Spacecraft Attitude and Reaction Wheel Desaturation Combined Control

Term project (AE-649A) report

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Abstract

Two popular types of spacecraft actuators are reaction wheels and magnetic torque coils. Magnetic torque coils are particularly interesting because they can be used for both attitude control and reaction wheel momentum management (desaturation control). Although these two tasks are performed at the same time using the same set of actuators, most design methods deal with only one of the these tasks or consider these two tasks separately. In this project, a design with these two tasks in mind is formulated as a single problem. A linear quadratic regulator design method is then proposed to solve this problem. A simulation example is provided to describe the benefit of the new strategy.

Keywords - Spacecraft**,** Reaction wheel, magnetic torque coil, local vertical local horizontal frame, linear quadratic regulator

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Abbreviations

IGRF	International Geomagnetic Reference Field
PTVR	Periodic Time Varying Riccati Equation
LVLH	Local Vertical Local Horizontal
DARE	Discrete-time Algebraic Riccati Equation

Chapter 1

Introduction

1.1 Spacecraft

A **spacecraft** is a vehicle or machine designed to fly in outer space. A type of artificial satellite, spacecraft are used for a variety of purposes, including communications, Earth observation, meteorology, navigation, space colonization, planetary exploration, and transportation of humans and cargo.

A Spacecraft needs an attitude control subsystem to be correctly oriented in space and respond to external torques and forces properly. The attitude control subsystem consists of sensors and actuators, together with controlling algorithms. The attitude-control subsystem permits proper pointing for the science objective, sun pointing for power to the solar arrays and earth pointing for communications.

1.2 Reaction wheels

Reaction/momentum wheels are flywheels used to provide attitude control authority and stability on spacecraft. By adding or removing energy from the flywheel, torque is applied to a single axis of the spacecraft, causing it to react by rotating. By maintaining flywheel rotation, called momentum, a single axis of the spacecraft is stabilized. Several reaction/momentum wheels can be used to provide full three-axis attitude control and stability.

1.3 Magnetic torque coil (Magnetorquer)

A **magnetic torquer** (also known as a **torque rod**) is a satellite system for attitude control, detumbling, and stabilization built from Electromagnetic coils. The magnetorquer creates a magnetic dipole that interfaces with an ambient magnetic field, usually Earth's, so that the counter-forces produced provide useful torque.

1.4 Problem definition

Spacecraft attitude control and reaction wheel desaturation are normally regarded as two different control system design problems. The desired torques is in a three-dimensional space, magnetic torque coils can only generate torques in a two- dimensional plane [2].

The designs proposed earlier considered only momentum management but not attitude control. Since both attitude control and reaction wheel desaturation are performed at the same time using the same magnetic torque coils, the control system design should consider these two design objectives at the same time.

For a Low Earth Orbit spacecraft that uses Earth's magnetic field, the reference frame for the spacecraft is most likely local vertical local horizontal (LVLH) frame.

In this project, a more attractive design method is proposed, which considers as many factors as practical. The controlled attitude is aligned with LVLH frame and the orbit is aligned with the equator, i.e. zero orbit inclination. A general reduced quaternion model, including reaction wheels, magnetic torque coils, the geomagnetic field along the orbit and its interaction with magnetic torque coils, is proposed.

A single-objective function, which considers the performance of both attitude control and reaction wheel management at the same time, is suggested. This objective function is optimized using the solution of a Riccati equation described, which leads to an optimal control. Based on the Riccati equations, we show that the design can be calculated in an efficient way and the designed controller is optimal for both the spacecraft attitude control and for the reaction wheel momentum management at the same time.

We provide a simulation test to demonstrate that the designed system achieves more accurate attitude than the optimal control system that uses only magnetic torques. Moreover, the designed controller based on linear quadratic regulator (LQR) method works on the nonlinear spacecraft system.

Chapter 2

Spacecraft model for attitude and Reaction wheel desaturation control

For our problem of controlling the spacecraft attitude as well as reaction wheel desaturation using Magnetic torque coils we are taking the model developed by Y. Yang [1].

For spacecraft using Earth's magnetic torques, the nadir pointing model is probably the most desired one. Therefore, the attitude of the spacecraft is represented by the rotation of the spacecraft body frame relative to the local vertical and local horizontal frame.

The nonlinear model of system is given by

$$\begin{aligned}
&\mathbf{J}\dot{\boldsymbol{\omega}} = \mathbf{f}(\boldsymbol{\omega}, \boldsymbol{\Omega}, \mathbf{q}) + \mathbf{t}_g - [-\mathbf{I}, \mathbf{b}^\times]\mathbf{u} + \mathbf{t}_d \\
&\mathbf{J}_w\dot{\boldsymbol{\Omega}} = -\mathbf{t}_w \\
&\dot{\mathbf{q}} = \mathbf{g}(\mathbf{q}, \boldsymbol{\omega}).
\end{aligned}$$

