Question-01

```
A1=[-0.8 0.04 -0.88 0; 0 -0.18 0.035 -0.81; 1 -0.2 -0.6 0; 1 0 0 0];
B1=[0 -1.193; 225 0; 0 -0.035; 0 0];
C1=eye(4);
D1=zeros(4,2);


%[b,a] = ss2tf(A1,B1,C1,D1,1)
fprintf('input : 1-δth, 2-δe')
```

```
input : 1-δth, 2-δe
```

```
fprintf('States/Output : 1-q, 2-u_sp, 3-α, 4-θ')
```

```
States/Output : 1-q, 2-u_sp, 3-α, 4-θ
```

```


m = ss(A1,B1,C1,D1);
sys = tf(m)
```

```
sys =

  From input 1 to output...
                        9 s^2 + 45 s
   1:  ---------------------------------------------
       s^4 + 1.58 s^3 + 1.619 s^2 + 0.2814 s + 0.162

                  225 s^3 + 315 s^2 + 306 s
   2:  ---------------------------------------------
       s^4 + 1.58 s^3 + 1.619 s^2 + 0.2814 s + 0.162

                       -45 s^2 - 27 s
   3:  ---------------------------------------------
       s^4 + 1.58 s^3 + 1.619 s^2 + 0.2814 s + 0.162

                          9 s + 45
   4:  ---------------------------------------------
       s^4 + 1.58 s^3 + 1.619 s^2 + 0.2814 s + 0.162

  From input 2 to output...
       -1.193 s^3 - 0.8997 s^2 - 0.1317 s - 2.697e-35
   1:  ----------------------------------------------
       s^4 + 1.58 s^3 + 1.619 s^2 + 0.2814 s + 0.162

             -0.001225 s^2 + 0.9236 s + 0.5548
   2:  ---------------------------------------------
       s^4 + 1.58 s^3 + 1.619 s^2 + 0.2814 s + 0.162

        -0.035 s^3 - 1.227 s^2 - 0.2198 s - 0.1944
   3:  ---------------------------------------------
       s^4 + 1.58 s^3 + 1.619 s^2 + 0.2814 s + 0.162

              -1.193 s^2 - 0.8997 s - 0.1317
```

```
   4:  ----------------------------------------------
       s^4 + 1.58 s^3 + 1.619 s^2 + 0.2814 s + 0.162

Continuous-time transfer function.
```

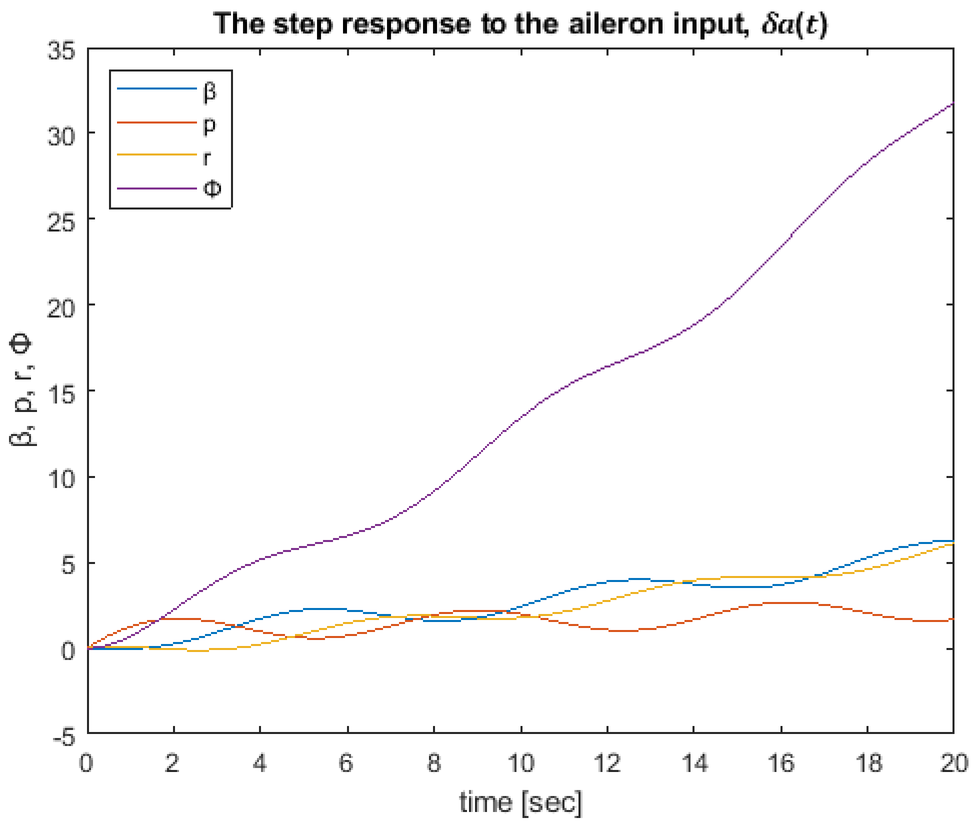
```
%sys = tf (b,a,'InputName', 'δth', 'δe', 'OutputName', {'q','u','a','t'})
```

