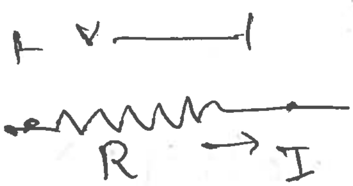
A. Power Dissipation in resistive circuit

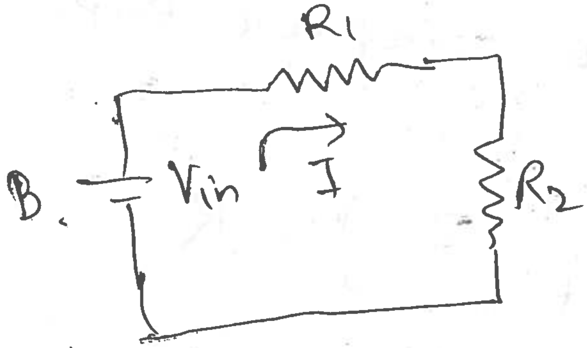
The total power consumed/created by a circuit ~~depends on~~ equals the sum of each elements power.

$$P = \sum_{K} V_K i_K$$

by convention, positive value of '$V_k i_k$' means consumption of power, and negative value means generation of power.



Power consumption across resistor $= \frac{V^2}{R} = I^2 R$ (always +ve)



$$I = \frac{V_{in}}{R_1 + R_2}$$

$$P_{R_1} = + \frac{V_{in}^2 R_1}{(R_1+R_2)^2}$$

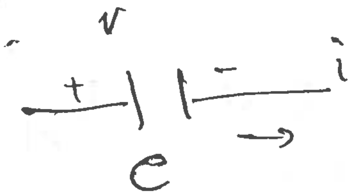
$$P_{R_2} = \frac{V_{in}^2 R_2}{(R_1+R_2)^2}$$

$$P_B = (-) V_{in} I_{in}$$

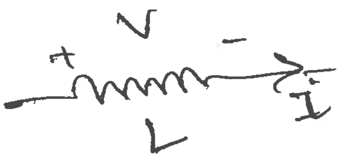
$$= (-) \frac{V_{in}^2}{R_1+R_2}$$

Thus, $P_B + P_{R_1} + P_{R_2} = 0$

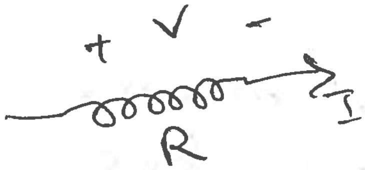
B. Circuits with Capacitors and inductors



$$\boxed{i = C\frac{dv}{dt}}$$



$$\boxed{V = L\frac{di}{dt}}$$

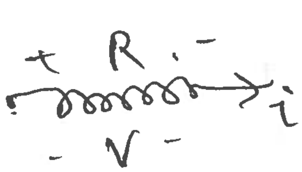


$$\boxed{V = IR}$$

Let us assume that the ~~voltage~~ all the sources in the circuit are complex exponentials.

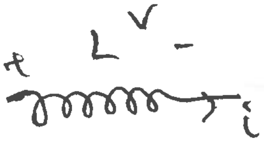
$$v_{in}(t) = V_{in}\, e^{j2\pi f t}$$

this implies that all the voltages and current are also complex exponentials.



