = 0.59375$

$x_6 = 0.5781$

$x_7 = 0.5703$

$x_8 = 0.5664$

$x_9 = 0.5684$

$x_{10} = 0.5674$

$x_{11} = 0.5669$

$x_{12} = 0.5672$

$x_{13} = 0.5671$

hence, the root correct to 3 decimal places = Ans $\boxed{0.567}$

Q3

Q3 $f(x) = x^3 - 2x - 5$ Regular Falsi

first lets find b/w which two numbers root lie

$f(2) = -1$ $f(3) = 16$

hence the root lies b/w 2 and 3

$\therefore$ take $a = 2$, $b = 3$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= \frac{2(16) - 3(-1)}{16 - (-1)} = \frac{35}{17}$$

$$x_1 = 2.0588$$

$$f(x_1) = -0.397 < 0$$

Therefore the root lies b/w 2.0588 and 3

Again we will use the same formula

$$x_2 = \frac{2.0588(16) - 3(-0.397)}{16 - (-0.397)} = 2.081$$

Proceed like this we get

$x_3 = 2.089$

$x_4 = 2.092$

$x_5 = 2.093$

$x_6 = 2.094$

Ans

Q4

Q4 $f(x) = \cos x - xe^x = 0$ Regula Falsi

first lets find b/w which two will the root lie

$f(0) = 1$ and $f(1) = -2.177$

$a = 0$ and $b = 1$ ∴ The root lies between 0 and 1.

$$x_1 = \frac{0(-2.177) - 1(1)}{-2.177 - 1} = 0.314$$

$f(x_1) = f(0.314) = 0.51986$

The root lies b/w 0.314 and 1

hence, $$x_2 = \frac{0.314(-2.177) - 1(0.519)}{-2.177 - 0.519}$$

$$x_2 = 0.44673$$

Proceed like this we get

$x_3 = 0.49402$,

$x_4 = 0.50995$,

$x_5 = 0.51520$,

$x_6 = 0.51692$,

Ans

Q5

Q5 $2x - \log_{10} x = 7$ by iteration method

seperate x first

$$x = \frac{7 + \log_{10} x}{2} \qquad \phi(x) = \frac{\log_{10} x + 7}{2}$$

value when $x = 1$

$$\phi(1) = \frac{0+7}{2} = 3.5$$

next iter, $\phi_2(x) = \frac{\log_{10} 3.5 + 7}{2} = 3.772$

, " , $\phi_3(x) = \frac{\log_{10} 3.772 + 7}{2} = 3.78825$

$$\phi_4(x) = \frac{\log_{10} 3.78825 + 7}{2} = 3.7892$$

$$\phi_5(x) = \frac{\log_{10} 3.7892 + 7}{2} = 3.7893$$

$\therefore$ $x = 3.7893$ is the root of the Eq^n

Ans

Q6

Q6 $f(x) \Rightarrow x\sin x + \cos x = 0$ Newton-Raphson method

$$f(x) = x\sin x + \cos x$$
$$f'(x) = x\cos x + \cancel{\sin x} \cancel{(-\sin x)} = x\cos x$$

formula $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Initial guess : $x_0 = \pi$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = \pi - \frac{\pi \cdot \sin(\pi) + \cos(\pi)}{\pi\cos\pi}$$

$$x_1 = 2.8233$$

Similarly, $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$$x_2 = 2.7986$$
$$x_3 = 2.7984$$
$$x_4 = 2.7984$$

$\therefore$ $x = 2.7984$ can be the root of $f(x) \Rightarrow x\sin x + \cos x = 0$

Ans

Q7

Q $f(x) \Rightarrow x - e^{-x} = 0$ Newton-Raphson

$$f(x) = x - e^{-x}$$
$$f'(x) = 1 + e^{-x}$$

$f(0) = -1$ $f(1) = 0.63212$

$\therefore$ a root lies in b/w 0 and 1

Let $x_0 = 1$

formula $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$= 1 - \frac{0.63212}{1.3679}$$

$$x_1 = 0.5379$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.5379 + \frac{0.0461}{1.584}$$

$$= 0.56714$$

Proceeding,

$\therefore$ we can say $x = 0.567$ Ans can be taken as a root for $f(x)$

Q8

Q8 (a) Square root of a number using newton-raphson method.

Let n be the number

$$\Rightarrow x = \sqrt{n}$$

$$\Rightarrow x^2 = n$$

$$f(x) = x^2 - n = 0$$

$$f'(x) = 2x$$

by formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$\Rightarrow x_{n'+1} = x_{n'} - \frac{x_{n'}^2 - n}{2x_{n'}}$$

$$x_{n'+1} = \frac{1}{2}\left(x_{n'} + \frac{n}{x_{n'}}\right)$$

using this formula square root of any number can be found upto any precision.