Question-02

```
A2=[-0.05 -0.003 -0.98 0.2; -1 -0.75 1 0; 0.3 -0.3 -0.15 0; 0 1 0 0];
B2=[0 0; 1.7 -0.2; 0.3 -0.6; 0 0];
C2=eye(4);
D2=zeros(4,2);
```

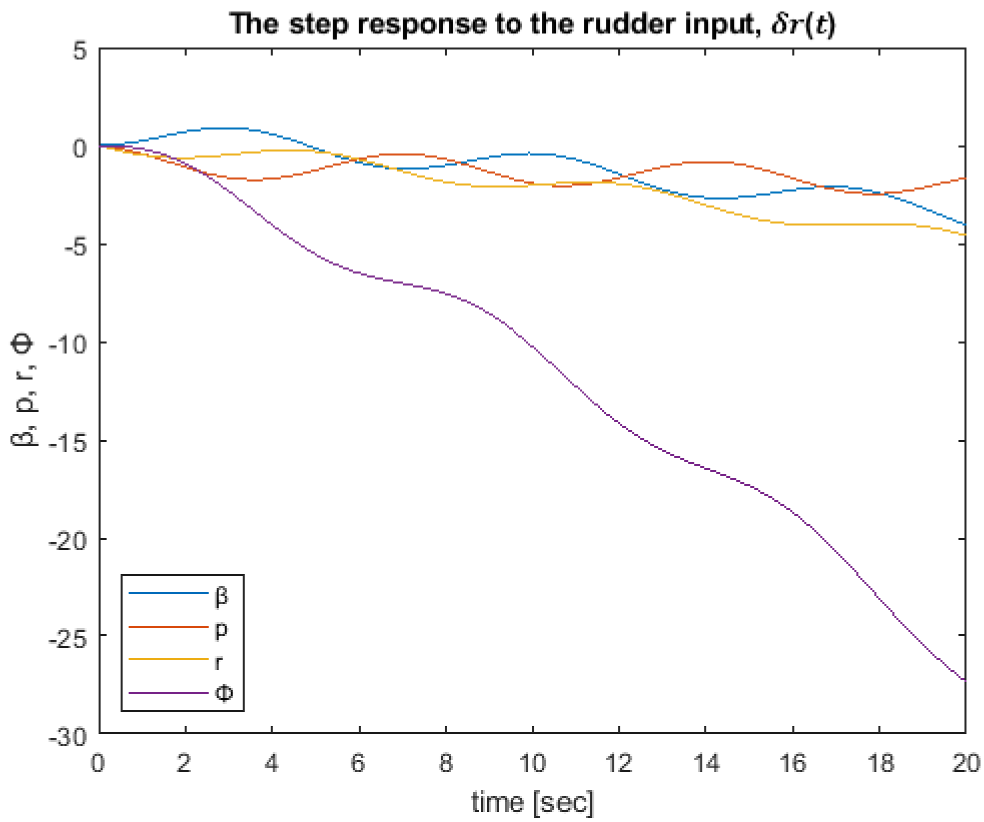
Question-02 (a)

```
t = 0:0.01:20;
[y,x,t] = step(A2,B2,C2,D2,1,t);
plot(t,x)
legend({'β','p','r','Φ'},'Location','northwest')
title('The step response to the aileron input, δa(t)')
xlabel('time [sec]')
ylabel('β, p, r, Φ')
```