$$v = V e^{j2\pi f t} \qquad Z = \frac{V}{I} = R$$

$$i = \left(\frac{V}{R}\right) e^{j2\pi f t}$$

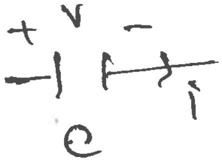


if $i(t) = I e^{j2\pi f t}$

then, $v(t) = LI\,(2j\pi f)\, e^{j2\pi f t}$

$$v(t) = L\,(2\pi f j)\, i(t)$$

$$Z = \frac{V(t)}{I(t)} = L\,(2\pi f)\, j$$



if $v(t) = V e^{j2\pi f t}$

$$i(t) = CV(2\pi f j)\, e^{j2\pi f t}$$

$$= C(2\pi f j)\, v(t)$$

$$Z = \frac{v(t)}{i(t)} = \frac{1}{2\pi f C j}$$

~~Thus, V-I relationship for all circuit element~~

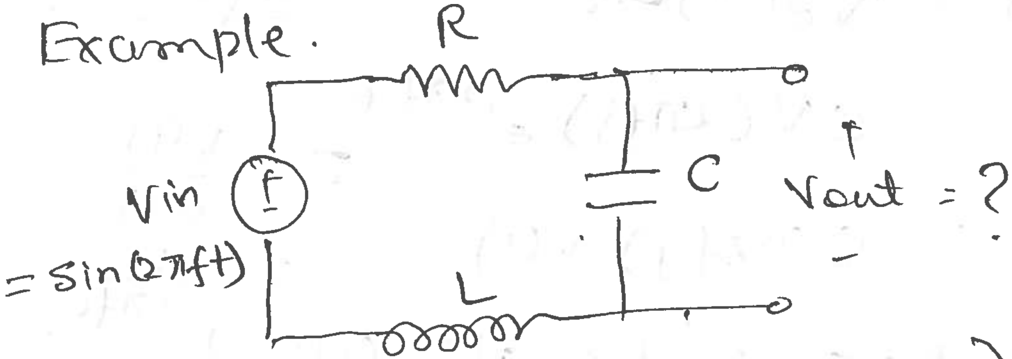
Thus, major consequence of assuming complex exponential voltage and currents is that the ratio $Z = \frac{V}{I}$ for each element does not depend on time, but does depend on source frequency.

All circuit Laws (KCL, KVL) applies under this formulation.

Steps to solve circuits having RLC.

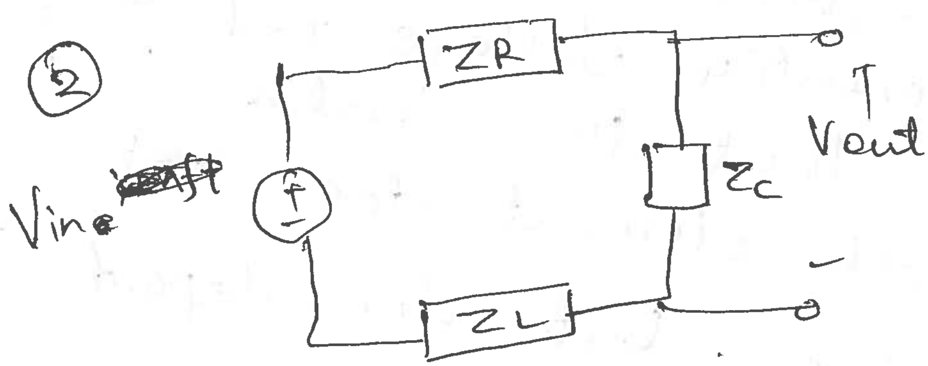
1. Even if it's not, pretend the source is a Complex exponential
2. Solve circuit using the impedence method - (series, parallel, KCL, KVL etc) in frequency domain.
3. Get the output for the actual input.

Example.



① $V_{in} = V_{in}\, e^{j2\pi f t}$ (assume)

②



$$Z_R = R$$

$$Z_C = \frac{1}{2\pi f j C} = \frac{1}{sC}$$

$$Z_L = 2\pi f j L = sL$$

$$V_{out} = \frac{Z_C}{Z_R + Z_L + Z_C} V_{in}$$

$$= \left[\frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} \right] V_{in}$$

$$V_{out} = \frac{V_{in}}{RCs + LCs^2 + 1} \quad ; \; s = 2\pi f j$$

$$V_{out} = \frac{V_{in}}{RC(2\pi f j) - LC(4\pi^2 f^2) + 1}$$

$$= \frac{V_{in}}{(1-4\pi^2 f^2 LC) + 2\pi RCfj} \quad \text{(cartesian)}$$

$$V_{out} = \frac{V_{in}}{\sqrt{[1-4\pi^2 f^2 LC]^2 + (2\pi RCf)^2}} \; \cancel{exp} \left[e^{j \underbrace{\tan^{-1}\left[\frac{2\pi fRC}{4\pi^2 f^2 LC - 1}\right]}_{\theta}} \right]$$

(polar)