(b) Reciprocal of a number using newton-raphson

let N be the number

$$x = \frac{1}{N} \Rightarrow \frac{1}{x} - N = 0 \quad [f(x)]$$

$$f'(x) = -\frac{1}{x^2}$$

formula $$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$= x_i - \frac{\left(\frac{1}{x_i} - N\right)}{-\frac{1}{x_i^2}}$$

$$x_{i+1} = x_i(2 - x_i n)$$

using this formula we can find the reciprocal of a number to any accuracy

Q9

Q9 $f(x) = x\tan x + 1$ real root using newton raphson

$f'(x) = x\sec^2 x + \tan x$

$f(2) = -3.37 < 0$

$f(3) = +0.57 > 0$

The root lies b/w 2 and 3

Take $x_0 = \frac{2+3}{2} = 2.5$

Formula: $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 2.5 - \frac{(-0.86755)}{3.14808}$

$x_1 = 2.775$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

we can find $x_2 = 2.798$

$x_3 = 2.798$

$\therefore$ The real root of $x\tan x + 1 = 0$ is $\underline{2.798}$

Ans

Q10

Q 10 Approximate $\sqrt{7}$ to 20 decimal places

I will use Newton Raphson Method.

$[f(x)]$ $x^2 - 7 = 0$ $\sqrt{7}$ is root of eq.

Initial approximation is $x_0 = 3$

$$f'(x) = 2x$$

Using formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{x_0^2 - 7}{2x_0} = 3 - \frac{9-7}{6}$$

$$x_1 = 3 - \frac{1}{3} = \frac{8}{3} = 2.66666$$

$$x_2 = x_1 - \frac{x_1^2 - 7}{2x_1}$$

$$x_2 = \frac{8}{3} - 0.020833331$$

$$x_2 = 2.645833333$$

Similarly x_3 & x_4, x_5 are computed

$\sqrt{7} = 2.6457513110645905905$ Ans

Q11

Q11 Applying gaussian elimination to the augmented matrix for this system produces the following:

$$\begin{bmatrix} 0.143 & 0.357 & 2.01 & -5.17 \\ -1.31 & 0.911 & 1.99 & -5.46 \\ 11.2 & -4.3 & -0.605 & 4.42 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2.5 & 14.1 & -36.2 \\ -1.31 & 0.911 & 1.99 & -5.46 \\ 11.2 & -4.3 & -0.605 & 4.42 \end{bmatrix} \Longleftarrow$$ Divide first row by 0.143

$$\begin{bmatrix} 1 & 2.5 & 14.1 & -36.2 \\ 0 & 4.19 & 20.5 & -52.9 \\ 11.2 & -4.3 & -0.605 & 4.42 \end{bmatrix} \Longleftarrow$$ Add 1.31 times the first row to second row

$$\begin{bmatrix} 1 & 2.5 & 14.1 & -36.2 \\ 0 & 4.19 & 20.5 & -52.9 \\ 0 & -32.3 & -159. & 409. \end{bmatrix} \Longleftarrow$$ Add -11.2 times the first row to third row

$$\begin{bmatrix} 1 & 2.5 & 14.1 & -36.2 \\ 0 & 1 & 4.89 & -12.6 \\ 0 & -32.3 & -159. & 409. \end{bmatrix} \Longleftarrow$$ Divide second row by 4.19

$$\begin{bmatrix} 1 & 2.5 & 14.1 & -36.2 \\ 0 & 1 & 4.89 & -12.6 \\ 6 & 0 & -1 & 2 \end{bmatrix} \Longleftarrow$$ Add 32.3 times the second row to third row

Thus $x_3 = -2$, using substitution we get $x_2 = 2.82$
$x_1 = -0.950$

Ans.

Q12

Q12 Applying gaussian elimination to the augmented matrix for the system produce the following

$$\left[\begin{array}{cccc} 1 & 0.5 & 0.33 & 1 \\ 0.5 & 0.33 & 0.25 & 1 \\ 0.33 & 0.25 & 0.2 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc} 1 & 0.5 & 0.33 & 1 \\ 0 & 0.08 & 0.08 & 0.5 \\ 0.33 & 0.25 & 0.2 & 1 \end{array}\right] \Leftarrow$$ Divide first row by ~~half~~ 2 Subtract that from second row

$$\left[\begin{array}{cccc} 1 & 0.5 & 0.33 & 1 \\ 0 & 0.08 & 0.08 & 0.5 \\ 0 & 0.08 & 0.09 & 0.67 \end{array}\right] \Leftarrow$$ Add -0.33 times the first row to third row

$$\left[\begin{array}{cccc} 1 & 0.5 & 0.33 & 1 \\ 0 & 0.08 & 0.08 & 0.5 \\ 0 & 0 & 0.01 & 0.17 \end{array}\right] \Leftarrow$$ Subtract second row from third row

