ORF 535 Final Project

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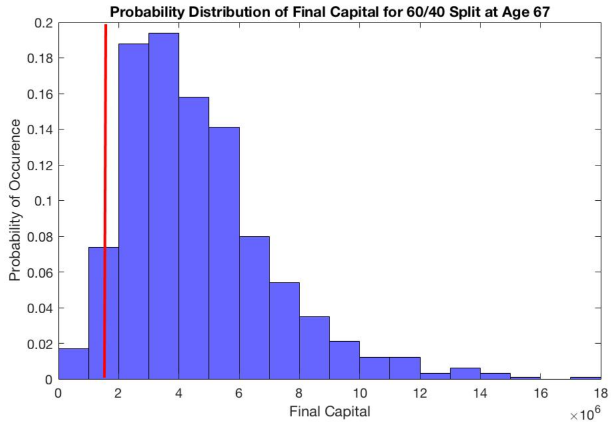
1 Scenario Generation

The 1000 scenarios were generated according to the procedure detailed at the end of the final project. The three sets of data (FTSE, Barclays Bonds, and SP Index) were split by the period they belonged to (normal or crash), and two different mean return and covariance matrices were computed. Using a transition matrix to determine the evolution of the regime over time, 1000 scenarios of 50 years were generated in a Monte Carlo scheme for each of the 3 assets. Using the initial weight of 60% stocks / 40% bonds, the initial salary of $125,000 with a growth rate of 4%, and a saving rate of 16%, the overall annual return was computed for each scenario every year. Thus, the returns at the end of year 50 represented 1000 samples of the final capital Lisa would have at age 81. Some descriptive statistics for the returns are below:

The probability that Lisa would have $7.5 million was **14.6%** at age 67 and **32.4%** at age 71. From a cursory look, we can see that retiring at age 67 given her current strategy does not seem feasible. It's also worth looking at the worst case scenarios more closely, since this that is the key result we want to avoid the most. We can first look at what the 5th percentile of returns is to get a sense of what the worst case scenario might look like. This is known as the "Value at Risk", or VaR, and represents the amount Lisa would stand to lose in the 5th percentile of scenarios. The 5% VaR was **-$1,537,700** at age 67 and **-$2,072,500** at age 71. Note that these numbers are negative because VaR represents the expected loss, and even in the 5th percentile of scenarios, Lisa stands to gain capital. However, this doesn't tell us anything about the extent of the loss beyond the 5th percentile. To get a more holistic sense of the worst case scenarios, we can look at the average of all of the cases that are as good as or worse than the VaR we just computed. This is known as the Conditional Value at Risk, or CVaR. The 5% CVaR was **-$1,156,200** at age 67 and **-$1,483,000** at age 71. It's interesting that the VaR changes by half a million dollars just by working for 4 years, but the CVaR changes by a much less amount. This points to the fact there will continually be scenarios with extremely poor returns causing the CVaR to change much less than the VaR as the number of years increases. To understand this risk measures visually, we can look at the distribution below.



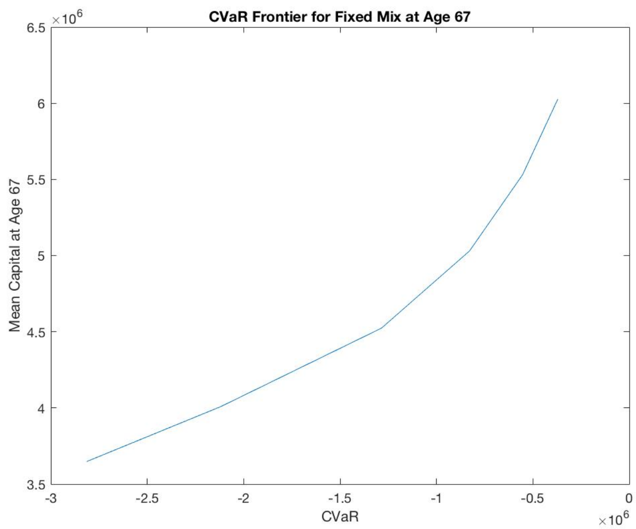
This plot shows the probability of Lisa achieving certain levels of final capital when she retires at 67 with a fixed 60/40 split. The red bar represents the 5th percentile of scenarios, i.e. 5% of the time she will perform at or to the left that line. This is the 5% Value at Risk. We can also understand the Conditional Value at Risk to be the average value we see to the left of the red bar. That is, if Lisa's performance is in the bottom 5 percent, how much capital can she expect to end up with? We will continue to use the CVaR as a primary measure of risk, and try to aim for a value that would Lisa could survive on even in the worst case.

