

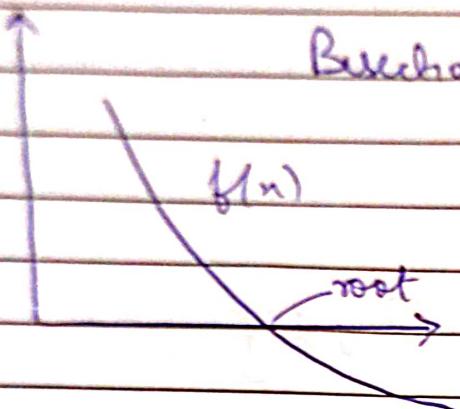
Assignment - 1

Roll No.: IMT2020045

Q1



Q1



Bisection method first lets find b/w which two numbers root lies

$$f(x) = x^3 - x - 1 = 0$$

$$f(1) = 1^3 - 1 - 1 = -1 < 0$$

$$f(2) = 2^3 - 2 - 1 = 5 > 0$$

∴ A root of $f(x)$ lies in 1 and 2

$$x_1 = \frac{1+2}{2} = \frac{3}{2} = 1.5$$

$$f(x_1) = f(1.5) = (1.5)^3 - 1.5 - 1 = 0.875 > 0$$

Root lies in 1 and 1.5

$$x_2 = \frac{1+1.5}{2} = 1.25$$

$$f(x_2) = f(1.25) = (1.25)^3 - 1.25 - 1 = -0.29 < 0$$

Hence root lies in 1.25 and 1.5

$$x_3 = \frac{1.25+1.5}{2} = 1.375$$

Similarly, we get $x_4 = 1.3125$

$$x_5 = 1.34375$$

$$x_6 = 1.328125 \text{ etc.}$$

Ans

Q2

$$f(x) = x e^x - 1 = 0$$

Bisection method

Q2

first lets find out b/w which two pts
the root lies

$$f(0) = 0 \cdot e^0 - 1 = -1 < 0$$

$$f(1) = 1 \cdot e^1 - 1 = 1.7183 > 0$$

Hence, a root lies in 0 and 1

$$x_1 = \frac{0+1}{2} = 0.5$$

$$f(0.5) = 0.5 e^{0.5} - 1 = -0.178 \\ < 0$$

Hence the root lies b/w 0.5 and 1

$$x_2 = \frac{0.5+1}{2} = 0.75$$

Proceed like this we get

$$x_3 = 0.625$$

$$x_4 = 0.5625$$

$$x_5 = 0.59375$$

$$x_6 = 0.5781$$

$$x_7 = 0.5703$$

$$x_8 = 0.5664$$

$$x_9 = 0.5684$$

$$x_{10} = 0.5674$$

$$x_{11} = 0.5669$$

$$x_{12} = 0.5672$$

$$x_{13} = 0.5671$$

hence, the root correct to 3 decimal places = 0.567

Q3

Q3 $f(x) = x^3 - 2x - 5$ Regular False

for lets find b/w which two numbers root lie

$$f(2) = -1 \quad f(3) = 16$$

hence the root lies b/w 2 and 3
 \therefore take $a=2, b=3$

$$\begin{aligned} x_1 &= \frac{a f(b) - b f(a)}{f(b) - f(a)} \\ &= \frac{2(16) - 3(-1)}{16 - (-1)} = \frac{35}{17} \end{aligned}$$

$$x_1 = 2.0588$$

$$f(x_1) = -0.3979 < 0$$

Therefore the root lies b/w 2.0588 and 3
 Again we will use the same formula

$$x_2 = \frac{2.0588(16) - 3(-0.397)}{16 - (-0.397)} = 2.081$$

Proceed like this we get

$$x_3 = 2.089$$

$$x_4 = 2.092$$

$$x_5 = 2.093$$

$$x_6 = 2.099$$

Ans

Q4

(Q4) $f(x) = \cos x - x e^x = 0$ Newton Raphson

first lets find b/w which have same root
line

$f(0) = 1$ and $f(1) = 2.177$
 $a=0$ and $b=1$: The root lies
between 0 and 1.