We get $x_3 = 17$

$x_2 = -10.75$

$x_1 = 0.76$ // Ans

[0.76, -10.75, 17]

This solⁿ does not match with [3, -24, 30] due to rounding off at each step but when we put these three values in the equation we nearly get approximately correct results

Q13

Q13 $4x_1 + 2x_2 - 2x_3 = 0$ Solve these Equations

$x_1 - 3x_2 + x_3 = 7$

$3x_1 - x_2 + 4x_3 = 5$

$(x_1, x_2, x_3) = (0,0,0)$ Initial approx.

→ Gauss - Seidel Method

from above equation $x_1 = \frac{1}{4}(2\cancel{x_3} - 2\cancel{x_2})$

$x_2 = -\frac{1}{3}(7 - x_1 + x_3)$

$x_3 = \frac{1}{4}(5 - 3x_1 + x_2)$

Initial guess = (0, 0, 0)

1st approximation.

$x_1 = \frac{1}{4}(0 - 2(0) + 2(0)) = 0$

$x_2 = -\frac{1}{3}(7 - (0) + 0) = -2.33$

$x_3 = \frac{1}{4}(5 - 3(0) + (-2.33)) = 0.67$

2nd approximation

$x_1 = \frac{1}{4}(0 - 2(-2.33) + 2(0.67)) = 1.5$

$x_2 = -\frac{1}{3}(7 - (1.5) + 0.67) = -2.06$

$$x_3 = \frac{1}{4}\left(5 - 3(1.5) + (-2.06)\right) = -0.39$$

3rd approximation

$$x_1 = \frac{1}{4}\left[0 - 2(-2.06) + 2(-0.39)\right) = 0.83$$

$$x_2 = -\frac{1}{3}\left(7 - (0.83) + (-0.39)\right) = -1.93$$

$$x_3 = \frac{1}{4}\left(5 - 3(0.83) + (-1.93)\right) = 0.14$$

4th approximation

$$x_1 = 1.03$$
$$x_2 = -2.04$$
$$x_3 = -0.04$$

5th approximation

$$\left.\begin{aligned} x_1 &= 1 \\ x_2 &= -1.99 \\ x_3 &= 0 \end{aligned}\right\} \longrightarrow \boxed{\begin{aligned} &\text{Approx Sol}^n \\ &x_1 = 1 \\ &x_2 = -2 \\ &x_3 = 0 \end{aligned}}$$

→ Jacobi

We use the same set of equations

1st approximation

$$x_1 = \frac{1}{4}\left[0 - 2(0) + 2(0)\right] = 0$$

$$x_2 = -\frac{1}{3}\left(7 - (0) + 0\right) = -2.33$$

$$x_3 = \frac{1}{4}\left[5 - 3(0) + 0\right] = 1.25$$

2nd approximation

$$x_1 = \frac{1}{4}\left[0 - 2(-2.33) + 2(1.25)\right] = 1.79$$

$$x_2 = -\frac{1}{3}\left[7 - (0) + 1.25\right] = -2.75$$

$$x_3 = \frac{1}{4}\left[5 - 3(0) + (-2.33)\right] = 0.67$$

Similar approximation is done for more approximation but it doesn't nicely converge even in 15 iterations

Q15

Q15 $x_0 = (1, 1, 1)$ power method with scaling

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

1st iter $\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

2nd iter $\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

3rd iter $\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

4th iter $\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

No chance of convergence since no approximation can be done

$\therefore$ can't find eigenvalue using the initial approximation

$x_0 = (1, 1, 1)$ Power method with scaling

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$$

1st iter

$$\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} = -3 \begin{bmatrix} -0.33 \\ -0.33 \\ 1 \end{bmatrix}$$

2nd iter

$$\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \begin{bmatrix} -0.33 \\ -0.33 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.99 \\ -2.99 \\ -3 \end{bmatrix} = -3 \begin{bmatrix} 0.99 \\ 0.99 \\ 1 \end{bmatrix}$$

3rd iter

$$\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \begin{bmatrix} 0.99 \\ 0.99 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.97 \\ 0.97 \\ -3 \end{bmatrix} = -3 \begin{bmatrix} 0.32 \\ 0.32 \\ 1 \end{bmatrix}$$