Or

$$\begin{bmatrix} \mathbf{J} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_w & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \dot{\boldsymbol{\Omega}} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial \boldsymbol{\omega}} & \frac{\partial \mathbf{f}}{\partial \boldsymbol{\Omega}} & \frac{\partial \mathbf{f}}{\partial q} + \mathbf{T} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \frac{\partial \mathbf{g}}{\partial \boldsymbol{\omega}} & \mathbf{0} & \frac{\partial \mathbf{g}}{\partial \mathbf{q}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{\Omega} \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} \mathbf{I} & -\mathbf{b}^\times \\ -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{t}_w \\ \mathbf{m} \end{bmatrix} + \begin{bmatrix} \mathbf{t}_d \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

where,

- J = inertia of Spacecraft
- Ω = Angular velocity of Reaction wheel
- ω = Body rate
- J_ω = Inertia of Reaction wheel
- t_g = gravity gradient torque
- t_d = external disturbance
- b =magnetic field
- q = quaternion

We assume that the inertia matrix of a spacecraft **J** = diag (J1, J2, J3) is a diagonal matrix i.e. all the reaction wheels are exactly aligned with the body axes of the spacecraft (Not inclined with respect to body axes) [3].

The system states $\mathbf{x} = [\boldsymbol{\omega}^T, \boldsymbol{\Omega}^T, \mathbf{q}^T]^T$ and control inputs $\mathbf{u} = [\mathbf{t}_w^T, \mathbf{m}^T]^T$ the spacecraft control system model.

Chapter 3

Linearized model for attitude and Reaction wheel desaturation control

Using the linearization technique of [1], [3] and [4], we can express non-linear spacecraft model approximately as a linear function as follows:

$$
\begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \dot{\boldsymbol{\Omega}} \\ \dot{\mathbf{q}} \end{bmatrix}
=
\begin{bmatrix}
\mathbf{J}^{-1}\frac{\partial \mathbf{f}}{\partial \boldsymbol{\omega}} & \mathbf{J}^{-1}\frac{\partial \mathbf{f}}{\partial \boldsymbol{\Omega}} & \mathbf{J}^{-1}\left(\frac{\partial \mathbf{f}}{\partial q} + \mathbf{T}\right) \\
\mathbf{0} & \mathbf{0} & \mathbf{0} \\
\frac{\partial \mathbf{g}}{\partial \boldsymbol{\omega}} & \mathbf{0} & \frac{\partial \mathbf{g}}{\partial \mathbf{q}}
\end{bmatrix}
\begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{\Omega} \\ \mathbf{q} \end{bmatrix}
+
\begin{bmatrix}
\mathbf{J}^{-1} & -\mathbf{J}^{-1}\mathbf{b}^{\times} \\
-\mathbf{J}_w^{-1} & \mathbf{0} \\
\mathbf{0} & \mathbf{0}
\end{bmatrix}
\begin{bmatrix} \mathbf{t}_w \\ \mathbf{m} \end{bmatrix}
+
\begin{bmatrix} \mathbf{J}^{-1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}
\mathbf{t}_d
$$

$$
=
\begin{bmatrix}
0 & 0 & \omega_0\frac{J_1-J_2+J_3}{-J_1} & 0 & 0 & \frac{\omega_0 J_{w_3}}{-J_1} & 8\omega_0^2\frac{J_3-J_2}{J_1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 6\omega_0^2\frac{J_3-J_1}{J_2} & 0 \\
\omega_0\frac{J_1-J_2+J_3}{J_3} & 0 & 0 & \frac{\omega_0 J_{w_1}}{J_3} & 0 & 0 & 0 & 0 & 2\omega_0^2\frac{J_1-J_2}{J_3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}
$$

$$
+
\begin{bmatrix}
J_1^{-1} & 0 & 0 & 0 & \frac{b_3}{J_1} & -\frac{b_2}{J_1} \\
0 & J_2^{-1} & 0 & -\frac{b_3}{J_2} & 0 & \frac{b_1}{J_2} \\
0 & 0 & J_3^{-1} & \frac{b_2}{J_3} & -\frac{b_1}{J_3} & 0 \\
-J_{w_1}^{-1} & 0 & 0 & 0 & 0 & 0 \\
0 & -J_{w_2}^{-1} & 0 & 0 & 0 & 0 \\
0 & 0 & -J_{w_3}^{-1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} t_{w_1} \\ t_{w_1} \\ t_{w_1} \\ m_1 \\ m_2 \\ m_3 \end{bmatrix}
+
\begin{bmatrix} t_{d_1}/J_1 \\ t_{d_2}/J_2 \\ t_{d_3}/J_3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
:= \mathbf{Ax} + \mathbf{Bu} + \mathbf{d}.
$$

where,

- Inertia matrix of spacecraft: $J = \mathrm{diag}\,(J_1, J_2, J_3) = \begin{pmatrix} J1 & 0 & 0 \\ 0 & J2 & 0 \\ 0 & 0 & J3 \end{pmatrix}$
- Body rate with respect to LVLH: $\Omega = [\omega_1, \omega_2, \omega_3]^T$
- Orbit rate with respect to LVLH: $\omega_{lvlh} = [0, \omega_0, 0]^T$

- Inertia matrix of reaction wheel:

$$J_w = \text{diag}\,(J_{w1}, J_{w2}, J_{w3}) = \begin{pmatrix} Jw1 & 0 & 0 \\ 0 & Jw2 & 0 \\ 0 & 0 & Jw3 \end{pmatrix}$$