$$x_1 = \frac{0(-2.177)}{-2.177 - 1} = f(1) = 0.314$$

$$f(x_1) = f(0.314) = 0.51986$$

The root lies b/w 0.314 and 1

$$\text{hence, } x_2 = \frac{0.314(-2.177)}{-2.177 - 0.519} = f(0.519)$$

$$x_2 = 0.44673$$

Proceed like this we get

$$x_3 = 0.49402,$$

$$x_4 = 0.50995,$$

$$x_5 = 0.51520,$$

$$x_6 = 0.51692,$$

Ans

Q5

Q5

$$2x - \log_{10} x = 7$$

by iteration method

separate x first

$$x = \frac{7 + \log_{10} x}{2} \quad \varphi(n) = \frac{\log_{10} x + 7}{2}$$

value when $x=1$

$$\varphi(1) = \frac{0+7}{2} = 3.5$$

$$\text{next iter, } \varphi_2(x) = \frac{\log_{10} 3.5 + 7}{2} = 3.772$$

$$\dots, \varphi_3(n) = \frac{\log_{10} 3.772 + 7}{2} = 3.78825$$

$$\varphi_4(n) = \frac{\log_{10} 3.78825 + 7}{2} = 3.7892$$

$$\varphi_5(n) = \frac{\log_{10} 3.7892 + 7}{2} = 3.7893$$

$\therefore x = 3.7893$ is the root of the

Eqⁿ

Ans

Q6

$$f(x) \Rightarrow$$

$$x \sin x + \cos x = 0$$

Newton-Raphson
method

$$f(x) = x \sin x + \cos x$$

$$f'(x) = \sin x + x \cos x + \cos x (-\sin x) = x \cos x$$

formula $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$

Initial guess: $x_0 = \cancel{\pi}$ π

$$x_1 = x_0 + \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = \pi + \frac{\pi \cdot \sin(\pi) + \cos(\pi)}{\pi \cos \pi}$$

$$x_1 = 2.8233$$

Similarly, $x_2 = x_1 + \frac{f(x_1)}{f'(x_1)}$

$$x_2 = 2.7986$$

$$x_3 = 2.7984$$

$$x_4 = 2.7984$$

$\therefore x = 2.7984$ can be the root of
 $f(x) \Rightarrow x \sin x + \cos x = 0$

Q7

Q7

$$f(x) \Rightarrow x - e^{-x} = 0$$

Newton-Raphson

$$f(x) = x - e^{-x}$$

$$f'(x) = 1 + e^{-x}$$

$$f(0) = -1$$

$$f(1) = 0.63212$$

\therefore a root lies in b/w 0 and 1

$$\text{Let } x_0 = 1$$

$$\text{formula } x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{0.63212}{1.3679}$$

$$x_1 = 0.5379$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.5379 - \frac{0.0461}{1.584}$$

$$= 0.56714$$

Proceding,

\therefore we can say $x = 0.567$ can
be taken as a root for
 $f(x)$

Q8

(a) Square root of a number using newton-raphson method.

Let n be the number

$$\Rightarrow x = \sqrt{n}$$

$$\Rightarrow x^2 = n$$

$$f(x) = x^2 - n = 0$$

$$f'(x) = 2x$$

by formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n^2 - n}{2x_n}$$

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{n}{x_n} \right)$$

using this formula square root of any number can be found upto any precision.

(b) Reciprocal of a number using newton-raphson

let p be the number

$$x = \frac{1}{N} \Rightarrow \frac{1}{x} - N = 0 \quad [f(x)]$$

$$f'(x) = -\frac{1}{x^2}$$

formula $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$$= x_i - \left(\frac{\frac{1}{x_i} - N}{-\frac{1}{x_i^2}} \right)$$

$$= \frac{1}{x_i^2}$$

$$x_{i+1} = x_i (2 - x_i N)$$

using this formula we can find the reciprocal of a number to any accuracy

Q9

$$f(n) = x \tan n + 1 \quad \text{real root by Newton Raphson}$$

$$f'(n) = x \sec^2 n + \tan n$$

$$f(2) = -3.37 < 0$$

$$f(3) = +0.57 > 0$$

The root lies b/w 2 and 3

$$\text{Take } x_0 = \frac{2+3}{2} = 2.5$$

$$\text{Formula: } x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2.5 - \frac{(-0.86755)}{3.14809}$$

$$x_1 = 2.775$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\text{we can find } x_2 = 2.798$$

$$x_3 = 2.798$$

\therefore the real root of $x \tan n + 1 = 0$ is 2.798

Q10

Q10 Approximate $\sqrt{7}$ to 20 decimal places

We will use Newton Raphson Method.