4th iter

$$\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \begin{bmatrix} 0.32 \\ 0.32 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.04 \\ -1.04 \\ -3 \end{bmatrix} = -3 \begin{bmatrix} 0.34 \\ 0.34 \\ 1 \end{bmatrix}$$

looks like it is converging towards

$(1, 1, 3)$ $[(0.34, 0.34, 1) \text{ or } 3]$

lets see

$$\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \\ -9 \end{bmatrix} = -3 \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

Yes it is eigenvector

Q16

Q16 · A = LU

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix} = \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ * & 1 & 0 \\ * & * & 1 \end{bmatrix}} \cdot \overset{U}{\begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix}}$$

$R_2 - \frac{R_1}{4}$

$R_3 - \frac{3R_1}{4}$

$$\begin{bmatrix} 4 & 1 & 1 \\ 0 & 15/4 & -9/4 \\ 0 & 5/4 & -19/4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{3}{4} & * & 1 \end{bmatrix} \begin{bmatrix} & - & \end{bmatrix}$$

$R_3 - \frac{R_2}{3}$

$$\underset{\boxed{U}}{\begin{bmatrix} 4 & 1 & 1 \\ 0 & 15/4 & -9/4 \\ 0 & 0 & -4 \end{bmatrix}} = \underset{\boxed{L}}{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{3}{4} & \frac{1}{3} & 1 \end{bmatrix}} \begin{bmatrix} & - & \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{3}{4} & \frac{1}{3} & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 4 & 1 & 1 \\ 0 & 15/4 & -9/4 \\ 0 & 0 & -4 \end{bmatrix}}$$

This is the LU factorization of the given matrix

- Use this factorization to solve $b = (4, 4, 6)^T$

$\therefore$

$$\underset{A}{\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}} \underset{X}{\begin{bmatrix} a \\ b \\ c \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}}$$

A = LU

Use our LU factorization.

Theorem: Solve $LUX = B$ for X to solve the system. Let $UX = Y$.
First solve $LY = B$ for Y and then solve $UX = Y$ for X

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{3}{4} & \frac{1}{3} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}}_{B}$$

from here $y_1 = 4$
$y_2 = 3$
$y_3 = 2$

Now,
$$\underbrace{\begin{bmatrix} 4 & 1 & 1 \\ 0 & 15/4 & -9/4 \\ 0 & 0 & -4 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}}_{Y}$$

from here
$$\left.\begin{aligned} x_3 &= -1/2 \\ x_2 &= 1/2 \\ x_1 &= 1 \end{aligned}\right]$$
Solution of the equation

- LU decomposition is not always possible it is possible only when — the leading minors must be non-zero. leading minors are the determinant for the 1×1, 2×2 and 3×3 for the pivot area.

The matrix ~~Ba~~ $\begin{bmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{bmatrix}$ cannot have LU factori-

Since 1×1 $\begin{bmatrix} 1 \end{bmatrix}$ Determinant $= 1$

2×2 $\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ Determinant $1 \times 8 - 4 \times 2 = 0$ ✗

Determinant of 2×2 pivot $= 0$

Matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$

is invertible $\because |A| = 2(1-2) - 2(1-3) + 1(2-3)$

$= -2 + 4 - 1$

$= 1$

Since $|A| \neq 0$

and has no LU factorization

since 2×2 pivot $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

has determinant $= 0$

Now if we interchange rows and columns the matrix determinant remains same this is a common property. To make it LU decomposable lets do some interchange

New $A = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

This is now LU decomposible

Since 1×1 pivot $[2]$ Det $\neq 0$

2×2 pivot $\begin{bmatrix} 2 & 2 \\ 3 & 2 \end{bmatrix}$ Det $\neq 0$

3×3 pivot $\begin{bmatrix} 2 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Det $\neq 0$

Q17

Q17 $A = \begin{bmatrix} 0.7825 & 0.8154 & -0.1897 \\ -0.3676 & 2.2462 & -0.0573 \\ -0.1838 & 0.1231 & 1.9714 \end{bmatrix}$

$\lambda_1 = 2 \quad \lambda_2 = 2 \quad \lambda_3 = 1$

Power method converges for the given matrix since the matrix is diagonalizable.

