Real Analysis(H2) (MA4.101a)

IIIT-H, Semester Monsoon 22, Assignment 2

Submission deadline: 21$^{\text{th}}$ January 2023

---

1. Show that $\vec{V} = (2xy + z^2)\hat{i} + (2yz + x^2)\hat{j} + (2xz + y^2)\hat{k}$ is a conservative vector field and calculate a scalar field ϕ such that $u = \vec{\nabla}\phi$.

2. Prove the divergence theorem for a right circular cylinder. *Hint: Use cylindrical polar coordinate system. Make the axis of cylinder along the z axis.*

3. Prove the identity $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \vec{\nabla}^2\vec{A}$.

4. Show that for a complex number z
$$\lim_{z\to\infty} \frac{2z^3 - 1}{z^2 + 1} = \infty \tag{1}$$

5. Show that for a function of complex variable $f(z) = \sqrt{r}e^{i\theta/2}$, the derivative is
$$f'(z) = \frac{1}{2f(z)}\,. \tag{2}$$

6. Show that for a function of complex variable $f(z) = e^{-\theta}\cos(\ln r) + ie^{-\theta}\sin(\ln r)$, where $r > 0, 0 \leq r \leq 2\pi$, the derivative is
$$f'(z) = i\frac{f(z)}{z}\,. \tag{3}$$