

Assignment 5

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Problem 1

a)

```
library(ISLR)
fix(College)
sum(is.na(College)) # to determine missing elements
```

```
## [1] 0
```

```
library(leaps)
```

```
## Warning: package 'leaps' was built under R version 3.4.4
```

```
regit.full=regsubsets(Apps~., College,nvmax=19)
reg.summary=summary(regit.full)
reg.summary
```

```
## Subset selection object
## Call: regsubsets.formula(Apps ~ ., College, nvmax = 19)
## 17 Variables (and intercept)
##           Forced in Forced out
## PrivateYes      FALSE      FALSE
## Accept          FALSE      FALSE
## Enroll          FALSE      FALSE
## Top10perc       FALSE      FALSE
## Top25perc       FALSE      FALSE
## F.Undergrad     FALSE      FALSE
## P.Undergrad     FALSE      FALSE
## Outstate        FALSE      FALSE
## Room.Board      FALSE      FALSE
## Books           FALSE      FALSE
## Personal        FALSE      FALSE
## PhD             FALSE      FALSE
## Terminal        FALSE      FALSE
## S.F.Ratio       FALSE      FALSE
## perc.alumni     FALSE      FALSE
## Expend          FALSE      FALSE
## Grad.Rate       FALSE      FALSE
## 1 subsets of each size up to 17
## Selection Algorithm: exhaustive
##           PrivateYes Accept Enroll Top10perc Top25perc F.Undergrad
## 1  ( 1 )  " "          "*"    " "      " "        " "
## 2  ( 1 )  " "          "*"    " "      " "        " "
## 3  ( 1 )  " "          "*"    " "      " "        " "
## 4  ( 1 )  " "          "*"    " "      " "        " "
## 5  ( 1 )  " "          "*"    "*"      " "        " "
```

```

## 6 ( 1 ) " "      "*"      "*"      "*"      " "      " "
## 7 ( 1 ) " "      "*"      "*"      "*"      "*"      " "
## 8 ( 1 ) "*"      "*"      "*"      "*"      " "      " "
## 9 ( 1 ) "*"      "*"      "*"      "*"      "*"      " "
## 10 ( 1 ) "*"      "*"      "*"      "*"      "*"      " "
## 11 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"
## 12 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"
## 13 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"
## 14 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"
## 15 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"
## 16 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"
## 17 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"
##
##      P.Undergrad Outstate Room.Board Books Personal PhD Terminal
## 1 ( 1 ) " "      " "      " "      " "      " "      " " " "
## 2 ( 1 ) " "      " "      " "      " "      " "      " " " "
## 3 ( 1 ) " "      " "      " "      " "      " "      " " " "
## 4 ( 1 ) " "      "*"      " "      " "      " "      " " " "
## 5 ( 1 ) " "      "*"      " "      " "      " "      " " " "
## 6 ( 1 ) " "      "*"      "*"      " "      " "      " " " "
## 7 ( 1 ) " "      "*"      "*"      " "      " "      " " " "
## 8 ( 1 ) " "      "*"      "*"      " "      " "      "*" " "
## 9 ( 1 ) " "      "*"      "*"      " "      " "      "*" " "
## 10 ( 1 ) " "      "*"      "*"      " "      " "      "*" " "
## 11 ( 1 ) " "      "*"      "*"      " "      " "      "*" " "
## 12 ( 1 ) "*"      "*"      "*"      " "      " "      "*" " "
## 13 ( 1 ) "*"      "*"      "*"      " "      " "      "*" " "
## 14 ( 1 ) "*"      "*"      "*"      " "      " "      "*" "*"
## 15 ( 1 ) "*"      "*"      "*"      " "      "*"      "*" "*"
## 16 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*" "*"
## 17 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*" "*"
##
##      S.F.Ratio perc.alumni Expend Grad.Rate
## 1 ( 1 ) " "      " "      " "      " "
## 2 ( 1 ) " "      " "      " "      " "
## 3 ( 1 ) " "      " "      "*"      " "
## 4 ( 1 ) " "      " "      "*"      " "
## 5 ( 1 ) " "      " "      "*"      " "
## 6 ( 1 ) " "      " "      "*"      " "
## 7 ( 1 ) " "      " "      "*"      " "
## 8 ( 1 ) " "      " "      "*"      " "
## 9 ( 1 ) " "      " "      "*"      " "
## 10 ( 1 ) " "      " "      "*"      "*"
## 11 ( 1 ) " "      " "      "*"      "*"
## 12 ( 1 ) " "      " "      "*"      "*"
## 13 ( 1 ) "*"      " "      "*"      "*"
## 14 ( 1 ) "*"      " "      "*"      "*"
## 15 ( 1 ) "*"      " "      "*"      "*"
## 16 ( 1 ) "*"      " "      "*"      "*"
## 17 ( 1 ) "*"      "*"      "*"      "*"

```

```
par(mfrow=c(2,2))
```

```

plot(reg.summary$rss,xlab="Number of Variables",ylab="RSS",type = "l",col="red")
b=which.min(reg.summary$rss)
points(b,reg.summary$rss[b],col="black", pch=20)