$$[f(x)] \quad x^2 - 7 = 0$$

$\sqrt{7}$ is root of $x^2 - 7 = 0$

Initial approximation is $x_0 = 3$

$$f'(x) = 2x$$

$$\text{Using formula } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{x_0^2 - 7}{2x_0} = 3 - \frac{9 - 7}{6}$$

$$x_1 = 3 - \frac{1}{3} = \frac{8}{3} = 2.66666$$

$$x_2 = x_1 - \frac{x_1^2 - 7}{2x_1}$$

$$x_2 = \frac{8}{3} - 0.020833331$$

$$x_2 = 2.64583333$$

Similarly x_3, x_4, x_5 are computed

$$\sqrt{7} = 2.6457513106459059050$$

Q11

Q11 Applying gaussian elimination to the augmented matrix for the system produces the following:

$$\left[\begin{array}{cccc} 0.143 & 0.357 & 2.01 & -5.17 \\ -1.31 & 0.911 & 1.99 & -5.46 \\ 11.2 & -4.3 & -0.605 & 4.42 \end{array} \right]$$

$$\left[\begin{array}{cccc} 1 & 2.5 & 14.1 & -36.2 \\ -1.31 & 0.911 & 1.99 & -5.46 \\ 11.2 & -4.3 & -0.605 & 4.42 \end{array} \right] \quad \begin{matrix} \leftarrow \text{Divide first} \\ \text{row by } 0.143 \end{matrix}$$

$$\left[\begin{array}{cccc} 1 & 2.5 & 14.1 & -36.2 \\ 0 & 4.19 & 20.5 & -52.9 \\ 11.2 & -4.3 & -0.605 & 4.42 \end{array} \right] \quad \begin{matrix} \leftarrow \text{Add } 1.31 \text{ times} \\ \text{the first row to} \\ \text{second row} \end{matrix}$$

$$\left[\begin{array}{cccc} 1 & 2.5 & 14.1 & -36.2 \\ 0 & 4.19 & 20.5 & -52.9 \\ 0 & -32.3 & -159. & 409. \end{array} \right] \quad \begin{matrix} \leftarrow \text{Add } -11.2 \text{ times} \\ \text{of first row to} \\ \text{third row} \end{matrix}$$

$$\left[\begin{array}{cccc} 1 & 2.5 & 14.1 & -36.2 \\ 0 & 1 & 4.89 & -12.6 \\ 0 & -32.3 & -159. & 409. \end{array} \right] \quad \begin{matrix} \leftarrow \text{Divide second} \\ \text{row by } 4.89 \end{matrix}$$

$$\left[\begin{array}{cccc} 1 & 2.5 & 14.1 & -36.2 \\ 0 & 1 & 4.89 & -12.6 \\ 0 & 0 & -1 & 2 \end{array} \right] \quad \begin{matrix} \leftarrow \text{Add } 32.3 \text{ times} \\ \text{of second row to} \\ \text{third row} \end{matrix}$$

Hence $x_3 = -2$. Using substitution we get $x_2 = 2.82$

$$x_1 = -0.950$$

Ans.

Q12

Q12 Applying gaussian elimination to the augmented matrix for the system produce the following

$$\left[\begin{array}{cccc} 1 & 0.5 & 0.33 & 1 \\ 0.5 & 0.33 & 0.25 & 1 \\ 0.33 & 0.25 & 0.2 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc} 1 & 0.5 & 0.33 & 1 \\ 0 & 0.08 & 0.08 & 0.5 \\ 0.33 & 0.25 & 0.2 & 1 \end{array} \right] \leftarrow$$

Divide first row by half 2
Subtract half from each row

$$\left[\begin{array}{cccc} 1 & 0.5 & 0.33 & 1 \\ 0 & 0.08 & 0.08 & 0.5 \\ 0 & 0.08 & 0.09 & 0.67 \end{array} \right] \leftarrow$$

Add -0.52 times the first row to third row

$$\left[\begin{array}{cccc} 1 & 0.5 & 0.33 & 1 \\ 0 & 0.08 & 0.08 & 0.5 \\ 0 & 0 & 0.01 & 0.17 \end{array} \right] \leftarrow$$

Subtract second row from third row

We get

$$x_3 = 17$$

$$x_2 = -10.75$$

$$x_1 = 0.76 / \text{Ans}$$

$$[0.76, -10.75, 17]$$

The solⁿ does not match w/ $\{2, -21, 27\}$
due to rounding off at each step
but when we put these values
in the equation it would
get approximately correct result

