# Affine and Projective Transformations

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Affine and projective transformations are fundamental in computer vision and graphics for manipulating images and understanding geometric relationships. Each serves distinct purposes and is useful for different types of transformations.

$$x' = T.\overline{x}$$

1. **Affine:** A linear transformation that preserves parallelism and ratios of distances. Includes operations such as translation, scaling, rotation, and shearing. Used for image alignment, scaling, rotation, and basic geometric corrections.

$$A = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

   For $(x_i, y_i)$ mapping to $(x_i', y_i')$:

$$x_i' = a_{11}x_i + a_{12}y_i + t_x$$
$$y_i' = a_{21}x_i + a_{22}y_i + t_y$$
$$w_i' = 1$$

2. **Projective:** A transformation that can handle perspective distortion and is more general than affine. It preserves the straightness of lines but not necessarily parallelism. Used for tasks involving perspective changes like image stitching, correcting skew, and camera calibration.

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

   For $(x_i, y_i)$ mapping to $(x_i', y_i')$:

$$x_i' = h_{11}x_i + h_{12}y_i + h_{13}$$
$$y_i' = h_{21}x_i + h_{22}y_i + h_{23}$$
$$w_i' = h_{31}x + h_{32}y + h_{33}$$
$$\overline{x_i} = \frac{x_i'}{w_i'}, and\ \overline{y_i} = \frac{y_i'}{w_i'}$$