```

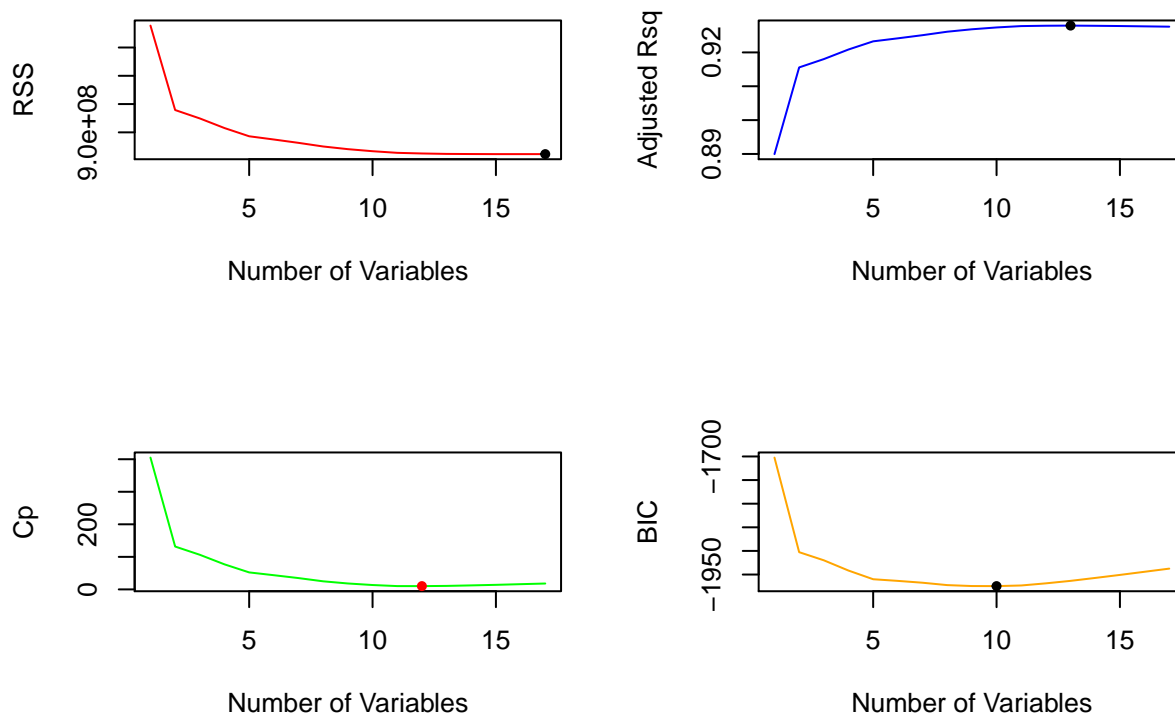
```
plot(reg.summary$adjr2,xlab="Number of Variables",ylab="Adjusted Rsq",type = "l",col="blue")
a=which.max(reg.summary$adjr2)
points(a,reg.summary$adjr2[a],col="black", pch=20)
print(a)
```

```
## [1] 13
```

```
plot(reg.summary$cp,xlab="Number of Variables",ylab="Cp",type = "l",col="green")
c=which.min(reg.summary$cp)
points(c,reg.summary$cp[c],col="red", pch=20)
print(c)
```

```
## [1] 12
```

```
plot(reg.summary$bic,xlab="Number of Variables",ylab="BIC",type = "l",col="orange")
d=which.min(reg.summary$bic)
points(d,reg.summary$bic[d],col="black", pch=20)
```



```
print(d)
```

```
## [1] 10
```

```
coef(regit.full,a)    #coeffecients of Adjusted R2
```

```
## (Intercept) PrivateYes Accept Enroll Top10perc
## -440.74148270 -484.77261885 1.58542302 -0.87824288 50.41461998
## Top25perc F.Undergrad P.Undergrad Outstate Room.Board
## -14.63667155 0.05762769 0.04642270 -0.08823311 0.14696204
```

```
##          PhD      S.F.Ratio      Expend      Grad.Rate
## -10.91804823  15.15475056   0.07786425   8.58578735
```

```
coef(regit.full,c)      #coeffecients of Cp
```

```
## (Intercept) PrivateYes      Accept      Enroll      Top10perc
## -157.28685883 -511.78760196  1.58691470 -0.88265385  50.41131660
##      Top25perc F.Undergrad P.Undergrad      Outstate      Room.Board
## -14.74735373  0.05945481  0.04593068 -0.09017643  0.14776586
##          PhD      Expend      Grad.Rate
## -10.70502848  0.07246655  8.63961002
```

```
coef(regit.full,d)      #coeffecients of BIC
```

```
## (Intercept) PrivateYes      Accept      Enroll      Top10perc
## -100.51668243 -575.07060789  1.58421887 -0.56220848  49.13908916
##      Top25perc      Outstate      Room.Board      PhD      Expend
## -13.86531103 -0.09466457  0.16373674 -10.01608705  0.07273776
##      Grad.Rate
##      7.33268904
```

As per the above analysis for:

1. Cp 12 variables are selected which include PrivateYes, Accept, Enroll, Top10perc, Top25perc, F.Undergrad, P.Undergrad, Outstate, Room.Board, PhD, Expend, Grad.Rate
2. BIC 10 variables are selected which include PrivateYes, Accept, Enroll, Top10perc, Top25perc, Outstate, Room.Board, PhD, Expend, Grad.Rate
3. Adj R² 13 variables are selected which include PrivateYes, Accept, Enroll, Top10perc, Top25perc, F.Undergrad, P.Undergrad, Outstate, Room.Board, PhD, S.F.Ratio, Expend, Grad.Rate

b) Forward Stepwise Selection

```
regit.fwd=regsubsets(Apps~., College,nvmax=19, method = "forward")
reg.summary.fwd=summary(regit.fwd)

par(mfrow=c(2,2))
plot(reg.summary.fwd$adjr2,xlab="Number of Variables",ylab="Adjusted Rsq",type = "l",col="blue")
a=which.max(reg.summary.fwd$adjr2)
points(a,reg.summary.fwd$adjr2[a],col="black", pch=20)
print(a)

## [1] 13

plot(reg.summary.fwd$cp,xlab="Number of Variables",ylab="Cp",type = "l",col="green")
c=which.min(reg.summary.fwd$cp)
points(c,reg.summary.fwd$cp[c],col="red", pch=20)
print(c)

## [1] 12

plot(reg.summary.fwd$bic,xlab="Number of Variables",ylab="BIC",type = "l",col="orange")
d=which.min(reg.summary.fwd$bic)
points(d,reg.summary.fwd$bic[d],col="black", pch=20)
print(d)

## [1] 10
```

```
coef(regit.full,a)      #coeffecients of Adjusted R2
```