Q13

Q13

$$4x_1 + 2x_2 - 2x_3 = 0$$

$$x_1 - 3x_2 + x_3 = 7$$

$$3x_1 - x_2 + 4x_3 = 5$$

Solve these
equations

$$(x_1, x_2, x_3) = (0, 0, 0) \quad \text{Initial approx.}$$

→ Gauss - Seidel Method

$$\text{from above equation } x_1 = \frac{1}{4} (2x_2 - 2x_3)$$

$$x_2 = \frac{1}{3} (7 - x_1 + x_3)$$

$$x_3 = \frac{1}{4} (5 - 3x_1 + x_2)$$

$$\text{Initial guess} = (0, 0, 0)$$

1st approximation

$$x_1 = \frac{1}{4} (0 - 2(0) + 2(0)) = 0$$

$$x_2 = \frac{1}{3} (7 - (0) + 0) = -2.33$$

$$x_3 = \frac{1}{4} (5 - 3(0) + (-2.33)) = 0.67$$

2nd approximation

$$x_1 = \frac{1}{4} (0 - 2(-2.33) + 2(0.67)) = 1.5$$

$$x_2 = \frac{1}{3} (7 - (1.5) + 0.67) = -2.06$$

$$x_3 = \frac{1}{4} (5 - 3(1.5) + (-2.06)) = -0.39$$

3rd approximation

$$x_1 = \frac{1}{4} [0 - 2(-2.06) + 2(-0.39)] = 0.83$$

$$x_2 = \frac{1}{3} (7 - 1(0.83) + (-0.39)) = -1.93$$

$$x_3 = \frac{1}{4} (5 - 3(0.83) + (-1.93)) = 0.15$$

4th approximation

$$x_1 = 1.035$$

$$x_2 = -2.04$$

$$x_3 = -0.04$$

5th approximation

$$\left. \begin{array}{l} x_1 = 1 \\ x_2 = -1.99 \\ x_3 = 0 \end{array} \right\} \text{closed}$$

Apprx Solⁿ

$$\left. \begin{array}{l} x_1 = 1 \\ x_2 = -2 \\ x_3 = 0 \end{array} \right.$$

→ Jacobi

We use the same set of equations

~~1st approximation~~

$$x_1 = \frac{1}{4} [0 - 2(0) + 2(0)] = 0$$

$$x_2 = -\frac{1}{3} [7 - (0) + 0] = -2.33$$

$$x_3 = \frac{1}{4} [5 - 3(0) + 0] = 1.25$$

~~2nd approximation~~

$$x_1 = \frac{1}{4} [0 - 2(-2.33) + 2(1.25)] = 1.79$$

$$x_2 = -\frac{1}{3} [7 - (0) + 1.25] = -2.75$$

$$x_3 = \frac{1}{4} [5 - 3(0) + (-2.33)] = 0.67$$

Similar approximation is done for more approximations but it doesn't nicely converge even in 15 iterations

Q15

Q15

$$x_0 = (1, 1, 1)$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

power method

with scaling

1st iter

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

2nd iter

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

3rd iter

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

4th iter

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

No chance of convergence. Thus no approximation can be done

can't find eigenvalues using the initial approximation

$$\rightarrow x_0 = (1, 1, 1)$$

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$$

Power method with scaling

1st iter $\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} = -3 \begin{bmatrix} -0.33 \\ -0.33 \\ 1 \end{bmatrix}$

2nd iter $\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \begin{bmatrix} 0.33 \\ -0.33 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix} = -3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

3rd iter $\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = -3 \begin{bmatrix} -0.33 \\ -0.33 \\ 1 \end{bmatrix}$

4th iter $\begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix} \begin{bmatrix} -0.33 \\ -0.33 \\ 1 \end{bmatrix} = -3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

No chance of convergence, since approx can be low

Can't find eigenvalues using the initial approx.