It's also worth taking a look at how much of Lisa's money she might stand to lose during the entire investment process. This is usually a more prominent measure of the real-time risk farther down the investment timeline and is abstracted away from the absolute amount of capital by dealing only with percentage changes. The largest percentage drop in capital during a scenario is referred to as the maximum drawdown for that scenario. The maximum drawdown over all scenarios was **83.40%** and the 5th percentile of the maximum drawdown was **70.03%**. This represents a clear danger of long-term investing in more volatile assets like stocks. Although they ostensibly provide the best return in the long run, there is a much greater risk of an extended period of decreasing capital, most likely during a crash regime. Even given that there is only a 50% chance of staying in a crash regime each year, the scenarios we have generated do show that there is a nontrivial probability of a long decline. This is something to keep in mind as Lisa figures out her investing strategy throughout time. Out of the **802** times Lisa was able to meet her goals sometime during the 50 year period, the mean time to reach her goal was **41.9** years and the standard deviation was **4.9** years. So given the 60/40 strategy, it does not seem reasonable to expect that Lisa would be able to graduate before working at least 40 years. To achieve a 75% chance of success would require working for **45** years, and for a 90% chance, it would take **48.1** years! That's quite a long time, given that Lisa would be age 79 by then. We can only hope that there would be a better strategy that would enable to her to retire earlier than that.

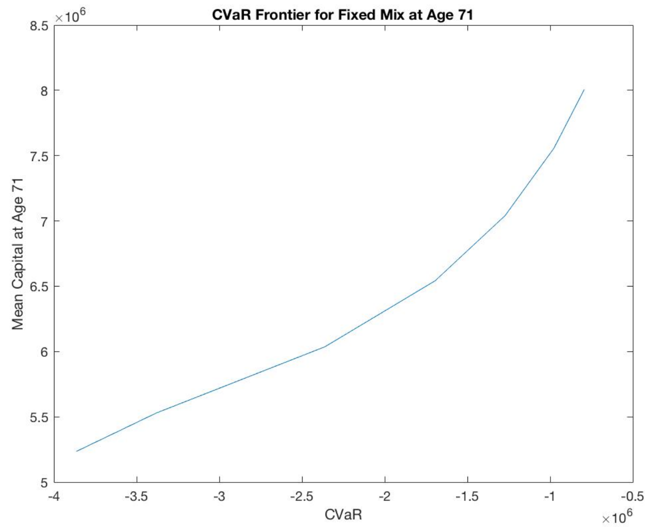
2 CVaR Frontiers

At this point we looked at different portfolios to determine tradeoffs between the mean return at potential retirement ages and the CVaR, or the average of the worst case scenarios below the 5th percentile. Because this problem was non-convex, I used gradations of 1% for each asset weight from 0% to 100% with the condition that all of the asset weights had to sum to 1. This resulted in **176,819** different portfolios being tested. Reasonably, it is not extremely important that the theoretical exact maximum is reached because there will be some slight variation in the real world due to the randomness of all of the scenario returns. Thus, it is a safe approximation to create a pseudogrid of 100 x 100 x 100 portfolios, filter by ones whose weights sum to 1, and then find the optimal portfolio out of this set.

The first CVaR frontier is shown below, and shows the relationship between desired mean capital at age 67 and minimum CVaR at that level.

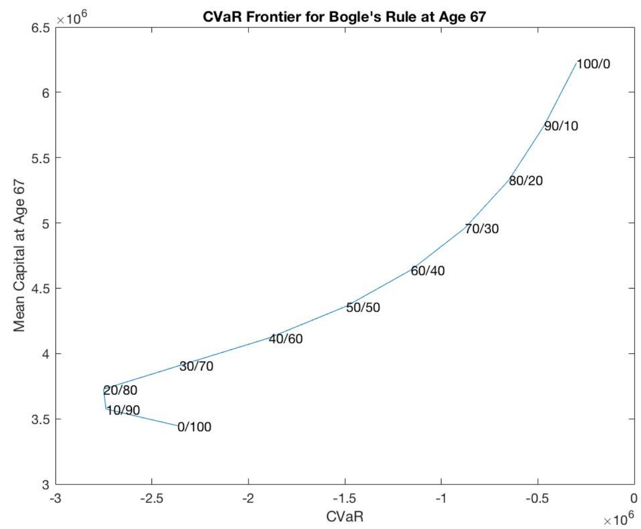


Unlike a typical efficient frontier between average and risk, this curve is actually concave up. What does this mean? In terms of the role of equity, the curve shows how powerful a higher amount of weight in equity can be to maximize capital. The graph clearly shows that relaxing the CVaR constraint has an increasingly powerful effect on the mean capital that can be achieved at age 67. In terms of the investing strategy that Lisa should take, this is a strong sign that stocks should be a significant amount of Lisa's portfolio. Yet, this analysis only projects until she is 67. What about farther down the line?