Question-02 (b)

```
[y,x,t] = step(A2,B2,C2,D2,2,t);
plot(t,x)
legend({'β','p','r','Φ'},'Location','southwest')
title('The step response to the rudder input, δr(t)')
xlabel('time [sec]')
ylabel('β, p, r, Φ')
```



Question-02 (c)

```
Co = ctrb(A2,B2);
P=rank(Co);
if P==4
    fprintf('The system is completely controllable using both the inputs.')
else
    fprintf('The system is not completely controllable using both the inputs.')
end
```

```
The system is completely controllable using both the inputs.
```

Question-02 (d)

```
B2=[0 0; 1.7 0; 0.3 0; 0 0];
Co = ctrb(A2,B2);
P=rank(Co);
if P==4
    fprintf('The aircraft is controllable using only the aileron inputs, δa.')
else
```

```
    fprintf('The aircraft is not controllable using only the aileron inputs, δa.')
end
```

The aircraft is controllable using only the aileron inputs, δa.

Question-02 (e)

```
B2=[0 0; 0 -0.2; 0 -0.6; 0 0];
Co = ctrb(A2,B2);
P=rank(Co);
if P==4
    fprintf('The aircraft is controllable using only the rudder input, δr.')
else
    fprintf('The aircraft is not controllable using only the rudder input, δr.')
end
```

The aircraft is controllable using only the rudder input, δr.

Question-02 (f)

```
C2=[0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 1];
Ob = obsv(A2,C2);
P=rank(Ob);
if P==4
    fprintf('The aircraft is observable with the bank-angle, ϕ(t), being the only measured output.')
else
    fprintf('The aircraft is not observable with the bank-angle, ϕ(t), being the only measured output.')
end
```

The aircraft is observable with the bank-angle, $\phi(t)$, being the only measured output.

Question-02 (g)

```
C2=[1 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];
Ob = obsv(A2,C2);
P=rank(Ob);
if P==4
    fprintf('The aircraft is observable with the sideslip-angle, β(t), being the only measured output.')
else
    fprintf('The aircraft is not observable with the sideslip-angle, β(t), being the only measured output.')
end
```

The aircraft is observable with the sideslip-angle, $\beta(t)$, being the only measured output.

Question-02 (h)

```
dp = [-1.5 0.05 -0.35+j*(1.5) -0.35-j*(1.5)];
K = place(A2,B2,dp)
```

```
K = 2×4
         0         0         0         0
   -1.0714   -4.9045   -0.3652    0.4882
```

Question-02 (i)

```
% *** Taking only aileron input ***
r=1;                                 % for step response
B2=[0 0; 1.7 0; 0.3 0; 0 0];
dp = [-1+j*1 -1-j*1 -15 -0.8];
K1 = place(A2,B2,dp);
AUG_2_i1 = A2 - B2*K1;


Co = ctrb(AUG_2_i1,B2);
P=rank(Co);
if P==4
    fprintf('The system is controllable using only the aileron input, δa(t), we can design a full-state feedback control.')
else
    fprintf('The system is not controllable using only the aileron input, δa(t), we can design a full-state feedback control.')
end
```

The system is controllable using only the aileron input, $\delta a(t)$, we can design a full-state feedback control.

```
fprintf('Control gain matrix using only the aileron input:\n')
```

Control gain matrix using only the aileron input:

```
Gm = K1   % gain matrix K
```

```
Gm = 2×4
   43.7313   12.8379  -16.5816   26.6107
         0         0         0         0
```

```
% *** Taking only rudder input ***
r=1;                                % for step response
B2=[0 0; 0 -0.2; 0 -0.6; 0 0];
dp = [-1+j*1 -1-j*1 -15 -0.8];
K2 = place(A2,B2,dp);
AUG_2_i2 = A2 - B2*K2;
```

```
Co = ctrb(AUG_2_i2,B2);
P=rank(Co);
if P==4
    fprintf('The system is completely controllable, we can design a full-state feedback control.')
else
    fprintf('The system is not completely controllable, we can not design a full-state feedback control.')
end
```

```
The system is completely controllable, we can design a full-state feedback control.
```

```
fprintf('Control gain matrix using only the rudder input:\n')
```

```
Control gain matrix using only the rudder input:
```