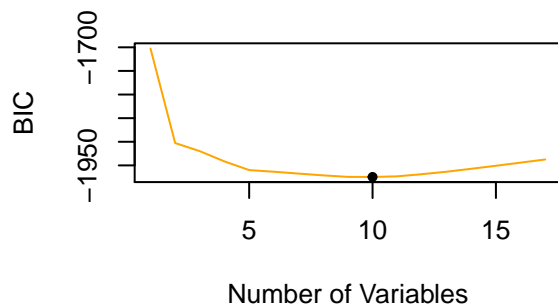
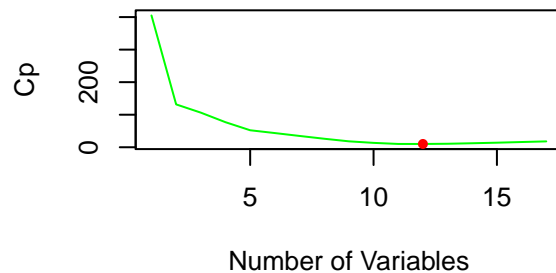
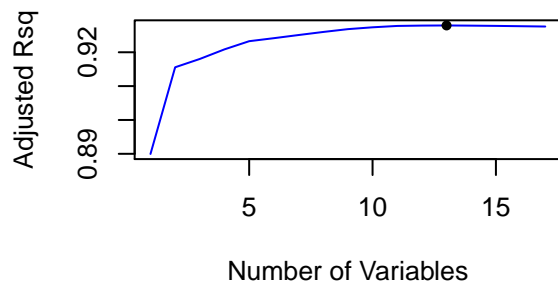
```
## (Intercept) PrivateYes Accept Enroll Top10perc
## -440.74148270 -484.77261885 1.58542302 -0.87824288 50.41461998
## Top25perc F.Undergrad P.Undergrad Outstate Room.Board
## -14.63667155 0.05762769 0.04642270 -0.08823311 0.14696204
## PhD S.F.Ratio Expend Grad.Rate
## -10.91804823 15.15475056 0.07786425 8.58578735
```

```
coef(regit.full,c)      #coeffecients of Cp
```

```
## (Intercept) PrivateYes Accept Enroll Top10perc
## -157.28685883 -511.78760196 1.58691470 -0.88265385 50.41131660
## Top25perc F.Undergrad P.Undergrad Outstate Room.Board
## -14.74735373 0.05945481 0.04593068 -0.09017643 0.14776586
## PhD Expend Grad.Rate
## -10.70502848 0.07246655 8.63961002
```

```
coef(regit.full,d)      #coeffecients of BIC
```

```
## (Intercept) PrivateYes Accept Enroll Top10perc
## -100.51668243 -575.07060789 1.58421887 -0.56220848 49.13908916
## Top25perc Outstate Room.Board PhD Expend
## -13.86531103 -0.09466457 0.16373674 -10.01608705 0.07273776
## Grad.Rate
## 7.33268904
```



b) Backward stepwise Selection

```
regit.bwd=regsubsets(Apps~., College,nvmax=19, method="backward")
reg.summary.bwd=summary(regit.bwd)

par(mfrow=c(2,2))
plot(reg.summary.bwd$adjr2,xlab="Number of Variables",ylab="Adjusted Rsq",type = "l",col="blue")
a=which.max(reg.summary.bwd$adjr2)
points(a,reg.summary.bwd$adjr2[a],col="black", pch=20)
print(a)

## [1] 13

plot(reg.summary.bwd$cp,xlab="Number of Variables",ylab="Cp",type = "l",col="green")
c=which.min(reg.summary.bwd$cp)
points(c,reg.summary.bwd$cp[c],col="red", pch=20)
print(c)

## [1] 12

plot(reg.summary.bwd$bic,xlab="Number of Variables",ylab="BIC",type = "l",col="orange")
d=which.min(reg.summary.bwd$bic)
points(d,reg.summary.bwd$bic[d],col="black", pch=20)
print(d)

## [1] 10
```

```
coef(regit.full,a)      #coeffecients of Adjusted R2
```

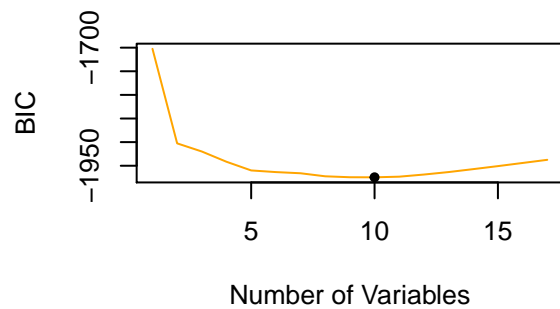
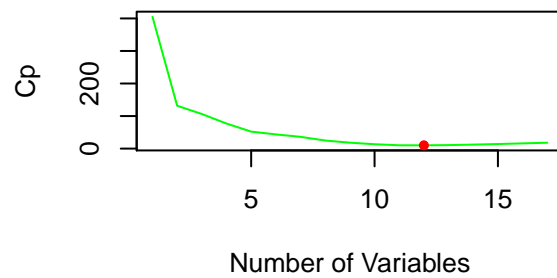
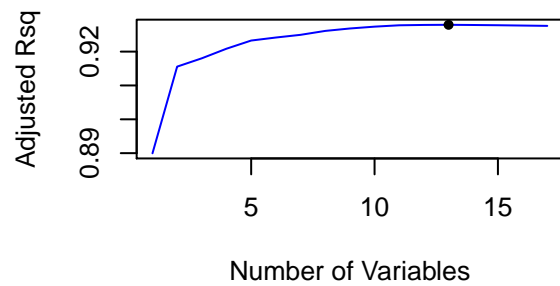
##	(Intercept)	PrivateYes	Accept	Enroll	Top10perc
##	-440.74148270	-484.77261885	1.58542302	-0.87824288	50.41461998
##	Top25perc	F.Undergrad	P.Undergrad	Outstate	Room.Board
##	-14.63667155	0.05762769	0.04642270	-0.08823311	0.14696204
##	PhD	S.F.Ratio	Expend	Grad.Rate	
##	-10.91804823	15.15475056	0.07786425	8.58578735	

```
coef(regit.full,c)      #coeffecients of Cp
```