We see the same effect here. The curve for retiring at age 71 looks almost exactly the same shape as that for age 67. However, there are some ways these two curves are different. Comparing the point series on the two graphs, we see that for a similar CVaR value, we can attain a higher mean end capital, which makes sense given that a longer time horizon with a consistently positive investing strategy should achieve a better return over a longer period of time for a certain level of risk. For example, if Lisa would want to end up at least **$2.5 million** upon retirement in the worst 5% of scenarios, she could expect a nest egg of around **$3.75 million** at 67, which is only half of her desired capital. Yet, the second figure shows that working for only four more years can yield a mean final capital of **$6 million**, which is significantly closer to her goal! In fact, Lisa would grow her retirement money by 60% in those four years, which makes retiring early seem like a much more lucrative option for her.

Before further inspection, it's important to look at other strategies before looking at a constant mix of assets throughout her working life. Sometimes it's more beneficial to slowly decrease the risk of Lisa's portfolio as she nears retirement because 1) she will need the money in less and less time and 2) in the event that her capital drops below her goal, she will have even less time to regrow her investment before retirement age. One implementation of this is Bogle's rule, where her portfolio will start out with more stocks and less bonds, and upweight bonds over stocks over time. The efficient frontier for Bogle's rule is seen below.



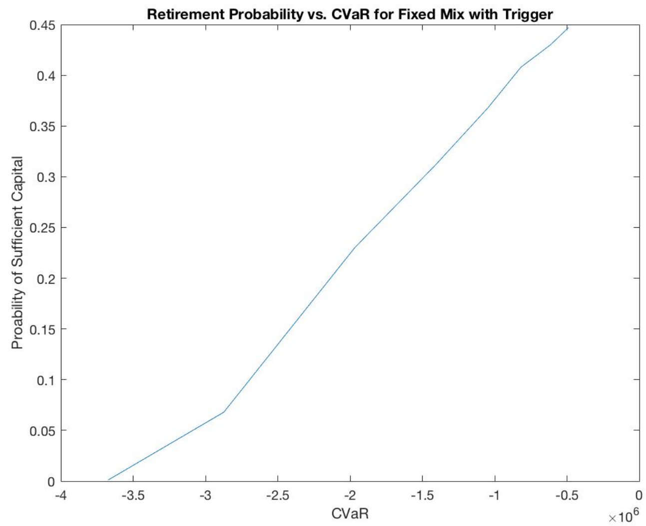
Above is the efficient frontier for Bogle's rule ending at age 67. Note that the way to read this chart is a bit different - instead of picking a specific point on the frontier, Lisa would start closer to the right side and move steadily along the line each year until reaching the left side, where she has her portfolio mostly leveraged in bonds. Over 1,000 simulated scenarios of the Bogle rule, Lisa was able to save an average of **$4,283,900** at an average worst case scenario of **$1,846,700** at age 67. At age 71, she was able to save an average of **$5,385,900** with an average worst case scenario of **$2,282,100**.

Looking at these efficient frontiers, I would posit that retiring at age 71 is a worthwhile goal because of the large compound gains to be realized during that time. Ideally, Lisa would have the maximum capital close to retirement, which means every subsequent year spent investing will be maximally profitable. Comparing the optimal fixed mix models to the Bogle's rule scenarios, the Bogle's rule seems to achieve a very similar return ($5.4 million vs. $6 million) with the same or slightly less risk level at age 71. So, there does not seem to be a significantly different result whether we use Bogle's rule or stick with the optimal fixed mix. This occurs because of the behavior of equity - while there is a greater probability that more capital can be lost with a higher leverage in equity, the overall returns of equity with respect to bonds seems to balance out that risk with reward.

It's potentially feasible that Lisa might be able to retire early, and it's very possible that she might prefer retiring early with sufficient capital rather than working longer and retiring later with more capital. As I understand her needs, anything about $7.5 million is icing on the cake, so it's worth exploring the option of moving all of her capital to cash in the event that she reaches the required capital.

