Complex Exponential and phasors

We know

$$A\cos(\omega t+\theta) = A\left[\frac{e^{j(\omega t+\theta)}+e^{-j(\omega t+\theta)}}{2}\right] \quad \text{— (I)}$$

$$\text{or } \mathrm{Re}\left\{A e^{j(\omega t+\phi)}\right\} \quad \text{— (II)}$$

$$= \mathrm{Re}\{\underbrace{A e^{j\phi}}_{\text{phasor}}\, e^{j\omega t}\}$$

The term $Ae^{j\phi}$ ~~ref~~ is called phasor.
It is understood that the term '$e^{j\omega}$' represent the time varying sinousoidal part.

Phasors make operation on complex exponential simple.

Example: Multiplication of a phasor by a complex exponential.

$$A\cos(\omega t+\theta) \longrightarrow \underset{\text{multiply}}{\otimes} \longrightarrow \boxed{Be^{j\phi}} \rightarrow ?$$

$$\cancel{\mathrm{Re}\{Ae^{j\omega t}e^{j\theta}\} \times Be^{j\phi}\}} =$$

$$= Re\left\{ (Ae^{j\theta})\cdot(Be^{j\phi})\, e^{j\omega t} \right\}$$

$$= Re\left\{ \underbrace{ABe^{j(\theta+\phi)}}_{} \; e^{j\omega t} \right\}$$

phasor got multiplied with the complex exponentials.

$$= AB\cos(\omega t + \theta + \phi\}$$

Exple-2 Sum of phasors. - Sum of two phasor lead to another phasor because sum of sinusoidals with same frequency is also a sinusoidal with ~~same~~ that frequency.

$$A_1\cos(\omega t+\theta_1) + A_2\cos(\omega t+\theta_2)$$

$$= A_3\cos(\omega t+\theta_3)$$

find A_3 and θ_3 using complex exponeil phasor method.

$$Re\left\{ A_1 e^{j\theta_1} e^{j\omega t} + A_2 e^{j\theta_2} e^{j\omega t} \right\}$$

$$= Re\left\{ \underbrace{(A_1 e^{j\theta_1} + A_2 e^{j\theta_2})}_{A_3 e^{j\theta_3}}\, e^{j\omega t} \right\} = A_3\cos(\omega t+\theta_3$$

Thus,

$$A_3 e^{j\theta_3} = A_1 e^{j\theta_1} + A_2 e^{j\theta_2}$$

$$= A_1 \cos\theta_1 + jA_1 \sin\theta_1 + A_2 \cos\theta_2 + jA_2 \sin\theta_2$$

$$= \underbrace{A_1 \cos\theta_1 + A_2 \cos\theta_2}_{m} + j\underbrace{(A_1 \sin\theta_1 + A_2 \sin\theta_2)}_{n}$$

$$A_3 = \sqrt{m^2 + n^2}$$

$$= \sqrt{(A_1 \cos\theta_1 + A_2 \cos\theta_2)^2 + (A_1 \sin\theta_1 + A_2 \sin\theta_2)^2}$$

$$\theta_3 = \tan^{-1}\left[\frac{n}{m}\right] = \tan^{-1}\left[\frac{A_1 \sin\theta_1 + A_2 \sin\theta_2}{A_1 \cos\theta_1 + A_2 \cos\theta_2}\right]$$

Example-3 Differentiation of phasors.

$$A\cos(\omega t+\theta) = Re\{\underbrace{Ae^{j\theta}}_{\text{phasor}} e^{j\omega t}\}$$

$$\frac{d}{dt}[A\cos(\omega t+\theta)] = \frac{d}{dt}\left(Re\{Ae^{j\theta} e^{j\omega t}\}\right)$$

$$= Re\left\{\frac{d}{dt}(Ae^{j\theta} e^{j\omega t})\right\}$$

$$= Re\{\underbrace{Aj\omega e^{j\theta}}_{\text{New phasor}} e^{j\omega t}\}$$

$$= Re\{A\omega e^{j\theta} e^{j\frac{\pi}{2}} e^{j\omega t}\}$$

$$= A\omega \cos(\omega t + \theta + \tfrac{\pi}{2}) = -A\omega \sin(\omega t+\theta)$$