- Notice that the linearized spacecraft model is in general a time varying system. The time variation of the system arises from an approximately periodic function of $\mathbf{b}(t) = \mathbf{b}(t + P)$, where

  Orbital Time period: $P = \frac{2\Pi}{\omega 0} = 2*\Pi *(a^3/GM)^{0.5}$

  $\{GM=3.986005*10^{14}\}$

- Angular rate vector: $\Omega = [\Omega_1, \Omega_2, \Omega_3]^T$
- International Geomagnetic reference field model (IGRF) :

  This magnetic field $\mathbf{b}(t)$ can be approximately expressed as follows [5]:

$$\begin{bmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \end{bmatrix} = \frac{\mu_f}{a^3} \begin{bmatrix} \cos(\omega_0 t)\sin(i_m) \\ -\cos(i_m) \\ 2\sin(\omega_0 t)\sin(i_m) \end{bmatrix}$$

  where i_m is the inclination of the spacecraft orbit with respect to the magnetic equator, $\mu_f = 7.9 \times 10^{15}$ Wbm is the field's dipole strength.

- Spacecraft magnetic torque coils induced magnetic moment in the spacecraft coordinates : $m = [m_1\ m_2\ m_3]^T$

Chapter 4

LQR design

Given the linearized spacecraft model which has the state variables composed of spacecraft quaternion **q**, the spacecraft rotational rate with respect to the LVLH frame $\boldsymbol{\omega}$, and the reaction wheel rotational speed , to control the spacecraft attitude and to manage the reaction wheel momentum are equivalent to minimize the following objective function:

$$\int_{0}^{\infty} (x_T Q x + u_T R u)\, dt$$

This is clearly an LQR design problem which has known efficient methods to solve.

4.1 Limiting case of zero inclination:

It was shown in [6] that a spacecraft without a reaction wheel in this orbit is not controllable. But for a spacecraft with three reaction wheels as we discussed in this paper, the system is fully controllable. The controllability condition can be checked by Kalman's test.

In this case, as we have seen from IGRF model of Earth geomagnetic field that the linear system is time invariant (i.e. the System matrix A and Input matrix B both are constants).

$$B = \begin{bmatrix} 0 \\ b2 \\ 0 \end{bmatrix}$$

Where, $b_2 = -\mu_f / a^3$

Therefore, a method for time-varying system is not appropriate for this simple problem. For this linear time invariant system, the optimal solution of is given by Riccati equation:

$$\mathbf{u = -K\, x = -R^{-1}\, B^{T}\, P}$$

where **P** is a constant positive semi definite solution of the following algebraic Riccati equation:

$$\mathbf{A^T P + PA - PBR^{-1} B^T P + Q = 0}$$

There is an efficient algorithms [22] for this system and a MATLAB function *lqr* implements this algorithm.

Chapter 5

Simulation test

Through the help of Simulation we would like to show that the proposed design achieves both attitude control and reaction wheel momentum management and would like to compare with the design [8] that does not use reaction wheels; our purpose is to show that using reaction wheels achieves better attitude pointing accuracy.
More important, we would like to demonstrate that the LQR design works very well for attitude and desaturation control for the nonlinear spacecraft.

The proposed algorithm has been tested using the arbitrary spacecraft model and orbit parameters which are well within the confined boundary defined earlier.

- Orbit inclination, $i_m = 0$
- Spacecraft inertia matrix, $\mathbf{J}$ = diag (250, 150, 100) kgm^2
- Circular orbit with altitude, h = 657 km.
- Orbital period, T = 5863 s (Calculated using time period formula)
- Orbital rate, ω_0 = 0.0011 rad/s.
- Initial quaternion error, q = [0.01 0.01 0.01]’
- Initial body rate vector, ω = [0.00001 0.00001 0.00001]’ rad/s
- Initial Reaction wheel speed vector,

  Ω = [0.00001 0.00001 0.00001]’ rad/s
- **Q** = diag ([0.001, 0.001, 0.001, 0.001, 0.001, 0.001, 0.02, 0.02, 0.02])
- **R** = diag([103, 103, 103, 102, 102, 102])

Here, **Q** and **R** both assumed arbitrarily.

