Signals & System

What signals we know?

① Sinusoidals

$$s(t) = A\cos(2\pi f_0 t + \theta)$$

② Complex Exponentials

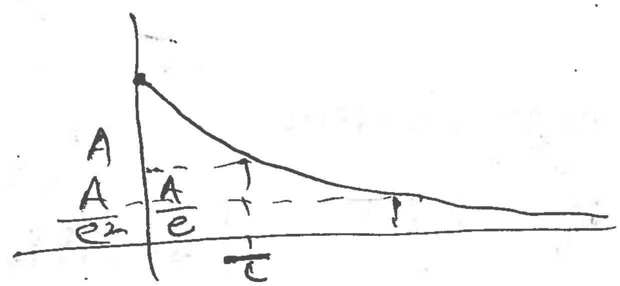
$$s(t) = A e^{j(2\pi f_0 t + \phi)}$$

Re

Im

③ Real exponential

$$s(t) = A e^{-\frac{t}{\tau}}$$



③ Unit step

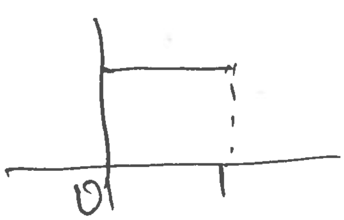
④ Unit Pulse

⑤ Square Wave

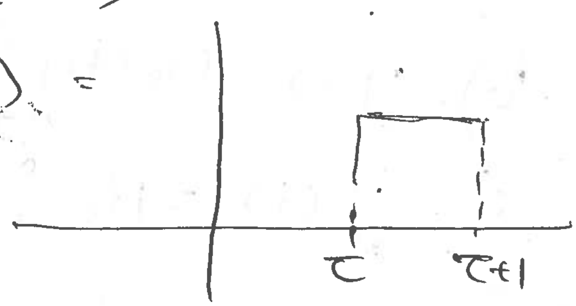
Example 1: Given $x(t)$, draw $x(t-\tau)$ (delay), $x(t+\tau)$ advance