Q16

Q16 • $A = LU$

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ * & 1 & 0 \\ * & * & 1 \end{bmatrix} \cdot \begin{bmatrix} a & * & * \\ 0 & b & * \\ 0 & 0 & c \end{bmatrix}$$

$$R_2 - \frac{R_1}{4} \quad \begin{bmatrix} 4 & 1 & 1 \\ 0 & 15/4 & -9/4 \\ 0 & 5/4 & -19/4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{3}{4} & 2 & 1 \end{bmatrix} \quad \boxed{-}$$

$$R_3 - 3R_1 \quad \boxed{\frac{1}{4}}$$

$$R_3 - \frac{R_2}{3} \quad \begin{bmatrix} 4 & 1 & 1 \\ 0 & 15/4 & -9/4 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{3}{4} & \frac{1}{3} & 1 \end{bmatrix} \quad \boxed{-}$$

$$\boxed{U} \quad \boxed{L}$$

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{3}{4} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & 1 \\ 0 & 15/4 & -9/4 \\ 0 & 0 & -4 \end{bmatrix}$$

$$L \quad U$$

This is the LU factorization of the given matrix

- Use this factorization to solve $b = (3, 4, 6)$

$$\therefore \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$$

$A \quad X \quad B$

$A = LU$

Use our LU factorization.

Theorem: Solve $LUX = B$ for X to solve the system. Let $UX = Y$.

First solve $LY = B$ for Y and then solve $UX = Y$ for X .

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & y_1 \\ \frac{1}{4} & 1 & 0 & y_2 \\ \frac{3}{4} & \frac{1}{3} & 1 & y_3 \end{array} \right] = \left[\begin{array}{c} 4 \\ 4 \\ 6 \end{array} \right]$$

$L \quad Y \quad B$

from here $y_1 = 4$

$y_2 = 3$

$y_3 = 2$

Now, $\left[\begin{array}{ccc|c} 4 & 1 & 1 & x_1 \\ 0 & \frac{1}{4} & -\frac{1}{4} & x_2 \\ 0 & 0 & -4 & x_3 \end{array} \right] = \left[\begin{array}{c} 4 \\ 3 \\ 2 \end{array} \right]$

$U \quad X \quad Y$

from here $x_3 = -\frac{1}{2}$

$x_2 = \frac{1}{2}$

$x_1 = 1$

Solved
the equation
for

LU decomposition is not always possible.
it is possible only when - the leading
minors must be non-zero. leading minors are
the determinant for the 1x1, 2x2 and 3x3
for the first term.

The matrix $\begin{bmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ 72 & 32 & 5 \end{bmatrix}$ cannot have LU factorization.

Since 1×1 matrix $\begin{bmatrix} 1 \end{bmatrix}$ has Determinant = 1

2×2 matrix $\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ Determinant $1 \times 8 - 4 \times 2 = 0$

Determinant of 2×2 pivot = 0

Matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$

$$\text{is invertible} \because |A| = 2(1-2) - 2(1-3) + 1(2-1) \\ = -2 + 4 - 1 \\ = 1$$

Since $|A| \neq 0$

and has no LU factorization

Since 2×2 pivot $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

has determinant = 0

Now, if we interchange row and column the matrix determinant doesn't change. This is a common property. In case of LU factorization its also same when changed.

$$\text{New } A = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

This is now LU decomposable

Since 1×1 pivot $\begin{bmatrix} 2 \end{bmatrix}$ $\neq \text{Det} \neq 0$

2×2 pivot $\begin{bmatrix} 2 & 2 \\ 3 & 2 \end{bmatrix}$ $\text{Det} \neq 0$

3×3 pivot $\begin{bmatrix} 2 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $\text{Det} \neq 0$

Q17

Q17

$$A = \begin{bmatrix} 0.7825 & 0.8154 & -0.1897 \\ -0.3676 & 2.2462 & -0.0573 \\ -0.1838 & 0.1231 & 1.9714 \end{bmatrix}$$

$$\lambda_1 = 2 \quad \lambda_2 = 2 \quad \lambda_3 = 1$$

Power method converges for the given matrix since the matrix is diagonalizable.

$$x_0 = (1, 3, 6)$$

1st iteration

$$x_1 = Ax_0 = \begin{bmatrix} 2.09 \\ 6.03 \\ 12.01 \end{bmatrix} = 12.01 \begin{bmatrix} 0.17 \\ 0.5 \\ 1 \end{bmatrix}$$

2nd iteration

$$x_2 = Ax_1 = \begin{bmatrix} 0.36 \\ 1.01 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 0.18 \\ 0.5 \\ 1 \end{bmatrix}$$

3rd iteration

$$x_3 = Ax_2 = \begin{bmatrix} 0.36 \\ 1.01 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 0.18 \\ 0.5 \\ 1 \end{bmatrix}$$