5.1 MATLAB code

5.1.1 For the case when Reaction wheel and Magnetic coils used simultaneously

```
w0=0.0011;                % Orbital rate [rad/s]

Mf=7.9*10^15;           % Magneti field's dipole strength [Wbm]

J1=250;
J2=150;
J3=100;
J=diag([J1 J2 J3]);     % Spacecraft inertia matrix

Jw1=2.5;
Jw2=1.5;
Jw3=1;
Jw=diag([Jw1 Jw2 Jw3]);    % Reaction wheel inertia matrix

Q=diag([0.001 0.001 0.001 0.001 0.001 0.001 0.02 0.02 0.02]);
R=diag([1000 1000 1000 100 100 100]);

a=657000;
A13=w0*(J1-J2+J3)/(-J1);
A16=(w0*Jw3)/(-J1);
A17=8*(w0^2)*(J3-J2)/J1;
A28=6*(w0^2)*(J3-J1)/J2;
A31=w0*(J1-J2+J3)/(J3);
A34=(w0*Jw1)/(J3);
A39=2*(w0^2)*(J1-J2)/J3;

A=[0 0 A13 0 0 A16 A17 0 0;0 0 0 0 0 0 0 A28 0;A31 0 0 A34 0 0 0 0 A39;0 0 0 0 0 0 0 0 0;0 0 0 0 0 0 0 0 0;0 0 0 0 0 0 0 0 0;0.5 0 0 0 0 0 0 0 0;0 0.5 0 0 0 0 0 0 0;0 0 0.5 0 0 0 0 0 0];         % vpa for more precise values

im = 0;         % Orbital inclination [rad]

%b1=(Mf/(a^3))*cos(w0*t)*sin(im)
%b2=(-(Mf/(a^3))*cos(im))
%b3=2*(Mf/(a^3))*sin(w0*t)*sin(im)

b1=0;
b2=-(Mf/(a^3));
b3=0;

B11=1/J1;
B15=b3/J1;
B16=-b2/J1;
B22=1/J2;
```

```
B24=-b3/J2;
B26=b1/J2;
B33=1/J3;
B34=b2/J3;
B35=-b1/J3;
B41=1/Jw1;
B52=1/Jw2;
B63=1/Jw3;

B=[B11 0 0 0 B15 B16;0 B22 0 B24 0 B26;0 0 B33 B34 B35 0;B41 0 0 0 0 0;0 B52 0 0 0 0;0 0 B63 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0;0 0 0 0 0 0];
D=zeros(9,6);
%[P,L,G] = dare(A,B,Q,R);        % Solution of disrete-time algebric Riccati equation

%K=(inv(R+(transpose(B))*P*B))*(transpose(B))*P*A     % Control gain

[K,P,E] = lqr(A,B,Q,R);

%u=K*x;                           % Control law
AUG=A-B*K;
sys=ss((AUG),eye(9),eye(9),eye(9));
v=[0.00001;0.00001;0.00001;0.00001;0.00001;0.00001;0.01;0.01;0.01];

t = 0:1:35000;
x = initial(sys,v,t);

x1 = [1 0 0 0 0 0 0 0 0]*x';
x2 = [0 1 0 0 0 0 0 0 0]*x';
x3 = [0 0 1 0 0 0 0 0 0]*x';
x4 = [0 0 0 1 0 0 0 0 0]*x';
x5 = [0 0 0 0 1 0 0 0 0]*x';
x6 = [0 0 0 0 0 1 0 0 0]*x';
x7 = [0 0 0 0 0 0 1 0 0]*x';
x8 = [0 0 0 0 0 0 0 1 0]*x';
x9 = [0 0 0 0 0 0 0 0 1]*x';

plot(t,x1,"LineWidth",1)
hold on
plot(t,x2,"LineWidth",1)
hold on
plot(t,x3,"LineWidth",1)
hold off

title('Spacecraft body rate ω1, ω2, and ω3   ')
xlabel('Time [seconds]')
ylabel('rad/s')
legend({'w1', 'w2', 'w3'},'Location','southeast')

plot(t,x4,"LineWidth",1)
hold on
plot(t,x5,"LineWidth",1)
```

```
hold on
plot(t,x6,"LineWidth",1)
hold off

title('Reaction wheel speed Ω1, Ω2 and Ω3   ')
xlabel('Time [seconds]')
ylabel('rad/s')
legend({'Ω1', 'Ω2', 'Ω3'},'Location','northeast')


plot(t,x7,"LineWidth",1)
hold on
plot(t,x8,"LineWidth",1)
hold on
plot(t,x9,"LineWidth",1)
hold off

title('Spacecraft attitude (quaternion) q1, q2 and q3   ')
xlabel('Time [seconds]')
%ylabel('rad/s')
legend({'q1', 'q2', 'q3'},'Location','northeast')
```