$x_0 = (1, 3, 6)$

1st iteration

$$x_1 = Ax_0 = \begin{bmatrix} 2.09 \\ 6.03 \\ 12.01 \end{bmatrix} = 12.01 \begin{bmatrix} 0.17 \\ 0.5 \\ 1 \end{bmatrix}$$

2nd iteration

$$x_2 = Ax_1 = \begin{bmatrix} 0.36 \\ 1.01 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 0.18 \\ 0.5 \\ 1 \end{bmatrix}$$

3rd iteration

$$x_3 = Ax_2 = \begin{bmatrix} 0.36 \\ 1.01 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 0.18 \\ 0.5 \\ 1 \end{bmatrix}$$

Same goes on for the next 2 iteration

But we have found out

the eigenvalue = 2

and eigenvector = $(0.18, 0.5, 1)^T$

Answer verified

Q18

Q18 $A = \begin{bmatrix} 5.4 & 0 & 0 \\ -113.0233 & -0.5388 & -0.6461 \\ -46.0567 & -6.4358 & -0.9612 \end{bmatrix}$

$\lambda_1 = 5.4, \quad \lambda_2 = 1.3, \quad \lambda_3 = -2.8$

$v_1 = (0.2, -4.1, 2.7)^T, \quad v_2 = (0, 1.3, -3.7)^T, \quad v_3 = (0, 2.6, 9.1)$

$x_0 = (0, 1, 1)$

To which eigenvector we converge?

1st iteration

$x_1 = Ax_0 = \begin{bmatrix} 0 \\ -1.1849 \\ -7.397 \end{bmatrix} = -7.397 \begin{bmatrix} 0 \\ 0.16 \\ 1 \end{bmatrix}$

2nd iteration

$x_2 = Ax_1 = \begin{bmatrix} 0 \\ -0.73 \\ -1.99 \end{bmatrix} = -1.99 \begin{bmatrix} 0 \\ 0.36 \\ 1 \end{bmatrix}$

3rd iteration

$x_3 = Ax_2 = \begin{bmatrix} 0 \\ -0.84 \\ -3.34 \end{bmatrix} = -3.34 \begin{bmatrix} 0 \\ 0.25 \\ 1 \end{bmatrix}$

4th iteration

$x_4 = Ax_3 = \begin{bmatrix} 0 \\ -0.78 \\ -2.57 \end{bmatrix} = -2.57 \begin{bmatrix} 0 \\ 0.3 \\ 1 \end{bmatrix}$

5^{th} iteration

$$x_5 = Ax_4 = \begin{bmatrix} 0 \\ -0.8 \\ -2.89 \end{bmatrix} = -2.89 \begin{bmatrix} 0 \\ 0.28 \\ 1 \end{bmatrix}$$

6^{th} iteration

$$x_6 = Ax_5 = \begin{bmatrix} 0 \\ -0.79 \\ -2.76 \end{bmatrix} = \underline{-2.76} \begin{bmatrix} 0 \\ 0.28 \\ 1 \end{bmatrix}$$

<u>Converged (approx)</u>

~~Ans~~ matches with $v_3 = (0, 2.6, 9.1)^T$, $\lambda_3 = -2.8$

Gershgorin's theorem

$$D_i = \{ z \in \mathbb{C} : |z - A_{ii}| \le \sum |A_{ij}| \}$$

for i^{th} row

Eigenvalues lie in $D_1 \cup D_2 \cup \dots \cup D_n$

$$A = \begin{bmatrix} 0.5 & 0 & 0.2 \\ 0 & 3.15 & -1 \\ 0.57 & 0 & -7.43 \end{bmatrix}$$

Intervals in which eigenvalue lie

$$\begin{cases} D_1 : & |z - 0.5| \le 0.2 \\ D_2 : & |z - 3.15| \le 1 \\ D_3 : & |z + 7.43| \le 0.57 \end{cases}$$

The matrix is diagonalizable, therefore power method can find a eigenvalue & eigenvector for it

$$x_0 = (1, 1, 1)$$

1st iteration $x_1 = Ax_0 = \begin{bmatrix} 0.7 \\ 2.15 \\ -6.86 \end{bmatrix} = -6.86 \begin{bmatrix} -0.1 \\ -0.31 \\ 1 \end{bmatrix}$

2nd iteration $x_2 = Ax_1 = \begin{bmatrix} 0.15 \\ -1.97 \\ -7.487 \end{bmatrix} = -7.487 \begin{bmatrix} -0.02 \\ 0.26 \\ 1 \end{bmatrix}$

3rd iteration $x_3 = Ax_2 = \begin{bmatrix} 0.19 \\ -0.18 \\ -7.4414 \end{bmatrix} = -7.4414 \begin{bmatrix} -0.02 \\ 0.09 \\ 1 \end{bmatrix}$

Looks like it is getting converged

(Approx) So the eigenvalue $= -7.44$

Corresp. eigenvector $= (-0.02, 0.09, 1)$

Ans