##	(Intercept)	PrivateYes	Accept	Enroll	Top10perc
##	-157.28685883	-511.78760196	1.58691470	-0.88265385	50.41131660
##	Top25perc	F.Undergrad	P.Undergrad	Outstate	Room.Board
##	-14.74735373	0.05945481	0.04593068	-0.09017643	0.14776586
##	PhD	Expend	Grad.Rate		
##	-10.70502848	0.07246655	8.63961002		

```
coef(regit.full,d)      #coeffecients of BIC
```

##	(Intercept)	PrivateYes	Accept	Enroll	Top10perc
##	-100.51668243	-575.07060789	1.58421887	-0.56220848	49.13908916
##	Top25perc	Outstate	Room.Board	PhD	Expend
##	-13.86531103	-0.09466457	0.16373674	-10.01608705	0.07273776
##	Grad.Rate				
##	7.33268904				



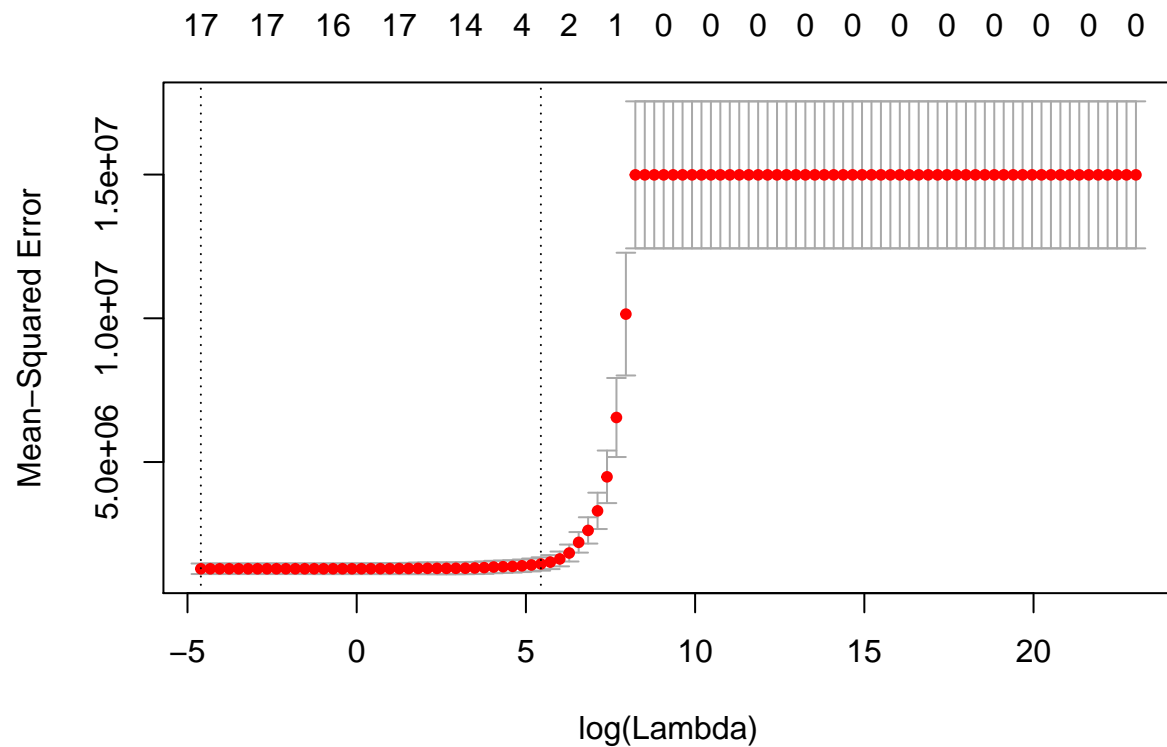
The answers for forward and backward stepwise selection and for a) part that is best subset selection are exactly same.

c)

```
library(glmnet)

## Warning: package 'glmnet' was built under R version 3.4.4
## Loading required package: Matrix
## Loading required package: foreach
## Warning: package 'foreach' was built under R version 3.4.4
## Loaded glmnet 2.0-16
x=model.matrix(Apps~.,College)[,-1]
y=College$Apps
grid=10^seq(10,-2,length=100)

cv.out=cv.glmnet(x,y,alpha=1,lambda = grid)
plot(cv.out)
```



```
bestlam=cv.out$lambda.min
bestlam
```

```
## [1] 0.01
```