$x(2t)$, $x(t/2)$, $x(2t-1)$

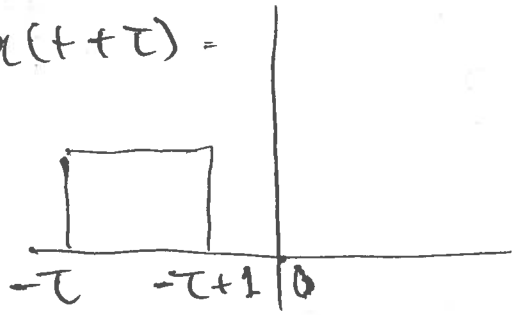
Ans - $x(t) =$



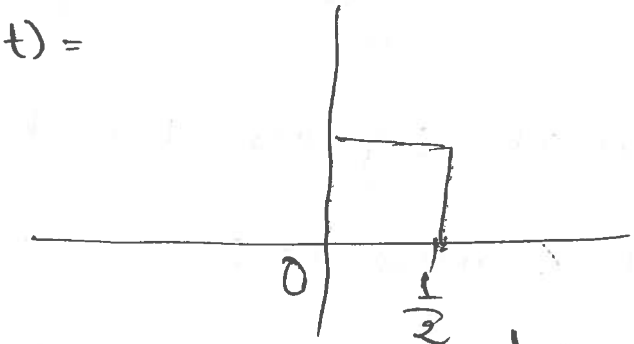
$x(t-\tau) =$



$x(t+\tau) =$

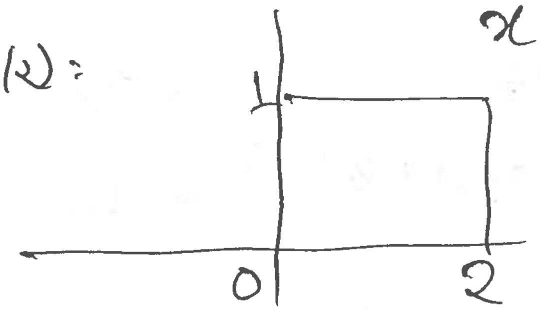


$x(2t) =$

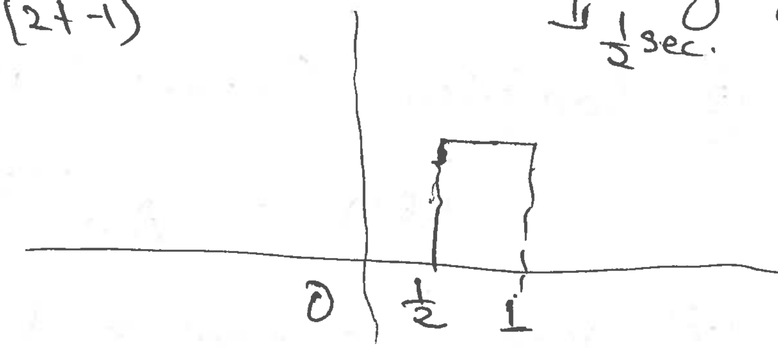


delay by $\frac{1}{2}$ sec.

$x(t/2) =$



$x(2t-1)$



Linear System –

if. $x_i(t) \longrightarrow$ System $\longrightarrow y_i(t)$ $\quad i = 1, 2 \ldots$

then $\sum a_i x_i(t) \longrightarrow$ System $\longrightarrow \sum a_i y_i(t)$

Example: Prove that these system are linear?

a) Amplifier with gain G (trivial)

b) $y(t) = [x(t)]^2$

c) $y(t) = \mathrm{Re}\{x(t)\}$

Time invariant System

if, $x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$

then, $x(t-\tau) \longrightarrow \boxed{\text{System}} \longrightarrow y(t-\tau)$

Example: is $y(t) = t\,x(t)$ a time invariant system?

$x(t) \longrightarrow \boxed{\quad} \longrightarrow t\,x(t)$

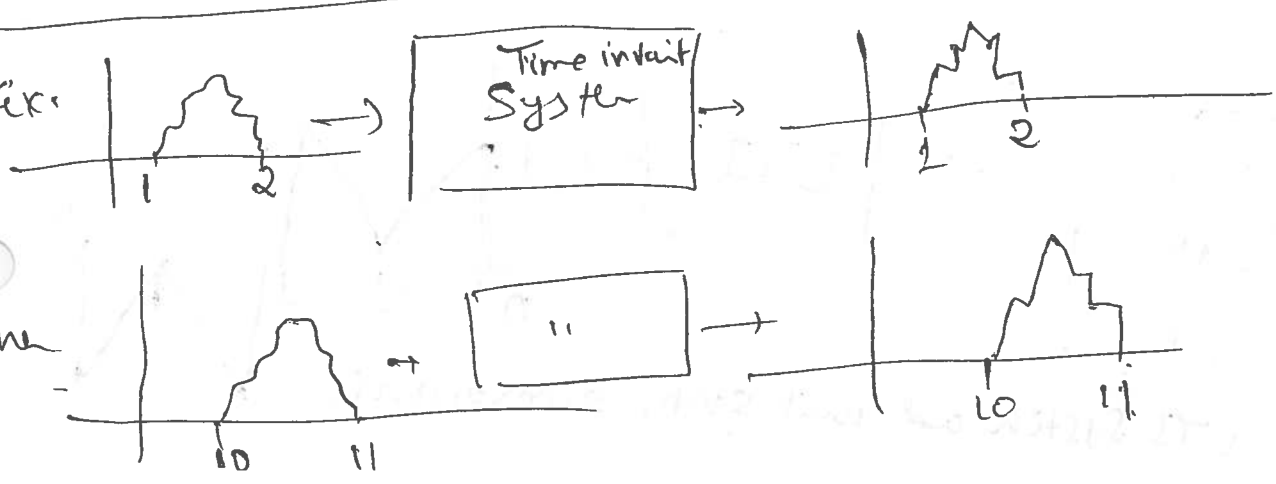
$x(t-\tau) \longrightarrow \boxed{\quad} \longrightarrow t\,x(t-\tau).$