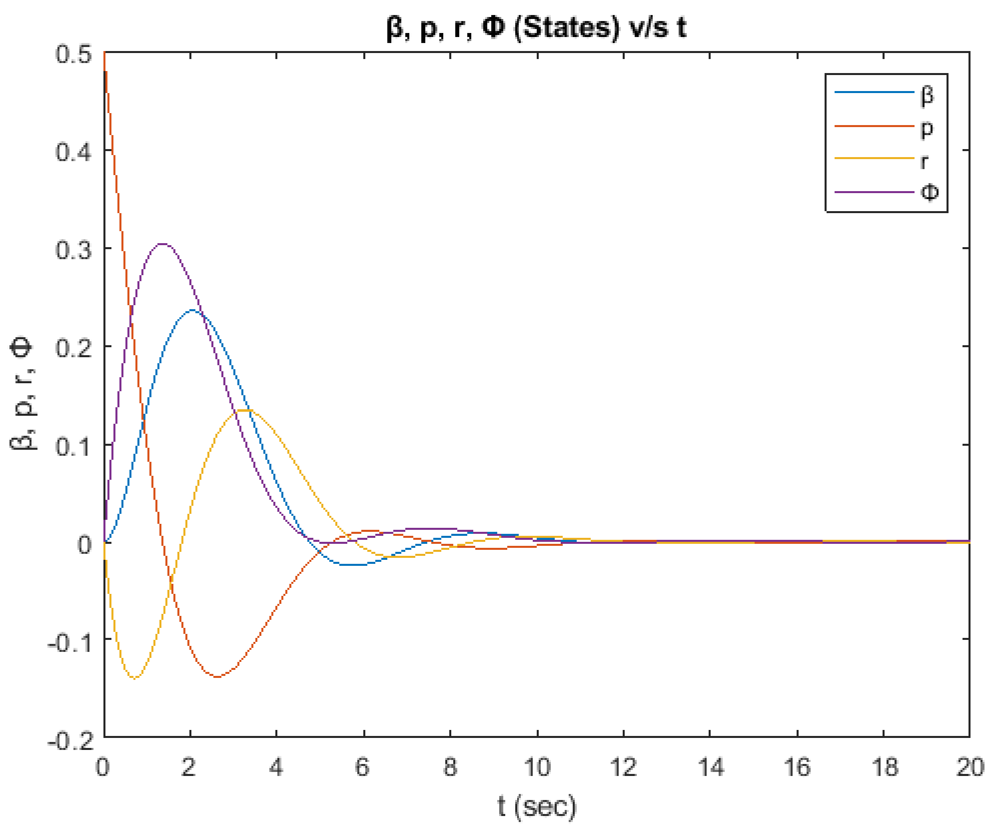
```
Gm = K2   % gain matrix K
```

```
Gm = 2×4
         0          0          0          0
    6.0592   -45.3680   -12.9607   -34.0302
```

Question-02 (j)