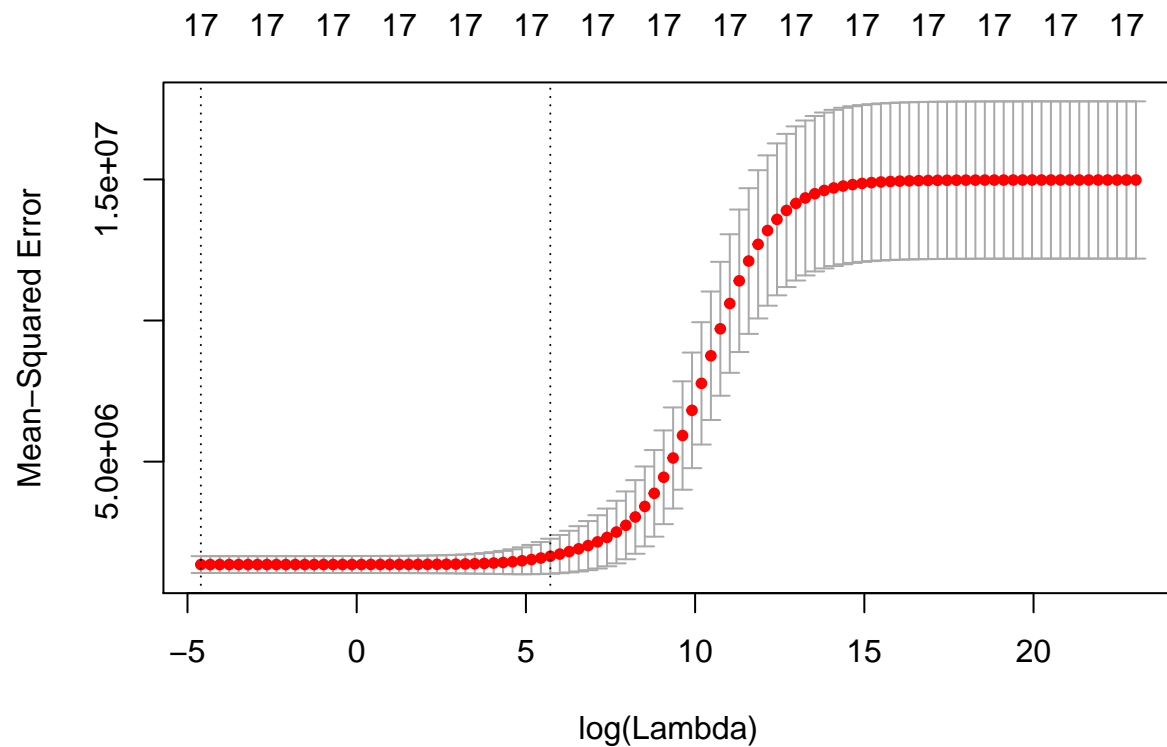
```
lasso.mod=glmnet(x,College$Apps,alpha=1)
predict(lasso.mod, s=bestlam,type = "coefficients")[1:18,]
```

```
## (Intercept) PrivateYes Accept Enroll Top10perc
## -471.39372069 -491.04485135 1.57033288 -0.75961467 48.14698891
## Top25perc F.Undergrad P.Undergrad Outstate Room.Board
## -12.84690694 0.04149116 0.04438973 -0.08328388 0.14943472
## Books Personal PhD Terminal S.F.Ratio
## 0.01532293 0.02909954 -8.39597537 -3.26800340 14.59298267
## perc.alumni Expend Grad.Rate
## -0.04404771 0.07712632 8.28950241
```

```
Lambda=0.3764936
```

d)

```
cv.out=cv.glmnet(x,y,alpha=0,lambda = grid)
plot(cv.out)
```

```
bestlam=cv.out$lambda.min
bestlam
```

```
## [1] 0.01
```

```
##refit using best lambda
```

```
lasso.mod=glmnet(x,College$Apps,alpha=0)
```

```
predict(lasso.mod, s=bestlam,type = "coefficients")[1:18,]
```

```
## (Intercept) PrivateYes Accept Enroll Top10perc
## -1.468326e+03 -5.278781e+02 1.004588e+00 4.313442e-01 2.580619e+01
## Top25perc F.Undergrad P.Undergrad Outstate Room.Board
## 5.501092e-01 7.258520e-02 2.420595e-02 -2.407454e-02 1.987732e-01
## Books Personal PhD Terminal S.F.Ratio
## 1.285477e-01 -8.146131e-03 -4.028284e+00 -4.811071e+00 1.302180e+01
## perc.alumni Expend Grad.Rate
## -8.544783e+00 7.589013e-02 1.126699e+01
```

```
lambda=0.01
```

e)

i)

```
set.seed(1)
attach(College)
```

```
x=model.matrix(Apps~.,College)[,-1]
y=College$Apps
train=sample(1:nrow(x),nrow(x)/2)
test=(-train)
y.test=College[test,]
x.train=College[train,]
```

```
regit.full=regsubsets(Apps~., data=x.train,nvmax=19)
reg.summary=summary(regit.full)
coef(regit.full,which.max(reg.summary$adjr2))
```

```
##      (Intercept)      PrivateYes      Accept      Enroll      Top10perc
##      84.95670099 -691.04103152      1.67873705      -0.86164941      66.92631417
##      Top25perc      Outstate      Room.Board      PhD      Expend
##      -22.35416377      -0.09482472      0.24520032     -10.14399113      0.03783190
##      Grad.Rate
##      6.45828153
```

```
a=which.max(reg.summary$adjr2)
a
```

```
## [1] 10
```

```
glm.fit=glm(Apps~Private+Accept+Enroll+Top10perc+Top25perc+Outstate+Room.Board+PhD+Expend+Grad.Rate,data=x.train,y=y.train)
pred.glm=predict(glm.fit,y.test)
mean((pred.glm-y.test$Apps)^2)
```

```
## [1] 1078371
```

The best model contains 10 predictors. Test Error=1078371

ii) Fit a lasso

```
train=model.matrix(Apps ~., data = x.train)
test = model.matrix(Apps ~., data = y.test)
fit.lasso = glmnet(train, x.train$Apps, alpha = 1, lambda = grid, thresh = 1e-12)
cv.lasso = cv.glmnet(train, x.train$Apps, alpha = 1, lambda = grid, thresh = 1e-12)
bestlam.lasso = cv.lasso$lambda.min
bestlam.lasso
```

```
## [1] 174.7528
```

```
pred.lasso=predict(fit.lasso,s=bestlam.lasso,newx=test)
mean((pred.lasso-y.test$Apps)^2)
```

```
## [1] 1117158
```