5.1.2 For the case when only Magnetic torque coils are used

```
w0=0.0011;              % Orbital rate [rad/s]

Mf=7.9*10^15;           % Magneti field's dipole strength [Wbm]

J11=250;
J22=150;
J33=100;
J=diag([J11 J22 J33]);     % Spacecraft inertia matrix

im = 0;           % Orbital inclination [rad]

%Q=diag([1.5 1.5 1.5 1 1 1])

%R=diag([0.008 0.008 0.008])

Q=diag([0.02 0.02 0.02 0.001 0.001 0.001]);
R=diag([100 100 100]);

a=657000;

f41 = (8*(J33-J22)*w0*w0)/J11
f46 = ((-J11+J22-J33)*w0)/J11
```

```
f64 = ((J11-J22+J33)*w0)/J33
f52 = (6*(J33-J11)*w0*w0)/J22
f63 = (2*(J11-J22)*w0*w0)/J33

A = [0 0 0 0.5 0 0; 0 0 0 0 0.5 0; 0 0 0 0 0 0.5; f41 0 0 0 0 f46; 0 f52 0 0 0 0; 0 0 f63 f64 0 0]         % vpa for more precise values

im = 0;

b1 = 0;
b2 = -(Mf/(a^3))*cos(im);
b3 = 0;

%b1=(Mf/(a^3))*cos(w0*t)*sin(im)
%b2=(-(Mf/(a^3))*cos(im))
%b3=2*(Mf/(a^3))*sin(w0*t)*sin(im)

b42 = b3/J11
b43 = -b2/J11
b51 = -b3/J22
b53 = b1/J22
b61 = b2/J33
b62 = -b1/J33

B=[0 0 0; 0 0 0; 0 0 0; 0 b42 b43; b51 0 b53; b61 b62 0]
%[P,L,G] = dare(A,B,Q,R);        % Solution of disrete-time algebric Riccati equation

%K=(inv(R+(transpose(B))*P*B))*(transpose(B))*P*A     % Control gain

[K,P,E] = lqr(A,B,Q,R)

%AUG2 = A - B*K;
sys=ss((A - B*K),eye(6),eye(6),eye(6))

v=[0.01; 0.01; 0.01; 0.00001; 0.00001; 0.00001]

t = 0:1:35000;
x = initial(sys,v,t);

x1 = [1 0 0 0 0 0]*x';
x2 = [0 1 0 0 0 0]*x';
x3 = [0 0 1 0 0 0]*x';
x4 = [0 0 0 1 0 0]*x';
x5 = [0 0 0 0 1 0]*x';
x6 = [0 0 0 0 0 1]*x';

plot(t,x1,"LineWidth",1)
hold on
plot(t,x2,"LineWidth",1)
hold on
plot(t,x3,"LineWidth",1)
```

```
hold off
title('Spacecraft attitude (quaternion) q1, q2 and q3   ')
xlabel('Time [seconds]')
%ylabel('rad/s')
legend({'q1', 'q2', 'q3'},'Location','northeast')

plot(t,x4,"LineWidth",1)
hold on
plot(t,x5,"LineWidth",1)
hold on
plot(t,x6,"LineWidth",1)
hold off

title('Spacecraft body rate ω1, ω2, and ω3   ')
xlabel('Time [seconds]')
ylabel('rad/s')
legend({'w1', 'w2', 'w3'},'Location','southeast')

```

Chapter 6

Simulation results

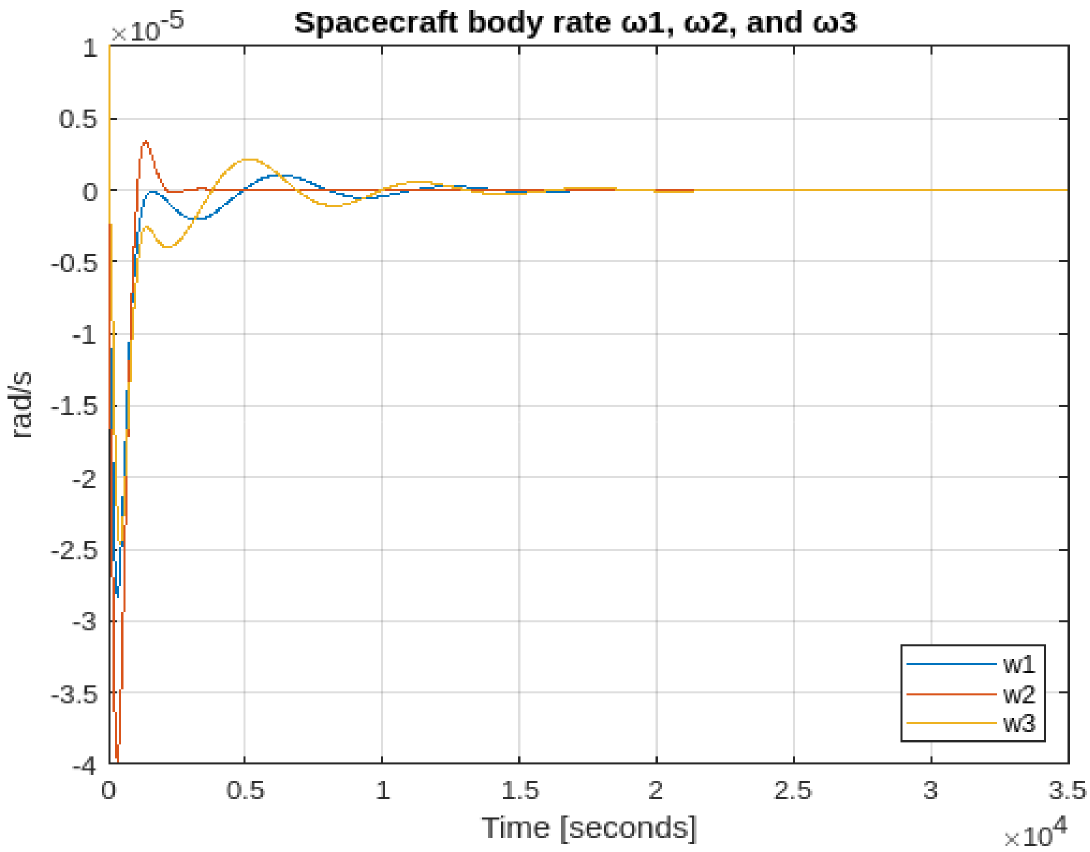

Fig. 1. Linear system body rate response $\omega 1$, $\omega 2$, and $\omega 3$

(Reaction wheel and Magnetic torque coil combined)