so, $S(x(t-\tau)) = t\,x(t-\tau)$

but, $y(t-\tau) = (t-\tau)\,x(t-\tau) \neq S(x(t-\tau))$

So, not time invariant

Ex:



then

Linear time invariant System

Example:-

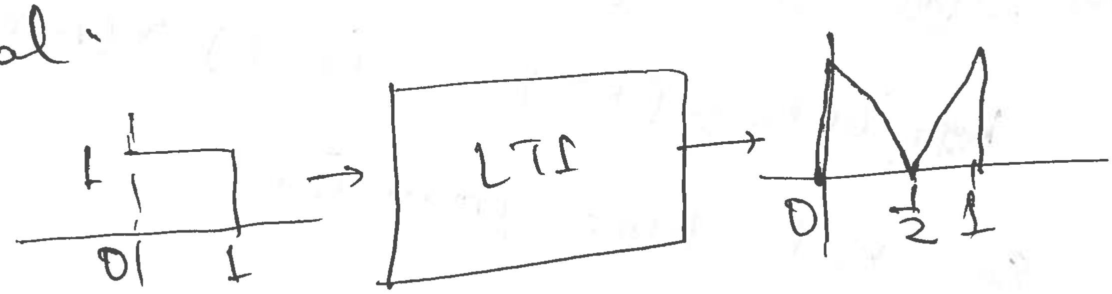
if $\delta(t) \rightarrow \boxed{\text{LTI.}} \rightarrow h(t)$

then $\sum_{i=1}^{N} A_i \delta(t-\tau_i) \rightarrow \boxed{\text{L.T.I.}} \rightarrow \sum_{i=1}^{N} A_i h(t-\tau_i)$

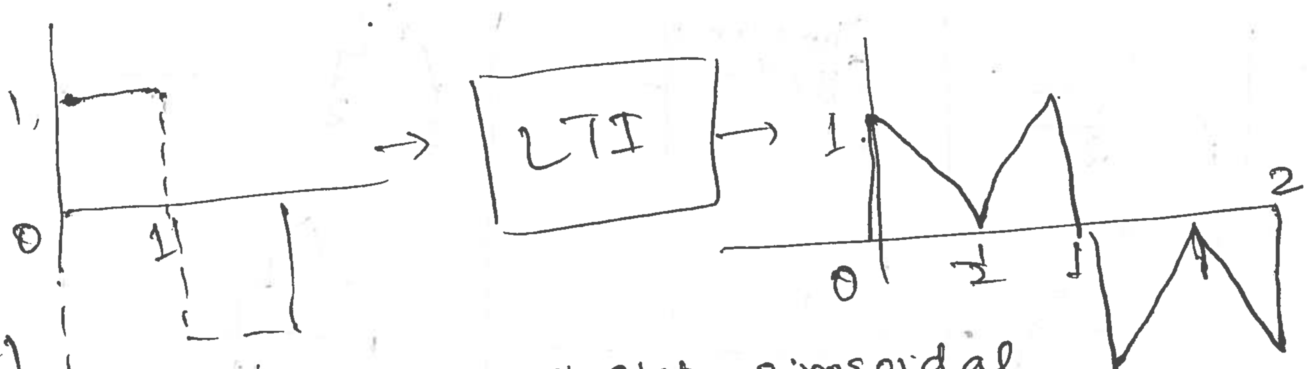
What does this mean?

If you can decompose a complicated signal as sum of weighted and delayed basic signal ($\delta(t)$), then you can easily know the response of the LTI system for the complicated signal.