Test Error=1038776

iii) Fit a ridge

```
train = model.matrix(Apps ~., data = x.train)
test = model.matrix(Apps ~., data = y.test)
```

```

grid=10^seq(10,-2,length=100)
fit.ridge = glmnet(train, x.train$Apps, alpha = 0, lambda = grid, thresh =
1e-12)
cv.ridge = cv.glmnet(train, x.train$Apps, alpha = 0, lambda = grid, thresh =
1e-12)
bestlam.ridge = cv.ridge$lambda.min
bestlam.ridge

```

```
## [1] 0.01321941
```

```

pred.ridge=predict(fit.ridge,s=bestlam.ridge,newx=test)
mean((pred.ridge-y.test$Apps)^2)

```

```
## [1] 1108509
```

Test Error=1108509 Based on the MSE, Lasso has a better performance amongst the three models

Problem 2

a)

```

set.seed(1)
train=sample(1:nrow(Carseats),nrow(Carseats)/2)
train.data=Carseats[train,]
test.data= Carseats[-train,]

```

b)

```
library(tree)
```

```
## Warning: package 'tree' was built under R version 3.4.4
```

```

tree.carseats=tree(Sales~.,data=train.data)
summary(tree.carseats)

```

```

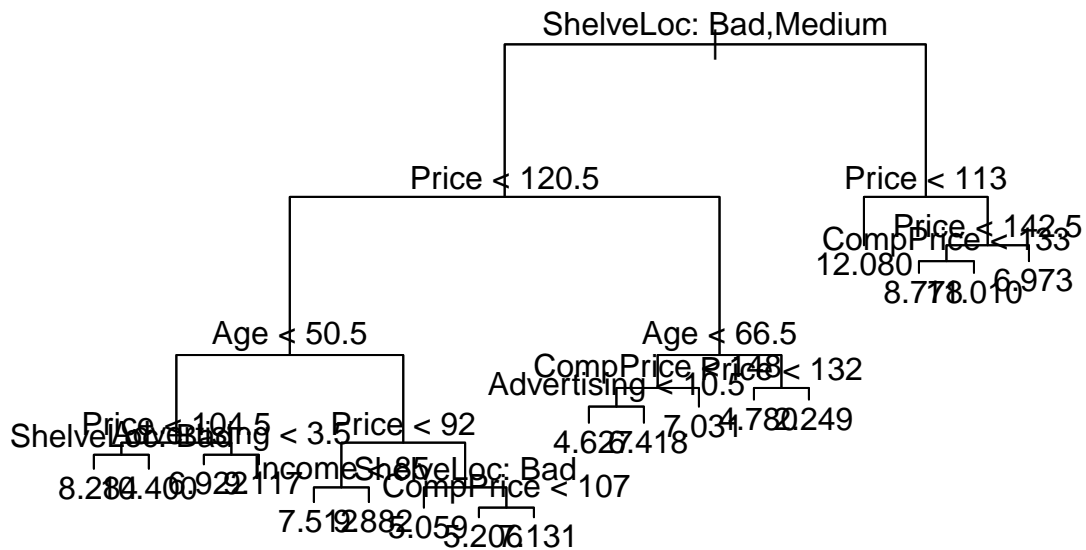
##
## Regression tree:
## tree(formula = Sales ~ ., data = train.data)
## Variables actually used in tree construction:
## [1] "ShelveLoc" "Price" "Age" "Advertising" "Income"
## [6] "CompPrice"
## Number of terminal nodes: 18
## Residual mean deviance: 2.36 = 429.5 / 182
## Distribution of residuals:
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## -4.2570 -1.0360 0.1024 0.0000 0.9301 3.9130

```

```

plot(tree.carseats)
text(tree.carseats,pretty = 0)

```



```

tree.pred=predict(tree.carseats,newdata=test.data)
mean((tree.pred-test.data$Sales)^2)      #Test MSE

## [1] 4.148897

```

c)