(3) $\cancel{V_{in} = Im[V_{in}]}$
$\cancel{\text{(actual)} \quad \text{assumed}}$

$$v_{out} = V_{out}\, e^{j2\pi ft}$$

$$= \frac{V_{in}}{\sqrt{(-)^2 + (-)^2}} \, e^{j\theta} \, e^{j2\pi ft}$$

(3) $v_{in} = \sin(2\pi ft) = Im\left[e^{j2\pi ft} \right]$

$$v_{out} = \frac{1}{\sqrt{(-)^2 + (-)^2}} \sin(2\pi ft + \theta)$$

Power in frequency domain

~~v(t) = |~~

$$P_{av} = \frac{1}{2} Re(VI^*)$$

$V, I \rightarrow$ Complex amplitude.

$$= \frac{1}{2} Re(Z)\, |I|^2$$

$$= \frac{1}{2} Re\left(\frac{1}{Z}\right) |V|^2$$

1) Only Real part of impedence contribute to long term power dissipation
2) Only the resister dissipate power.
3) L, C dissipate no power in long term.
4) these statement apply only for sinusoidal sources.

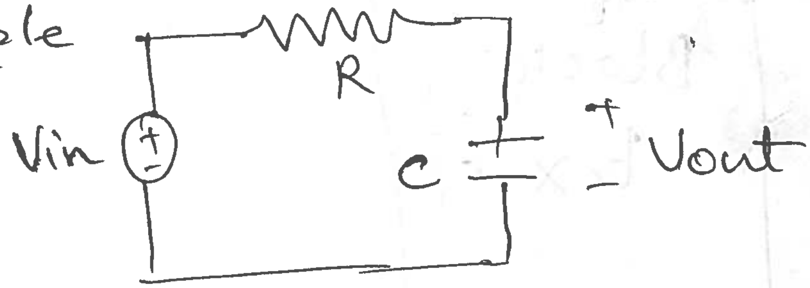
Transfer function

Implicit is that input and output are complex exponentials, and Vin, Vout are the complex amplitude.

$$\frac{V_{out}}{V_{in}} = H(f)$$

$$= \underset{\uparrow\ \text{magnitude}}{|H(f)|} \;\angle(H(f)) \rightarrow \text{phase.}$$