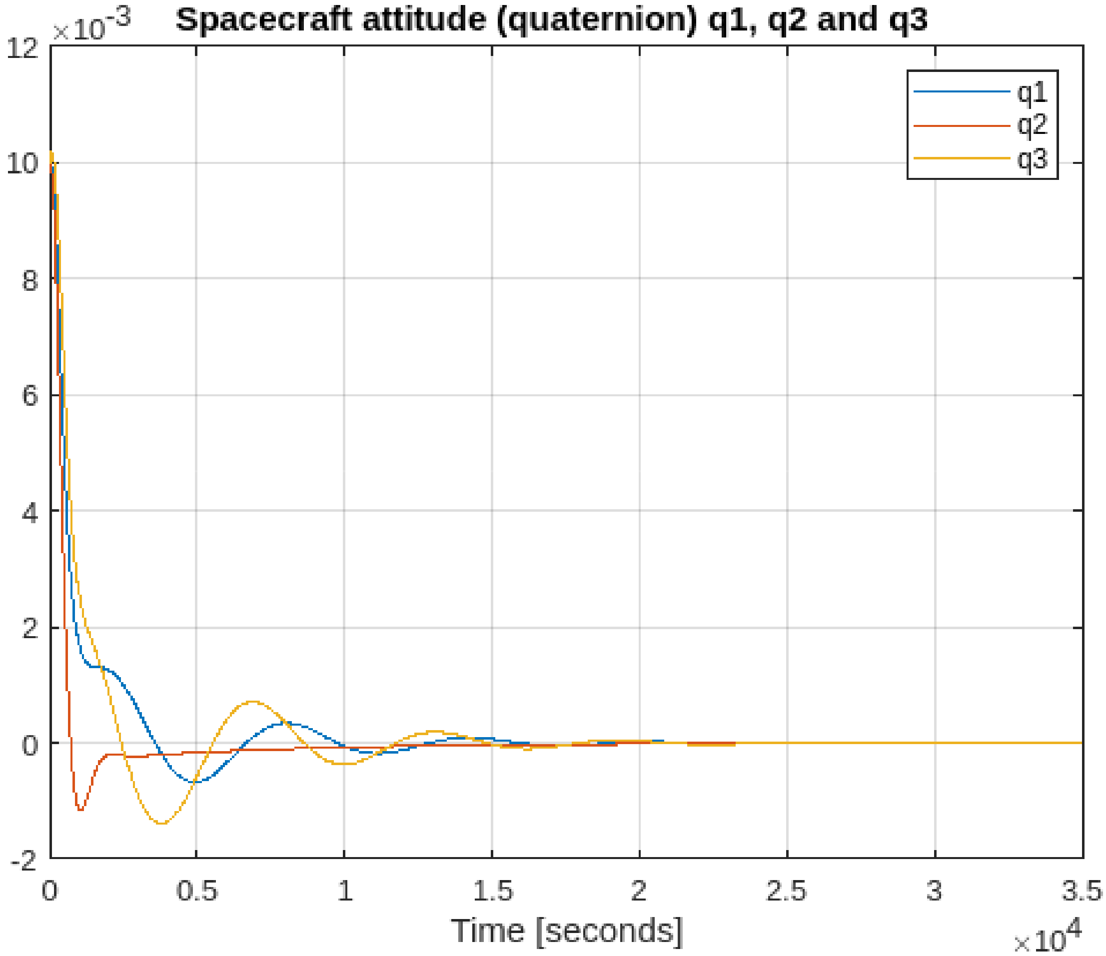

Fig. 2. Linear system attitude response *q*1, *q*2, and *q*3

(Reaction wheel and Magnetic torque coil combined)