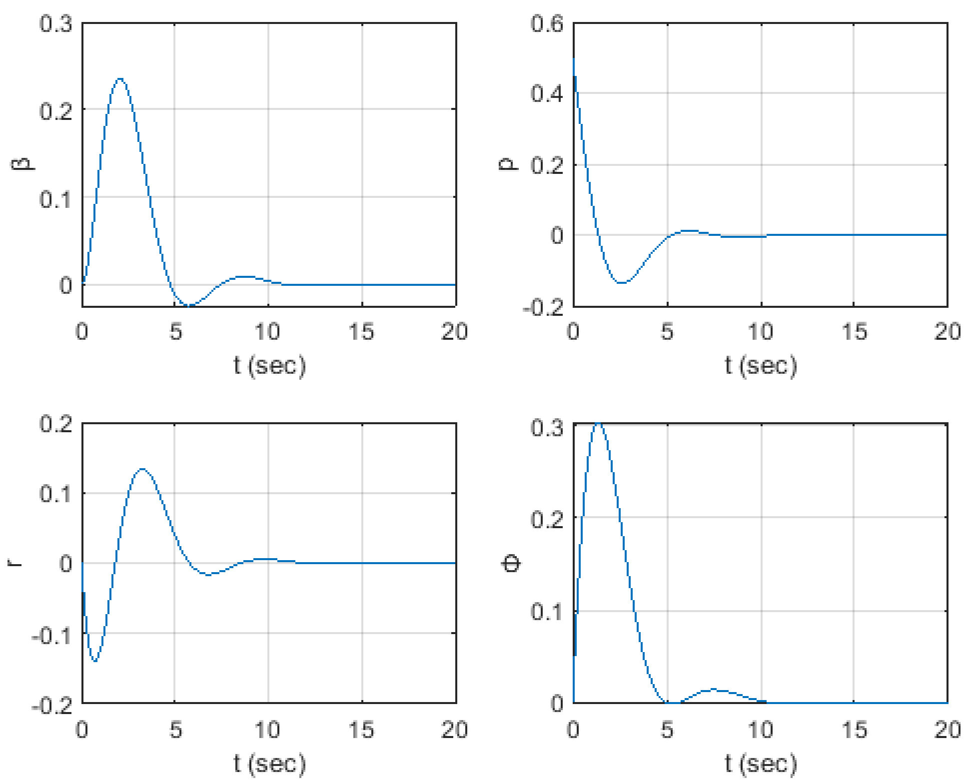
```
sys = ss(A2,B2,C2,D2);
Q2 = eye(4);
R2 = eye(1);

[Kn,S,P] = lqr(sys,Q2,R2);

t = 0:0.1:20;

AUG_j = A2 - B2*Kn;

sys = ss(AUG_j,eye(4),eye(4),eye(4));
v = [0; 0.5; 0; 0];
x=initial(sys,v,t);

x1 = [1 0 0 0]*x';
x2 = [0 1 0 0]*x';
x3 = [0 0 1 0]*x';
x4 = [0 0 0 1]*x';

plot(t,x1)
```

```
hold on
plot(t,x2)
hold on
plot(t,x3)
hold on
plot(t,x4)
hold off
title('β, p, r, Φ (States) v/s t')
xlabel('t (sec)')
ylabel('β, p, r, Φ')
legend('β','p','r','Φ')
```




```
subplot(2,2,1); plot(t,x1); grid
xlabel('t (sec)')
ylabel('β')


subplot(2,2,2); plot(t,x2); grid
xlabel('t (sec)')
```

```
ylabel('p')


subplot(2,2,3); plot(t,x3); grid
xlabel('t (sec)')
ylabel('r')


subplot(2,2,4); plot(t,x4); grid
xlabel('t (sec)')
ylabel('Φ')
```



Question-03

```
A3=[0 1 0 0; 0 0 -1 0; 0 0 0 1; 0 0 11 0];
B3=[0; 1; 0; -1];
C3=[1 0 0 0];
D3=[0];
```

Question-03 (a)

```
Q3 = 2*eye(4);
R3 = eye(1);
[K3,S3,P3] = lqr(A3,B3,Q3,R3);
fprintf('Dominant eigenvalue:')
```

```
Dominant eigenvalue:
```

```
eig(A3-B3*K3)
```

```
ans = 4×1 complex
  -4.1425 + 0.0000i
  -2.6871 + 0.0000i
  -1.0198 + 0.4800i
  -1.0198 - 0.4800i
```