Same goes on for the
eigenvalues next 2 choices

But we have found
the eigenvalue = 2

and eigenvector = $(0.18, 0.5, 1)^T$

Answer worked

Q18

Q18

$$A = \begin{bmatrix} 5.4 & 0 & 0 \\ -113.0233 & -0.5388 & -0.6461 \\ -46.0567 & -6.4358 & -0.9612 \end{bmatrix}$$

$$\lambda_1 = 5.4, \lambda_2 = 1.3, \lambda_3 = -2.8$$

$$v_1 = (0.2, -4.1, 2.7)^T, v_2 = (0, 1.3, -3.7)^T, v_3 = (0, 2.6, 9.1)^T$$

$$x_0 = (1, 1, 1)^T$$

To which eigenvector we converge?

1st iteration

$$x_1 = Ax_0 = \begin{bmatrix} 0 & 0 & 0 \\ -1.1849 & -7.397 & 0 \\ -7.397 & 0 & 0.16 \end{bmatrix} = -7.397 \begin{bmatrix} 0 \\ 0.16 \\ 1 \end{bmatrix}$$

2nd iteration

$$x_2 = Ax_1 = \begin{bmatrix} 0 \\ -0.73 \\ -1.99 \end{bmatrix} = -1.99 \begin{bmatrix} 0 \\ 0.36 \\ 1 \end{bmatrix}$$

3rd iteration

$$x_3 = Ax_2 = \begin{bmatrix} 0 \\ -0.84 \\ -3.34 \end{bmatrix} = -3.34 \begin{bmatrix} 0 \\ 0.25 \\ 1 \end{bmatrix}$$

4th iteration

$$x_4 = Ax_3 = \begin{bmatrix} 0 \\ -0.78 \\ -2.57 \end{bmatrix} = -2.57 \begin{bmatrix} 0 \\ 0.3 \\ 1 \end{bmatrix}$$

5th iteration

$$x_5 = Ax_4 = \begin{bmatrix} 0 \\ -0.8 \\ -2.89 \end{bmatrix} = -2.89 \begin{bmatrix} 0 \\ 0.28 \\ 1 \end{bmatrix}$$

6th iteration

$$x_6 = Ax_5 = \begin{bmatrix} 0 \\ -0.79 \\ -2.76 \end{bmatrix} = -2.76 \begin{bmatrix} 0 \\ 0.28 \\ 1 \end{bmatrix}$$

Converged (approx)

Ans

matches with $v_3 = (0, 2.6, 9.1)^T$

$$\lambda_3 = -2.8$$

Gershgorin's theorem

$$D_i = \left\{ z \in C : |z - A_{ii}| \leq \sum_{j \neq i} |A_{ij}| \right\}$$

for all rows

Eigenvalues lie in D_1, D_2, \dots, D_n

$$A = \begin{bmatrix} 0.5 & 0 & 0.2 \\ 0 & 3.15 & -1 \\ 0.57 & 0 & -7.43 \end{bmatrix}$$

Intervals in which

$$\text{Eigenvalues lie.} \quad \begin{cases} D_1: |z - 0.5| \leq 0.2 \\ D_2: |z - 3.15| \leq 1 \\ D_3: |z + 7.43| \leq 0.57 \end{cases}$$

The matrix is degenerable, therefore power method can find a eigenvalue & eigenvector for it

$$x_0 = (1, 1, 1)$$

1st iteration $x_1 = Ax_0 = \begin{bmatrix} 0.7 \\ 2.15 \\ -6.86 \end{bmatrix} = 6.86 \begin{bmatrix} -0.1 \\ -0.31 \\ 1 \end{bmatrix}$

2nd

-Iteration $x_2 = Ax_1 = \begin{bmatrix} 0.15 \\ -1.97 \\ -7.48 \end{bmatrix} = -7.487 \begin{bmatrix} -0.02 \\ 0.26 \\ 1 \end{bmatrix}$

3rd

Iteration $x_3 = Ax_2 = \begin{bmatrix} 0.19 \\ -0.18 \\ -7.4918 \end{bmatrix} = -7.4444 \begin{bmatrix} -0.02 \\ 0.08 \\ 1 \end{bmatrix}$

Looks like it is getting converged

(Appren) So the eigenvalue = -7.44

Corresp. Eigenvector = $(-0.02, 0.08, 1)$

Ans