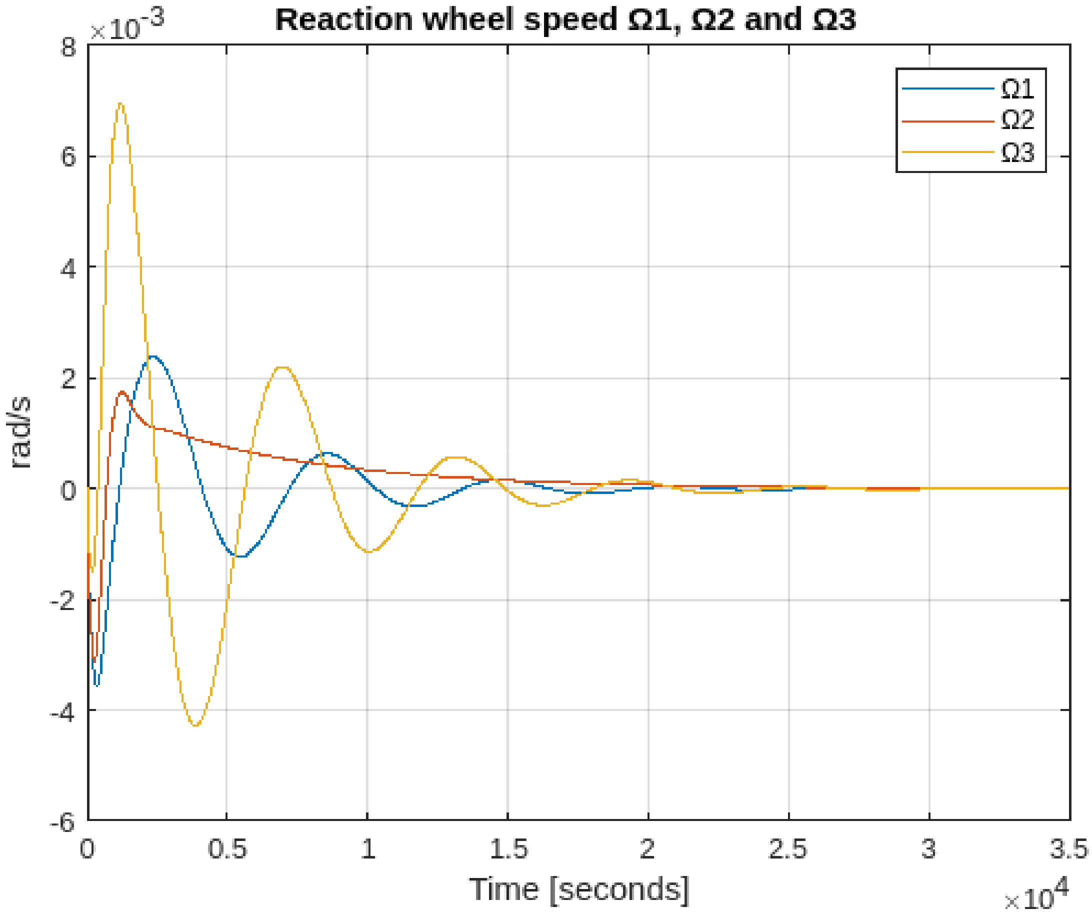

Fig. 3. Linear system reaction wheel response Ω1, Ω2, and Ω3

(Reaction wheel and Magnetic torque coil combined)

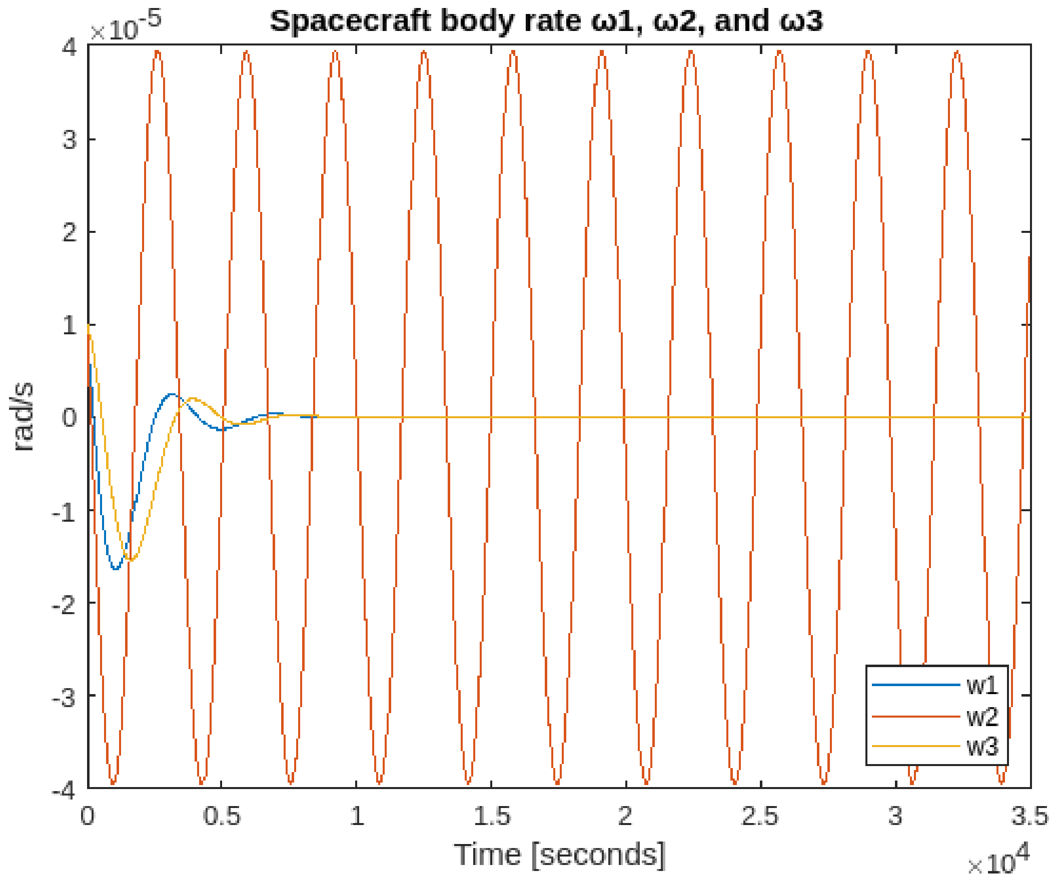

Fig. 4. Linear system body rate response $\omega 1$, $\omega 2$, and $\omega 3$

(Only Magnetic torque coil)