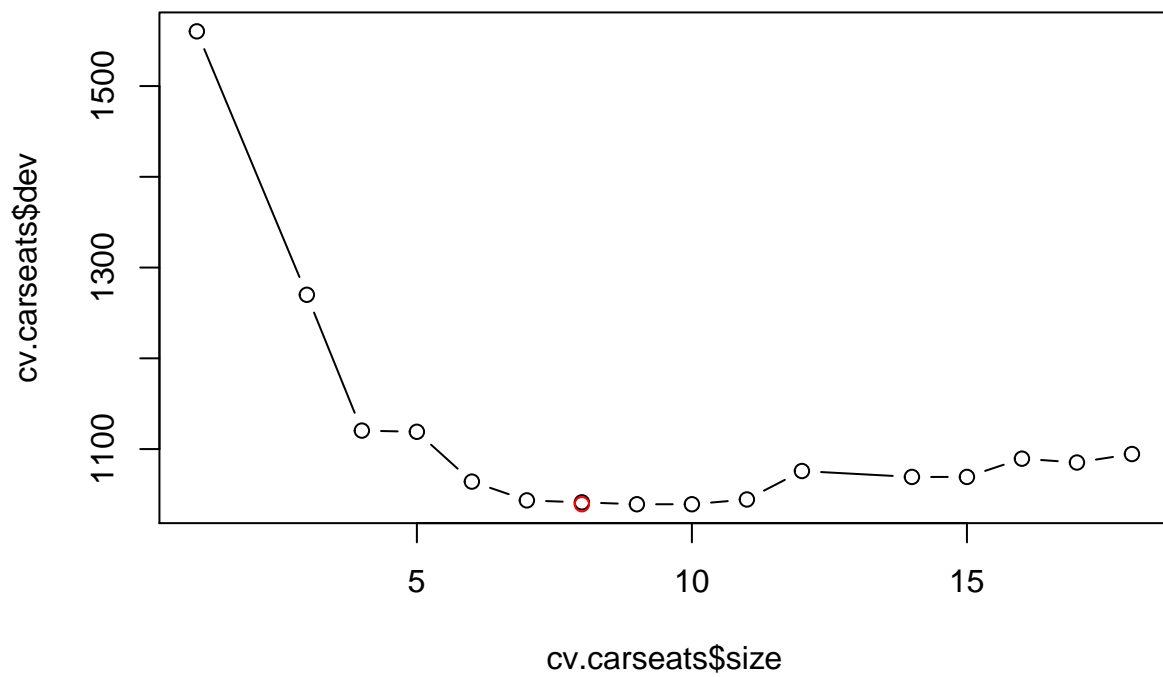
```

cv.carseats=cv.tree(tree.carseats)
plot(cv.carseats$size,cv.carseats$dev,type ="b")
tree.min=which.min(cv.carseats$dev)
tree.min

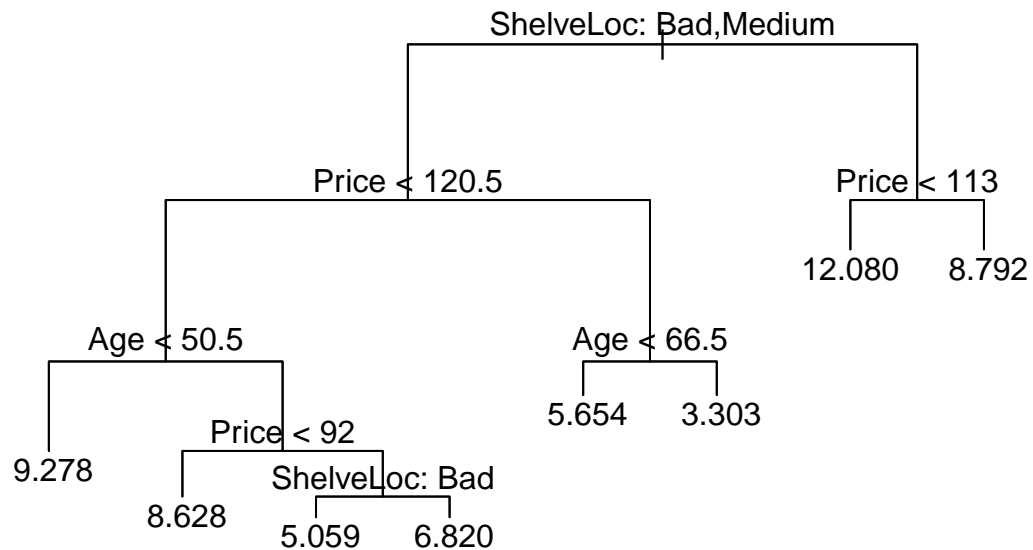
## [1] 8

points(tree.min,cv.carseats$dev[tree.min],col="red")

```



```
prune.carseats=prune.tree(tree.carseats,best=tree.min)
plot(prune.carseats)
text(prune.carseats, pretty=0)
```



```
tree.pred1=predict(prune.carseats,newdata=test.data)
mean((tree.pred1-test.data$Sales)^2)
```

```
## [1] 5.09085
```

MSE=5.09085 Pruning tree has increased test MSE from 4.148897 to 5.09085.

d)

```
library(party)
```

```
## Warning: package 'party' was built under R version 3.4.4
## Loading required package: grid
## Loading required package: mvtnorm
## Loading required package: modeltools
## Loading required package: stats4
## Loading required package: strucchange
## Warning: package 'strucchange' was built under R version 3.4.4
## Loading required package: zoo
## Warning: package 'zoo' was built under R version 3.4.4
##
```

```
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##      as.Date, as.Date.numeric
## Loading required package: sandwich
## Warning: package 'sandwich' was built under R version 3.4.4
library(randomForest)

## Warning: package 'randomForest' was built under R version 3.4.4
## randomForest 4.6-14
## Type rfNews() to see new features/changes/bug fixes.
attach(Carseats)
set.seed(1)
bag.carseats= randomForest(Sales ~ ., train.data, mtry = 10, importance =
TRUE)
yhat.bag= predict(bag.carseats, newdata = test.data)
mean((yhat.bag - test.data$Sales)^2)

## [1] 2.614642
importance(bag.carseats)

##              %IncMSE IncNodePurity
## CompPrice    16.4714051    126.605047
## Income        4.0561872     78.821925
## Advertising  16.2730251    122.793232
## Population    0.7711188     62.796112
## Price        54.5571815    512.940454
## ShelfLoc     42.4486118    320.749734
## Age          20.5369414    184.804253
## Education     2.7755968     42.427788
## Urban        -2.3962157      8.583232
## US           7.2258536     17.605661
```

Bagging process decreases test MSE to 2.61

e)

```
set.seed(1)
rf.carseats=randomForest(Sales~.,data=train.data,mtry=3)
xhat=predict(rf.carseats,newdata=test.data)
mean((xhat-test.data$Sales)^2)

## [1] 3.267852
importance(rf.carseats)

##              IncNodePurity
## CompPrice    134.77683
## Income       130.13842
## Advertising  139.26928
## Population   96.91406
```

```
## Price          376.12732
## ShelfLoc       240.96742
## Age            198.40681
## Education       64.60793
## Urban          16.08077
## US             32.57764
```