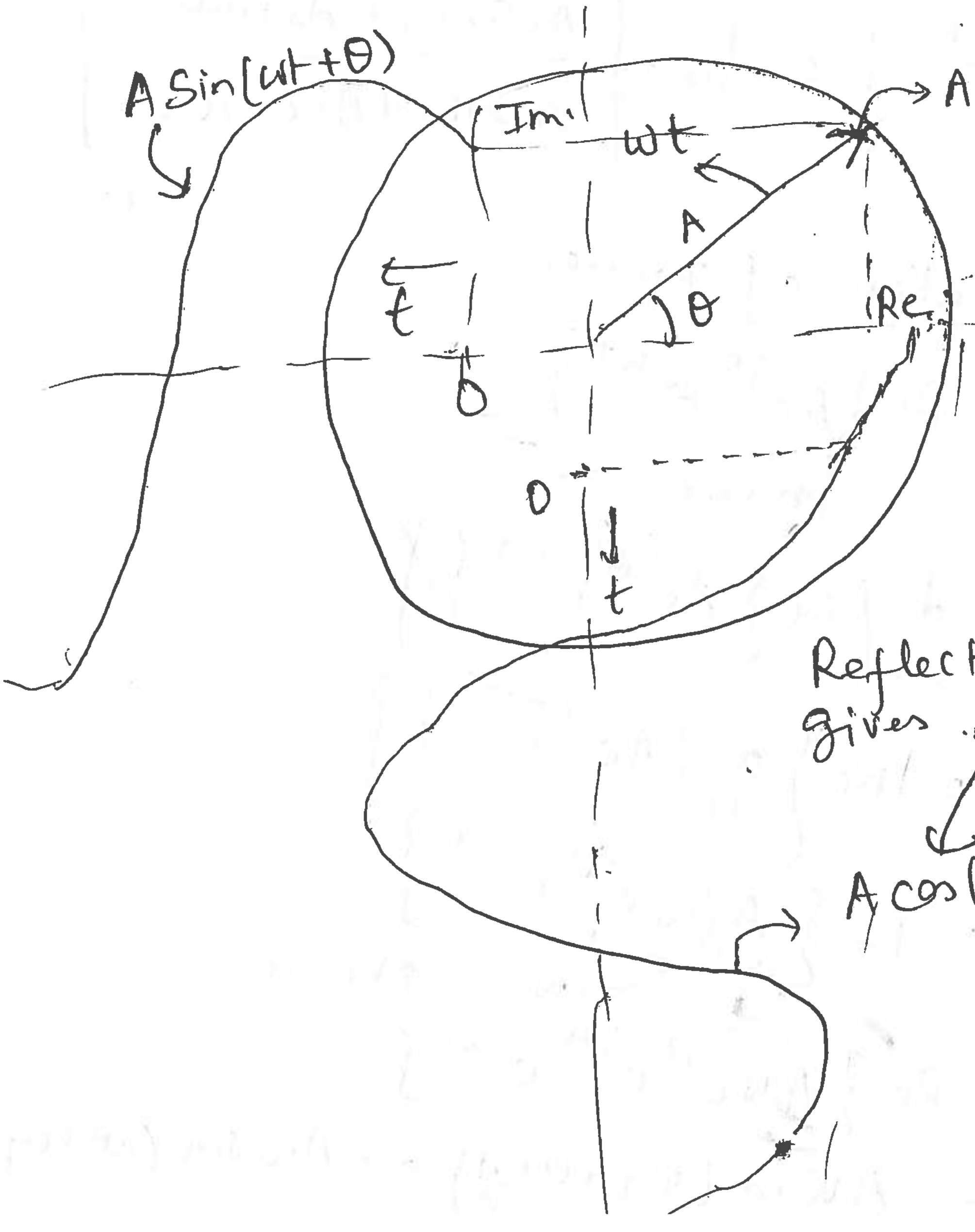
thus, ~~by differentiating~~,

thus, differentiation is equivalent to <u>multiply</u> phasor by $(j\omega)$.

Similarly, integration is equivalent to <u>dividing</u> phasor by $(j\omega)$

~~In Summary~~.

Geometric interpretation



$Ae^{j\theta}$ (phasor) is rotating at the rate ~~at~~ ω. At any time 't', the location is represented by $\underline{Ae^{j\theta}e^{j\omega t}}$.

Reflection on real axis gives

$A\cos(\omega t+\theta)$

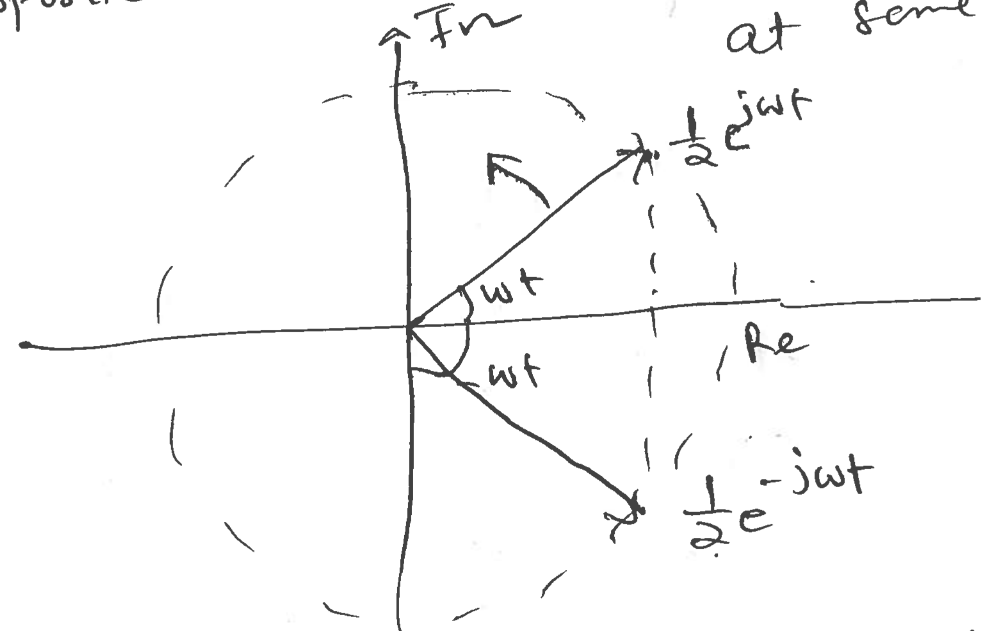
In summary, linear systems does not change the frequency 'w' of signals. eg addition, multiplication (amplifier), differentiation, integration and thus phasors are useful to anlyze the response of such linear systems.

Example - cos wt and negative frequency.

$$\cos(wt) = \frac{1}{2}\left[e^{jwt} + e^{-jwt}\right]$$

clearly, cos has two frequency 'w' and '-w'. In terms of ~~phasors~~ geometry, the correspond to two phasors rotating in opposite direction.

at some time 't'



Component on the real axis adds, and those on imaginary axis cancels.