Question-03 (b)

```
% *** First solve pole-placement problem ***
Ob = obsv(A3,C3)
```

```
Ob = 4×4
     1     0     0     0
     0     1     0     0
     0     0    -1     0
     0     0     0    -1
```

```
N = rank(Ob)
```

```
N = 4
```

```
if N==4
    fprintf('The system is completely Observable.')
else
    fprintf('The system is not completely Observable.')
end
```

```
The system is completely Observable.
```

```
% *** Desired characteristic polynomial ***
J0 = [-2 -3 -2+j*1 -2-j*1];
poly(J0)
```

```
ans = 1×5
     1     9    31    49    30
```

```
% *** Characteristic Polynomial ***
Ph = polyvalm(poly(J0),A3);


% *** The observer Gain Matrix ***
```

```
fprintf('The observer Gain Matrix:\n')
```

```
The observer Gain Matrix:
```

```
Ke = Ph*(inv(N'))*[0; 0; 0; 1]
```

```
Ke = 4×1
   -2.2500
  -10.5000
   37.0000
  123.0000
```

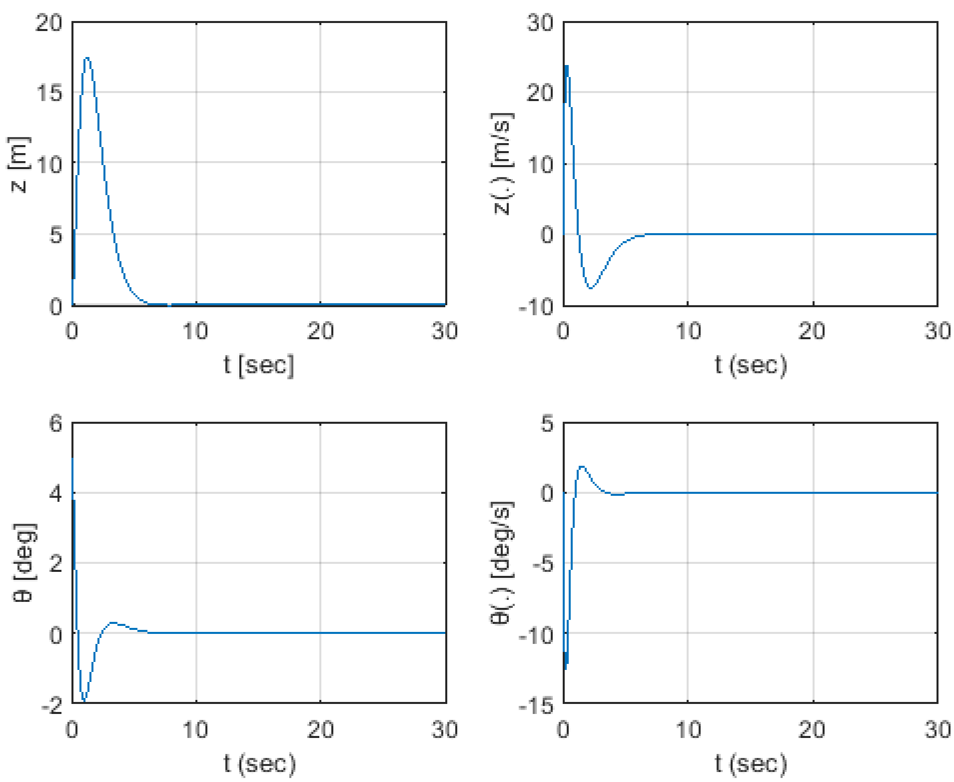
Question-03 (c)

```
% *** Output variable trajectory**
fprintf('Output variable trajectory:\n')
```

```
Output variable trajectory:
```