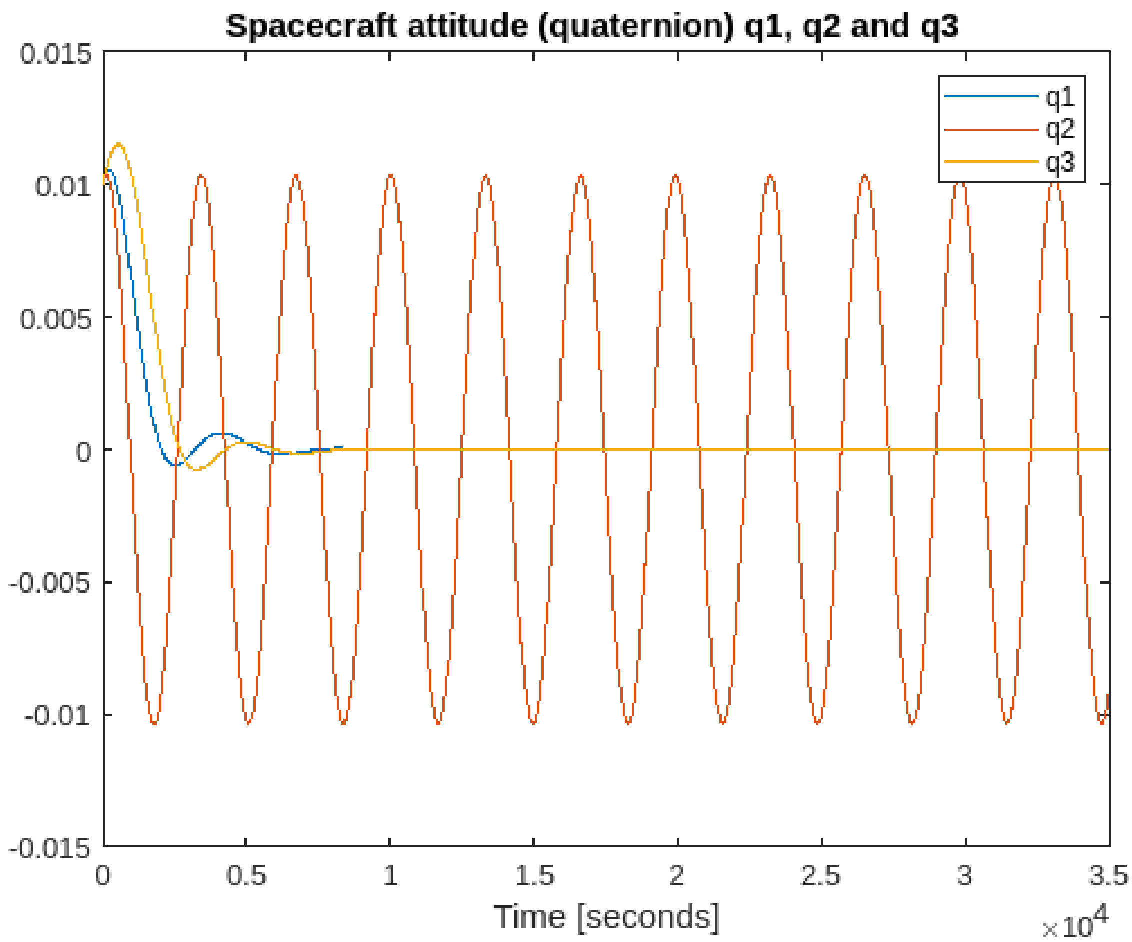

Fig. 5. Linear system attitude response *q*1, *q*2, and *q*3

(Only Magnetic torque coil)

- Comparing the response obtained here using both reaction wheels and magnetic torque coils and the response that uses magnetic torques only, we can see that both control methods stabilize the spacecraft, but using reaction wheels achieve much accurate nadir pointing.
- Also reaction wheel speeds approach to zero as *t* goes to infinity. Therefore, the second design goal for reaction wheel desaturation is achieved nicely.
- As magnetic torque coils can only generate torques in a two- dimensional plane, from simulation we can see that when only torque coils are used it can not asymptotically stabilize the spacecraft in at least one direction.

Chapter 7

Conclusion

In this project we investigated a LQR design method to control the spacecraft attitude to align the body frame with the LVLH frame and to desaturate the reaction momentum at the same time. A LQR based on Riccati equation is used for this purpose.

Simulation test is given to show that the design objective is achieved and the control system using both reaction wheels and magnetic torques accomplishes more accurate attitude in all the three axes than the control system that uses only magnetic torques.

Chapter 8

Future direction

In this project we consider the limiting case of spacecraft model when the orbit inclination is zero, but when the orbit is inclined with respect to the equator (i.e. orbit inclination is not zero) the control matrix B in the state space model will no longer remain constant rather it will vary with time according to IGRF model of and the components of magnetic field will come into picture. We can not solve the problem straightforward by using simple LQR and Riccati equation, instead we need to use Discrete Algebraic Riccati Equation (DARE) and Periodic Time-Varying Riccati (PTVR) equations (periodic because the orbit repeats after one orbital period.

Also the orbit considered is circular orbit, but when spacecraft orbiting in elliptical orbit, which is the common case for most of the satellites, then we need to modify the dynamics to include variation of distance from the Earth with respect to time as well.

Chapter 9

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