We used the same optimal portfolios as computed for the fixed mix models earlier, and installed a trigger that allowed her to move all of her money to cash as soon as her goal of $7.5 million was reached. Given that Lisa retires at age 71, we can see that her probability of retiring with sufficient capital increases, but so does her CVaR.

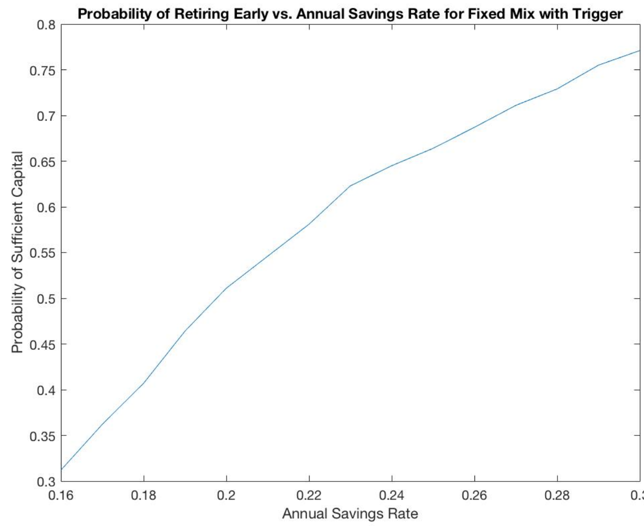
3 Maximizing Retirement Probability



From this plot, it seems no matter how much risk Lisa takes, she will be unlikely to retire at 71. Even at the highest risk level, where her portfolio consists of **98% stocks / 2% bonds** she is **45%** likely to have sufficient capital by the end of her 40th year of work. Without this trigger, this probability is **36.4%**, a **19.1%** difference. The Bogle's rule simulation shows an even worse fate. If we install an all-cash trigger on the Bogle's rule, at a CVaR of **-$2,563,900**, Lisa would only reach her goal with probability **16.5%**, compared to **16.0%** without the trigger. It seems that adding a trigger for the fixed mix model does significantly change the probability that she will be able to retire on time, but not for the Bogle's rule. This is intuitive because of the inherent risk in having a portfolio that is highly levered in equities, especially during a crash regime where the total capital can drop very sharply in value. Under Bogle's rule, however, the large weight in bonds later in her working life prevents larger drops in capital after she reaches her goal. Nevertheless, because the odds are so low under Bogle's rule, the fixed mix portfolio at the highest risk level is recommended with a trigger, because it affords her the highest chance of retirement.

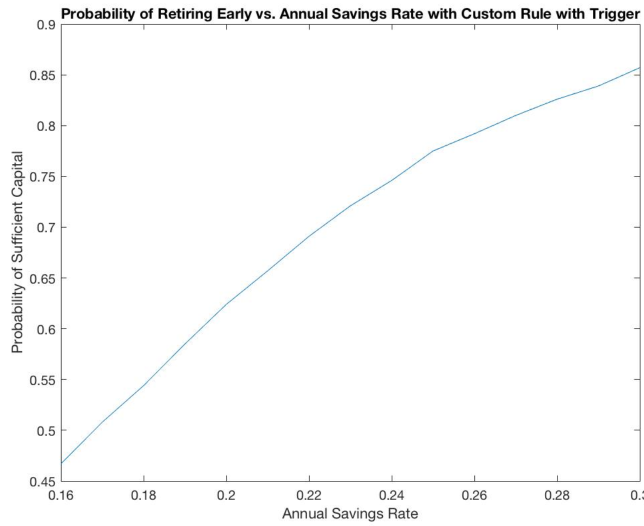
Also worth mentioning is the worst 5% of scenarios, which seem excessively poor at the highest risk level - only **$500,000!**. At minimum, Lisa should aim for **20%** of her goal to be met in the worst scenarios - this would correspond to a split of **62% stock / 38% bonds** and a CVaR of **$1.5 million**.

Given all of this information, it might be more sensible to save a larger proportion of annual savings every year to further boost the chances of retirement. Understandably, it's very hard to save a large proportion of annual salary. Yet, it will definitely be worth it if Lisa can actually retire on time, which is the highest priority. Below is a graph that details how likely she will be to retire on time relative to her savings rate. In this case, I included the cash trigger and plotted a range from **16%** to **30%** in **1%** increments.



