Assignment - 2

AE649A

Submission Due Date: 12th November 2022

Problem 1: Consider the following full set of longitudinal motion of a given aircraft at a given flight condition

$$\dot{q} = -0.6q - 0.2\dot{\alpha} - \alpha - 1.2\delta_e$$
$$\dot{u}_{sp} = 225\delta_{th} + 0.035\alpha - 0.81\theta - 0.18u_{sp}$$
$$\dot{\alpha} = q - 0.2u_{sp} - 0.6\alpha - 0.035\delta_e$$
$$\dot{\theta} = q$$

where the symbols carry their usual meaning. Answer the following

1. Find the transfer function $q(s)/\delta_e(s)$.
2. Also obtain the transfer functions $\alpha(s)/\delta_e(s)$ and $\theta(s)/\delta_e(s)$. Assume that the initial values of the states are zero.

[You can make use of MATLAB/Simulink for finding the solutions of the above problems]. If MATLAB/Simulink is used, codes need to be attached in solution sheet for the respective questions.

Marks: 20

Problem 2: A airplane is found to have poor handing qualities. Use state feedback control to provide the stability augmentation. The lateral equation of motion is given by

$$\begin{bmatrix} \dot{\beta}(t) \\ \dot{p}(t) \\ \dot{r}(t) \\ \dot{\phi}(t) \end{bmatrix} = \begin{bmatrix} -0.05 & -0.003 & -0.98 & .2 \\ -1.0 & -0.75 & 1.0 & 0 \\ 0.3 & -0.3 & -0.15 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta(t) \\ p(t) \\ r(t) \\ \phi(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1.7 & -0.2 \\ 0.3 & -0.6 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a(t) \\ \delta_r(t) \end{bmatrix}$$

For the above system, answer the following

a) Determine the step response, $x(t) = [\beta(t); p(t); r(t); \phi(t)]^T$, to the aileron input, $\delta_a(t) = u_s(t)$ (step).
b) Do the same as in (a) for rudder input, $\delta_r(t) = u_s(t)$ (step).
c) is the aircraft controllable using both the inputs?
d) is the aircraft controllable using only the aileron inputs, δ_a?
e) is the aircraft controllable using only the rudder input, δ_r?
f) is the aircraft observable with the bank-angle, $\phi(t)$, being the only measured output?
g) is the aircraft observable with the sideslip-angle, $\beta(t)$, being the only measured output?
h) Design a state-feedback control for the following desired eigenvalues:

$$\lambda_1 = -1.5; \lambda_2 = 0.05; \lambda_{3,4} = -0.35 \pm 1.5i$$

Assume the relative authority of the ailerons and rudder are $g_1 = 1.0$ and $g_2 = \frac{\delta_r}{\delta_a} = 0.33$.

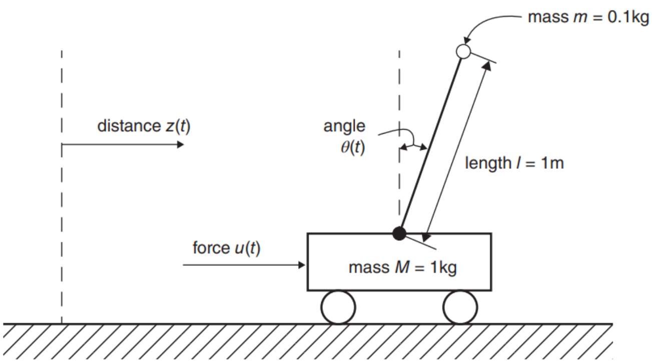
i) Can we design a full-state feedback control which places the closed-loop poles of the aircraft at $s_{1,2} = -1 \pm i, s_3 = -15; s_4 = -0.8$ using only one of the inputs? If so, which one, and what should be the control gain matrix?

j) Design the two-input optimal controller with $Q = I$ and $R = I$. Calculate and plot the initial response if the initial condition $x(0) = [0; 0.5; 0; 0]^T$.

[You can make use of MATLAB/Simulink for finding the solutions of the above problems]. If MATLAB/Simulink is used, codes need to be attached in solution sheet for the respective questions.

Marks: 50

Problem 3: A broom balancer is a control system which balances a broom on a arbitrary object, such as a booster rocket. Such a controlled system in shown in the following figure. Note that the



uncontrolled system here is fundamentally unstable, as the boom would fall if left uncontrolled. The governing equation describing this system is

$$(M+m)\ddot{z} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u$$

The above nonlinear system is linearized about the nominal or equilibrium points $z_n(t) = 0, \dot{z}_n(t) = 0, \dot{\theta}_n(t) = 0, and\ u_n(t) = 0$. The linear states space model with horizontal distance $z(t)$ as the output (controlled) variable is given by

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 11 & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} u; \qquad y = z = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

Answer the following

1. Design a full state feedback controller such that the controlled variable $z(t)$ is regulated reasonably well. Use the LQR method and select the output and control weightings such that the dominant eigenvalue has a real part of at least -1.
2. Now design an observer by determining the observer gain matrix K_e such that the observer closed loop matrix $A - K_e C$ has the closed loop eigenvalues at $\lambda = -2, \lambda = -3, \lambda = -2 + i, \lambda = -2 - i$.
3. Plot the output variable trajectory and the control trajectory for an initial condition of $\theta(0) = 5^0$ and all other initial variables are zero.

[You can make use of MATLAB/Simulink for finding the solutions of the above problems. If MATLAB/Simulink is used, codes need to be attached in solution sheet for the respective questions.]

Marks: 30