Fourier Transform

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The Fourier series and Fourier transform are mathematical tools used to represent and analyze functions, including images, in terms of their frequency components.

Fourier Transform Interpretation in Images:

- **Pixel Representation:** In the context of image processing, when we apply the Fourier transform (or Fourier series for periodic images), we are decomposing the image into its frequency components.
- **Spatial Frequencies:** Each pixel in the Fourier spectrum represents the magnitude of a spatial frequency component. Spatial frequencies describe how rapidly intensity varies in different directions across the image.
- **Low vs High Frequencies:**
  - **Low Frequencies:** Represent variations in intensity that change slowly across the image. These components correspond to large structures and smooth gradients in the image.
  - **High Frequencies:** Represent rapid changes in intensity, such as edges, textures, and fine details.
- **Interpretation:**
  - The position of a pixel in the Fourier spectrum corresponds to a specific spatial frequency along the horizontal and vertical axes.
  - The magnitude of that pixel indicates the strength (or amplitude) of that particular frequency component in the image.
- **Visualization:**
  - When visualizing the Fourier spectrum of an image, typically the center of the spectrum represents low frequencies, while the edges represent high frequencies.
  - Brighter pixels in the spectrum indicate stronger contributions from those frequencies in the original image.

2D Fourier Transform: The 2D Fourier transform of an image $f(x, y)$ is defined as: $F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp\left(-i2\pi(ux + vy)\right) dxdy$, here $F(u, v)$ represents the frequency components in the spatial domain, where u and v are the spatial frequencies along the horizontal and vertical axes respectively.