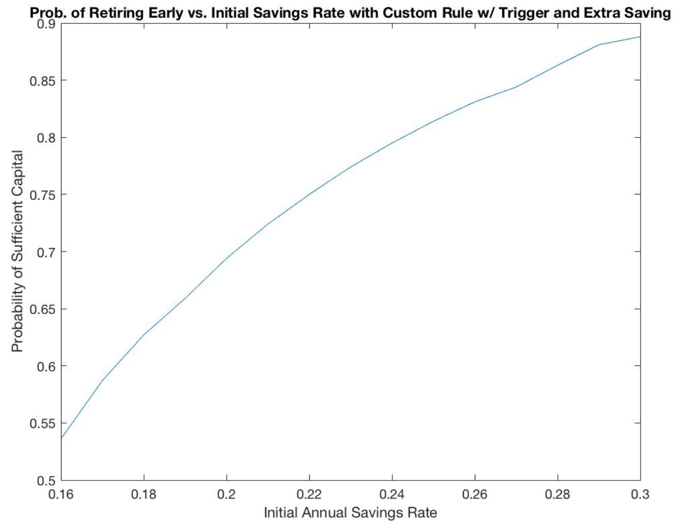
Obviously, a greater annual savings rate leads to a higher probability of being able to retire early. I understand that saving anything beyond 22% of salary may not feasible for a young professional in NYC, and at level it seems that Lisa would have roughly a **60%** chance of retiring within 40 years, which is a decent likelihood. I believe this is a reasonable strategy because of the way her salary will grow and her expenses will remain constant throughout time.

I previously mentioned that one of the dangers of investing in equities in spite of their high return over the long return is the large crashes that can occur close to retirement. To remedy this, I tried a strategy similar to Bogle's rule, but dependent on the current percent of the needed capital at any given point in time, rather than age. In other words, the farther away Lisa is from her goal, the more aggressive she should be in her investing strategy so that she can quickly gain more capital which can compound over the coming years. As she reaches her goal, it makes sense to take less risk to avoid losing the money she has already earned, and bonds should dominate the portfolio. Put quantitatively, the **percent of stock should equal 1 - her current capital / $7.5 million**. Running 1000 scenarios of this with the cash trigger yielded the curve below:



This is a big improvement on any strategy we've taken thus far. Using a savings rate of 22%, Lisa now has a comfortable **70%** chance of retiring within 40 years. Out of all of the successful scenarios, the average age she could retire was **65.5 years**, which is only **34.5** years of work! Her average capital raised at the end of 40 years, even after using a trigger that cut off investment after she reached her goal, was **$7,641,000**. At this point Lisa has plenty of information to make her decision about how much she would like to save. For example, if she decides to save the standard 16%, she'll have around a 50/50 chance of retiring in 40 years. Every extra 2% she saves would lead to an extra 5% probability of retiring, at least up until she's saving a quarter of her annual salary.

We also explored her chances in the event that she decided to save a larger percentage of her salary closer to retirement, say an extra 6% starting at age 55%. The above strategy implemented with this extra saving is shown below:



If Lisa were to boost her savings at age 55, she would have a **75%** chance of retiring in 40 years with an initial rate of **22%**. Even at the standard 16%, she would have almost a **55%** chance of retiring before age 71. Given the 22% rate, her average age of retirement would still be the same (around 66), but her average capital would now be **$8,071,700**, which is a **5.6%** improvement on just the custom strategy I described above. Not only that, but in the worst 5% of scenarios, she would still stand to gain an average of **$2,493,600**, which is significantly more than the benchmark of 1.5 million.

If Lisa is more adamant about the savings rate but still desires a higher probability of retiring, she can choose to work more than 40 years and retire at age 75 instead. While I personally would not recommend this option, she might have the inclination that she would WANT to work until she is age 75. If she does this but stays at the baseline of 16% savings rate, she would have a **68.3%** chance of retiring early without the extra savings, and a **75.1%** chance with the extra savings. In both cases, her average success came out to be at around age 69.

So, there are a myriad of options presented here, and ultimately I can not make the final decision. What Lisa decides to do will be dependent mostly on her choice of annual savings rate, her decision to save more later in life, and her level of desire to retire earlier in life rather than later. The options presented here cover all of these variables and give her a large set of strategies to choose from, but no matter which one she chooses to implement, she should have a decent shot at retiring successfully without having to compromise on the size of her final nest egg.

4 Appendix

I also played with including the Exxon stock in the portfolio calculations to see if it would at all outperform the current available assets and increase retirement probability or decrease the CVaR.