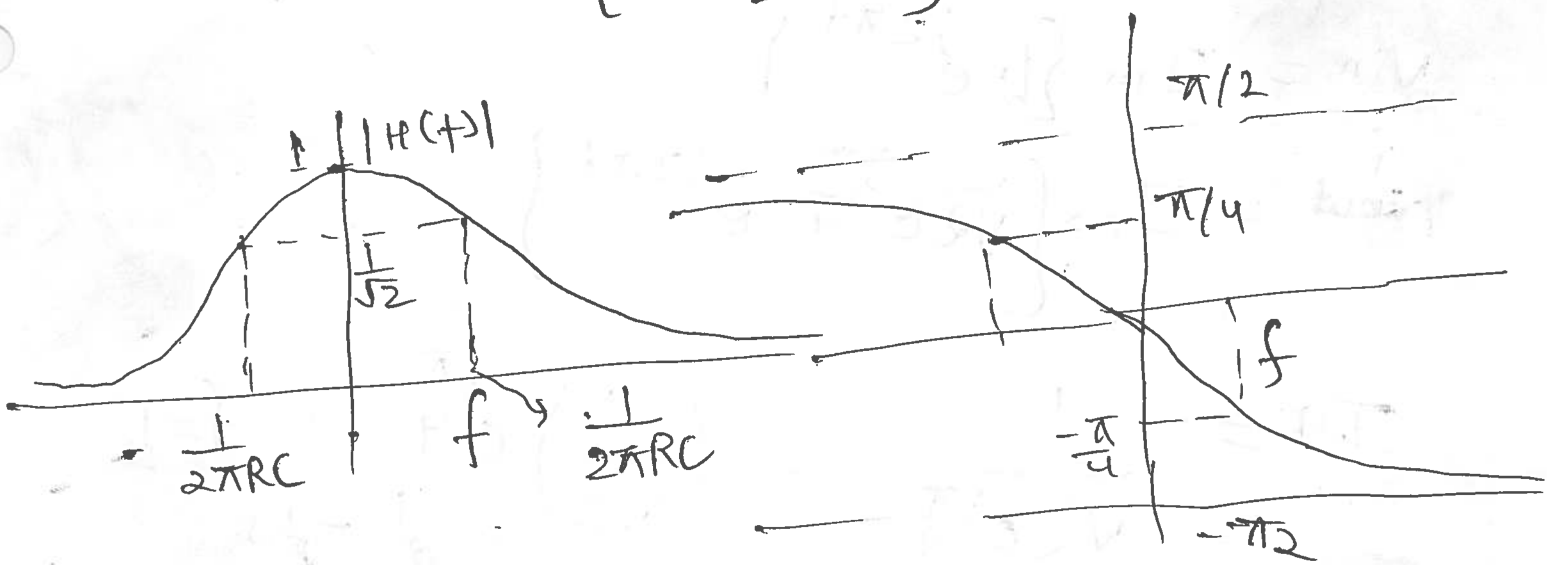
Example



$$H(f) = \frac{1}{j2\pi fRC + 1} =$$

$$|H(f)| = \frac{1}{\sqrt{1 + 4^2\pi^2 f^2 R^2 C^2}}$$

$$\angle H(f) = \tan^{-1}\left[\frac{-2\pi fRC}{1}\right]$$

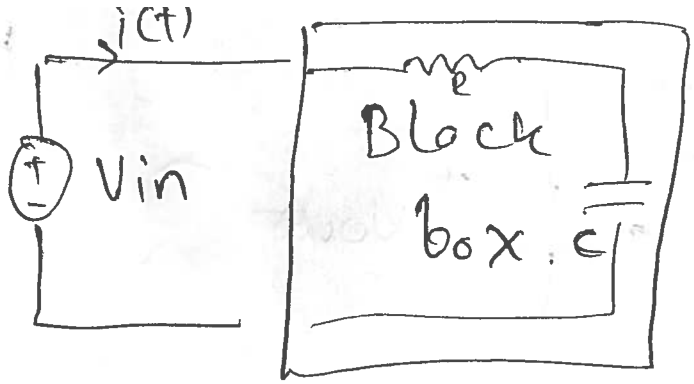


Note - 1) $|H(f)| = |H(-f)|$

2) $\angle H(-f) = -\angle H(f)$

3) cutoff frequency $= \frac{1}{2\pi RC}$

Ex. 1



a) $V_{in}(t) = \sin(2\pi \cancel{f} t)$

$i(t) = \sqrt{2}\cos\left(2\pi t - \frac{\pi}{4}\right)$

$\left(\because \sin\left(\frac{\pi}{2}+\theta\right) = \cos(\theta)\right)$

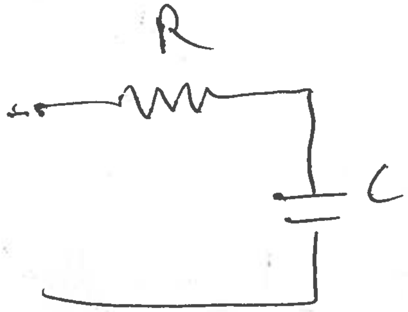
$= \sqrt{2}\sin\left(2\pi t + \frac{\pi}{4}\right)$

$V_{in} = \mathrm{Im}\left\{1\cdot e^{j2\pi t}\right\}$

$i_{\cancel{out}} = \mathrm{Im}\left\{\sqrt{2}e^{j\frac{\pi}{4}}\, e^{j2\pi t}\right\}$

$T.F = \dfrac{1}{\sqrt{2}e^{j\frac{\pi}{4}}} = \left(\dfrac{1}{\sqrt{2}}\right)e^{-j\frac{\pi}{4}}$ $\quad f = 1$

$= \frac{1}{2} - \frac{1}{2}j$



$T.F. = \dfrac{R}{} + \dfrac{1}{2\pi j f C}$

so, $R = \frac{1}{2}$, $\frac{1}{2} = \frac{1}{2\pi C}$

$\Rightarrow C = \frac{1}{\pi}$