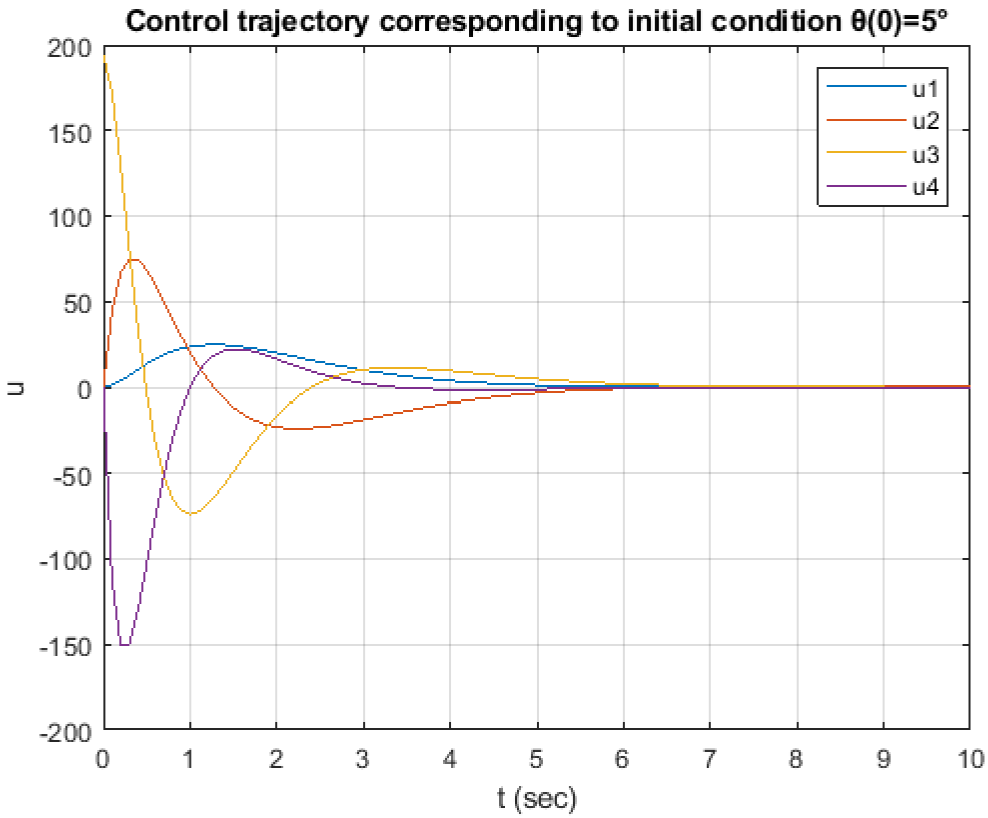
```
t = 0:0.1:30;


AUG_k = A3 - B3*K3;


sys = ss(AUG_k,eye(4),eye(4),eye(4));
v3 = [0; 0; 5; 0];
y = initial(sys,v3,t);


y1 = [1 0 0 0]*y';
y2 = [0 1 0 0]*y';
y3 = [0 0 1 0]*y';
y4 = [0 0 0 1]*y';


plot(t,y1)
hold on
plot(t,y2)
hold on
plot(t,y3)
hold on
plot(t,y4)
hold off
title('y (States) v/s t')
xlabel('t (sec)')
ylabel('y')
legend('z','z(.)','θ','θ(.)')


subplot(2,2,1); plot(t,y1); grid
xlabel('t [sec]')
ylabel('z [m]')
```

```
subplot(2,2,2); plot(t,y2); grid
xlabel('t (sec)')
ylabel('z(.) [m/s]')


subplot(2,2,3); plot(t,y3); grid
xlabel('t (sec)')
ylabel('θ [deg]')


subplot(2,2,4); plot(t,y4); grid
xlabel('t (sec)')
ylabel('θ(.) [deg/s]')
```




```
%*** Control trajectory ***
t = 0:0.1:10;
```

```
AUG_k = A3 - B3*K3;
sys = ss(AUG_k,eye(4),eye(4),eye(4));
v3 = [0; 0; 5; 0];
y = initial(sys,v3,t);


y1 = [1 0 0 0]*y';
y2 = [0 1 0 0]*y';
y3 = [0 0 1 0]*y';
y4 = [0 0 0 1]*y';


u1 = -K3(1,1)*y1;
u2 = -K3(1,2)*y2;
u3 = -K3(1,3)*y3;
u4 = -K3(1,4)*y4;


subplot(1,1,1); plot(t,u1); grid
hold on
plot(t,u2)
hold on
plot(t,u3)
hold on
plot(t,u4)
hold off
title('Control trajectory corresponding to initial condition θ(0)=5°')
xlabel('t (sec)')
ylabel('u')
legend('u1', 'u2', 'u3', 'u4')
```


Control trajectory corresponding to initial condition θ(0)=5°
u
t (sec)
u1
u2
u3
u4
200
150
100
50
0
-50
-100
-150
-200
0
1
2
3
4
5
6
7
8
9
10