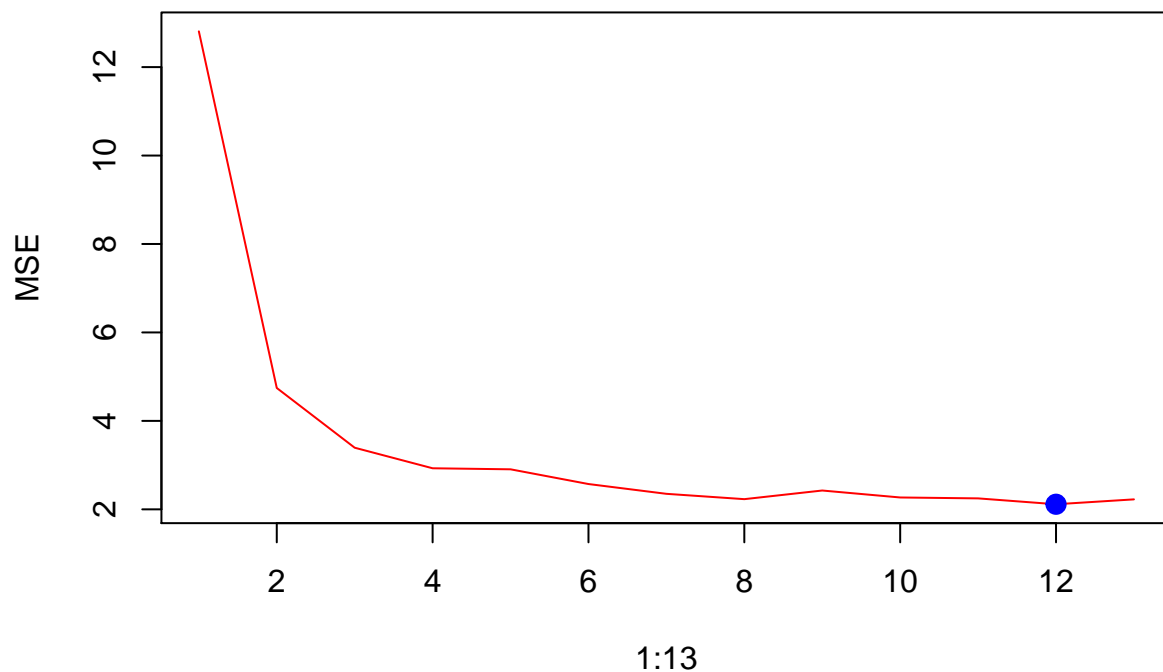
Test MSE=3.27

Problem 3

a)

```
library(MASS)
attach(Boston)
trainbos=sample(1:nrow(Boston),nrow(Boston)/2)
testbos=Boston[-trainbos,"medv"]
set.seed(2)
MSE=replicate(0,13)

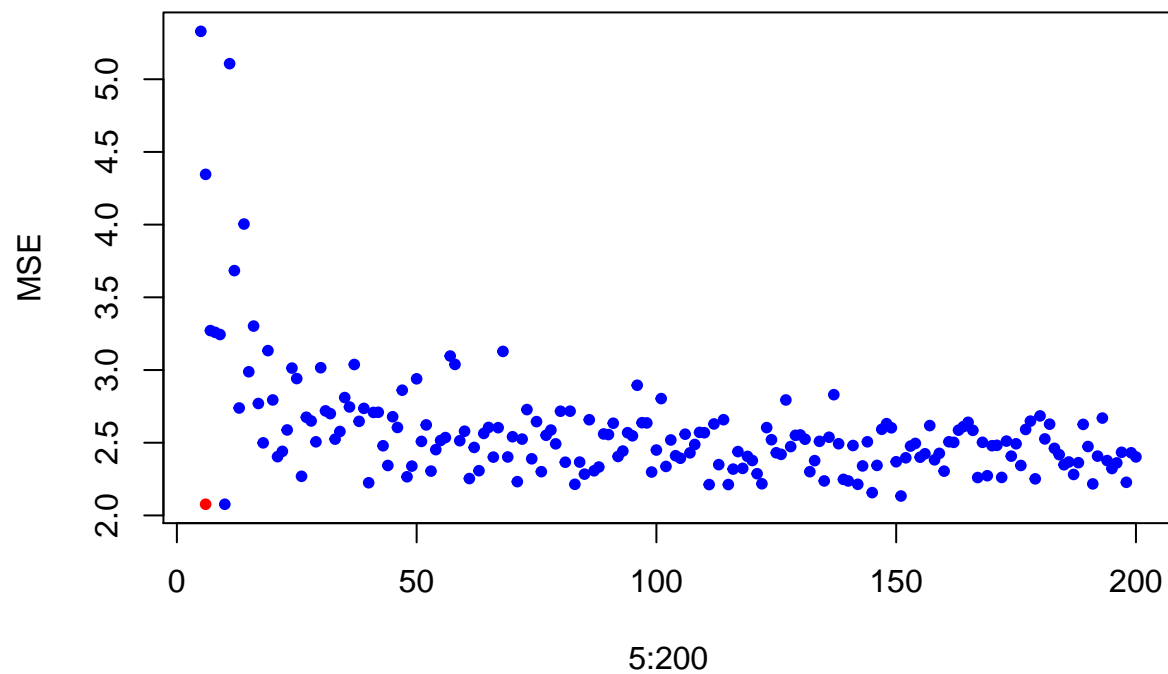
for (i in 1:13)
{
  rfbos=randomForest(medv~.,data=Boston[-trainbos,],mtry=i,ntree=100,importance=TRUE);
  ycaprfbos=predict(rfbos,newdata=Boston[-trainbos,]);
  MSE[i]=mean((ycaprfbos-testbos)^2)
}
plot(1:13,MSE,col="red",cex=2,pch=20,type="l")
minMSE=which.min(MSE)
points(minMSE,MSE[minMSE],col="blue",cex=2,pch=20)
```

From the above graph it is evident that $mtry=13$ using random forest.

b)

```
set.seed(5)
MSE=replicate(0,196)
for(i in 5:200)
{rfbos=randomForest(medv~.,data=Boston[-trainbos,],mtry=6,ntree=i,importance=TRUE);
ycaprfbos=predict(rfbos,newdata=Boston[-trainbos,]);
MSE[i-4]=mean((ycaprfbos-testbos)^2)}
plot(5:200,MSE,col="blue",cex=1,pch=20)
minMSE=which.min(MSE)
points(minMSE,MSE[minMSE],col="red",cex=1,pch=20)
```



From above plot is evident that for $\text{ntree}=70$ the MSE is lowest.