Example: if



then



Example - LTI System and unit step, sinusoidal

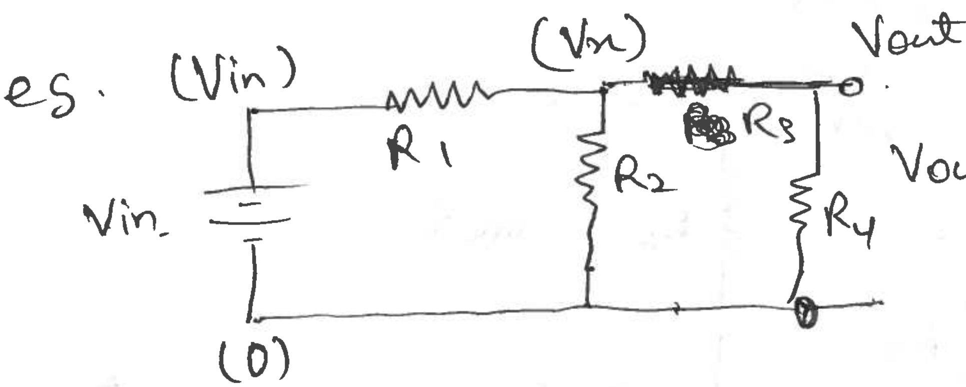
Circuit theory

Steps to solving a circuit.

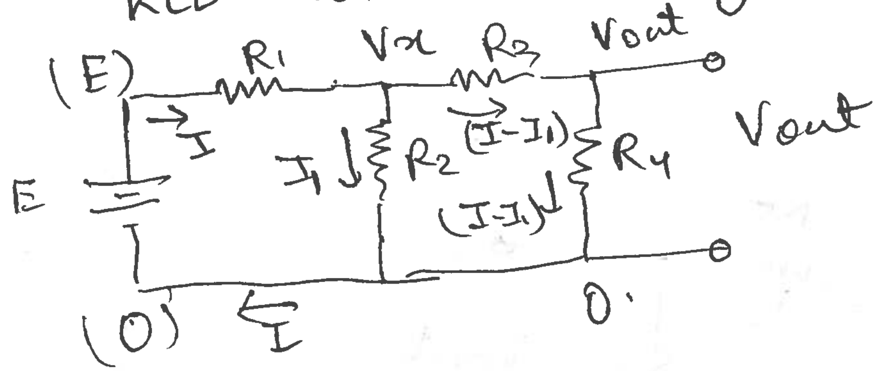
1. Assign voltage to nodes, all known and unknown voltage. Set one voltage as 0V, the choise is arbitrary but generally the negative terminal of voltage source is considered as 0V.

~~Vx = Vout~~

eg.



2. Assign current, with direction. Apply KCL while choosing variable names.



~~check current~~

3. Voltage drop in the direction of current flow so, apply KVC and ohms law with proper sign,

V. drop - -ve, V. Rise - +ve

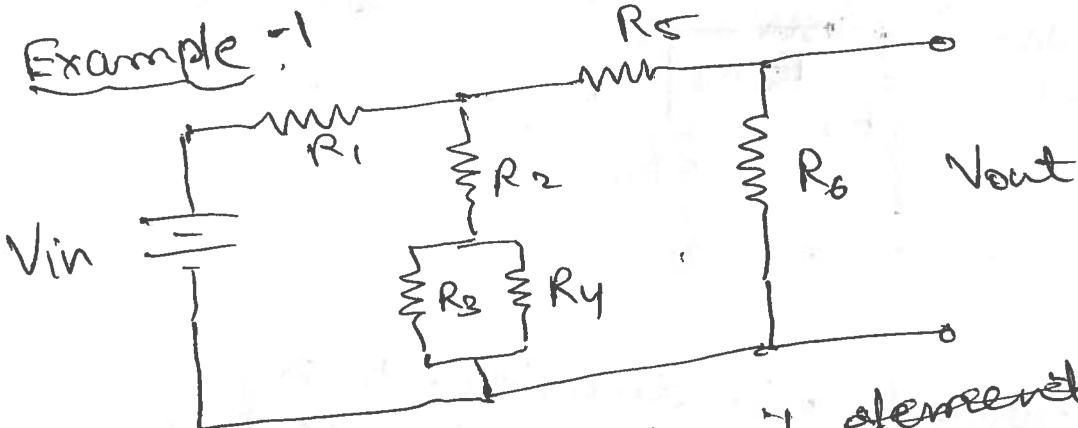
(4) If current flowing through two circuit element is same, they are in series

