The Fourier series and Fourier transform are mathematical tools used to represent and analyze functions, including images, in terms of their frequency components.

Fourier Transform Interpretation in Images:

- Pixel Representation: In the context of image processing, when we apply the Fourier transform (or Fourier series for periodic images), we are decomposing the image into its frequency components.
- Spatial Frequencies: Each pixel in the Fourier spectrum represents the magnitude of a spatial frequency component. Spatial frequencies describe how rapidly intensity varies in different directions across the image.
- Low vs High Frequencies:
  - **Low Frequencies**: Represent variations in intensity that change slowly across the image. These components correspond to large structures and smooth gradients in the image.
  - **High Frequencies**: Represent rapid changes in intensity, such as edges, textures, and fine details.
- Interpretation:
  - The position of a pixel in the Fourier spectrum corresponds to a specific spatial frequency along the horizontal and vertical axes.
  - The magnitude of that pixel indicates the strength (or amplitude) of that particular frequency component in the image.
- Visualization:
  - When visualizing the Fourier spectrum of an image, typically the center of the spectrum represents low frequencies, while the edges represent high frequencies.
  - Brighter pixels in the spectrum indicate stronger contributions from those frequencies in the original image.

2D Fourier Transform: The 2D Fourier transform of an image $f(x, y)$ is defined as:
$F(u, v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\exp\left(-i2\pi(ux + vy)\right) dxdy$, here $F(u, v)$ represents the frequency components in the spatial domain, where u and v are the spatial frequencies along the horizontal and vertical axes respectively.