If voltage across two circuit element is same, they are in parallel.

Simplify circuit as much as possible.

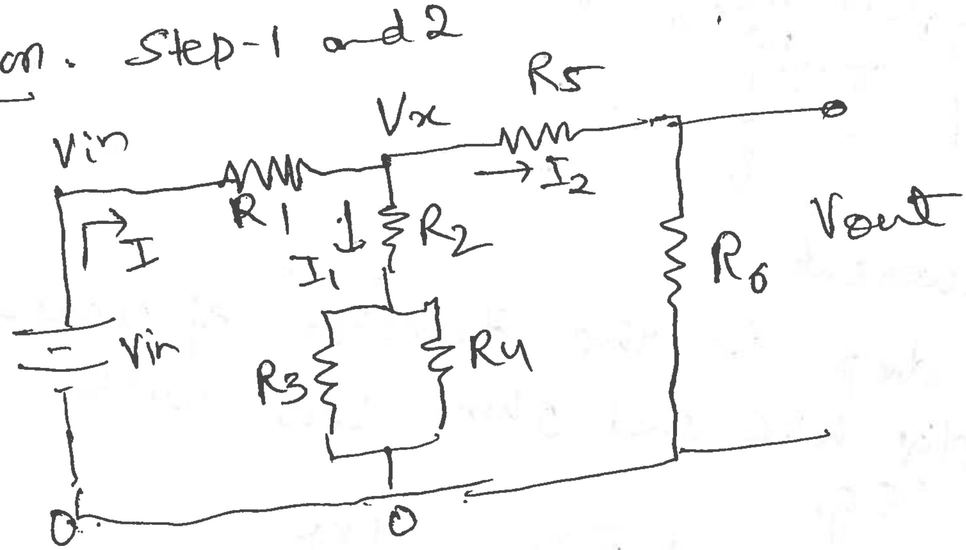
(5) Apply KVL and ohm law to determine all needed parameters. (if needed).

Example -1



Solve for all circuit ~~elements~~. current and voltage.

Solution. Step-1 and 2



Step-3, 4

$$V_x = V_{in} - IR_1$$

$$I = \frac{V_{in}}{\underbrace{R_1 + (R_2 + R_3||R_4) || (R_5+R_6)}_{\to R_{eq}}} \quad - ①.$$

$$\text{So, } V_x = V_{in}\left[\frac{(R_2 + R_3||R_4)||(R_5+R_6)}{R_{eq}}\right] \quad - ②$$

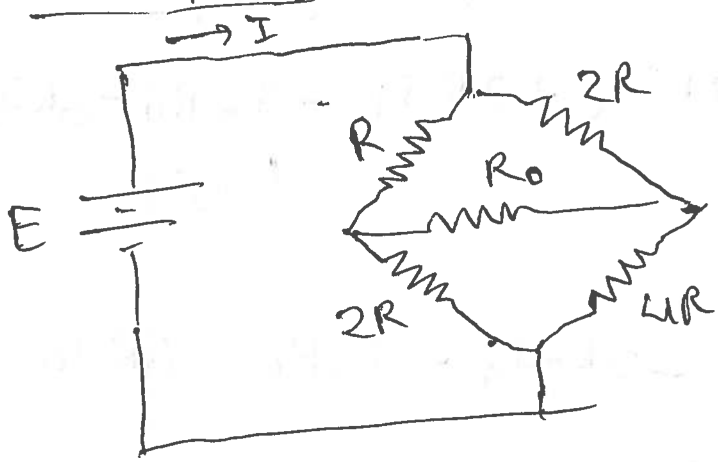
$$I_1 = \frac{V_x}{R_2 + R_3||R_4}, \quad I_2 = \frac{V_x}{R_5+R_6} \to ②$$

$$I_1 + I_2 = I \quad (\text{Kcl})$$

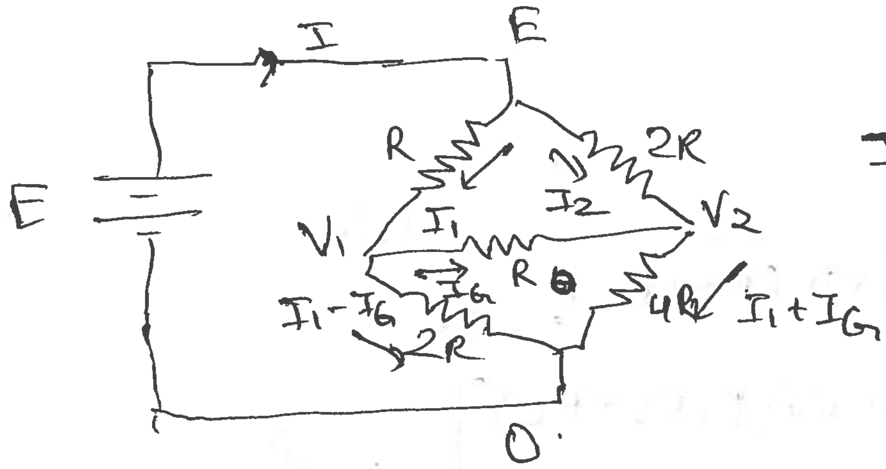
$$V_{out} = R_6 \times I_2 = \frac{R_6 V_x}{R_5+R_6} \quad (\text{Voltage divider}) \quad - ④$$

Example-2

find I,



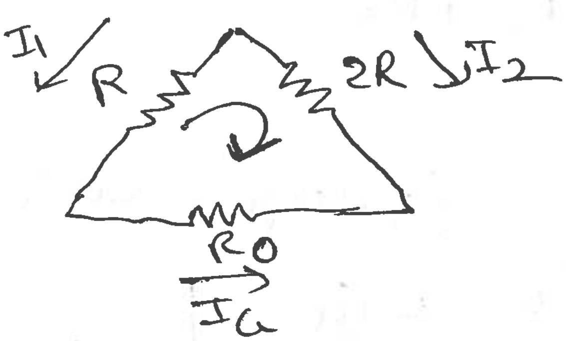
Step 1 and 2



$I_1 + I_2 = I$

Step. 3 & 4 $\longrightarrow$ So series or parallel

Step. 5 Apply KVL



$-2I_2R + R_0 I_G + I_1 R = 0$
(drop) (rise) (rise)
$\hookrightarrow$ (1)

$-I_G R_0 \quad -4R(I_2 + I_G) + 2R(I_1 - I_G)$
(drop) (drop) (rise)
$\hookrightarrow$ (2)

~~eq 2~~ $= -4RI_2 + 2RI_1 - I_G R_0 - 6RI_G =$
$\hookrightarrow$ (2).

$(-2) \times$ eq (1) + eq (2)
$\hookrightarrow \quad 0 + 0 - 2R_0 I_G - I_G R_0 - 6RI_G = 0$

$\Rightarrow \quad I_G = 0.$

As, $I_G = 0$, we can effectively remove R_0.

Thus, $I = \dfrac{E}{3R \| 6R} = \dfrac{E}{2R}$ Ans.