Problem Statement:

A propeller airplane weighs 45000 N and has a wing area of 28 m2. The drag polar is given by $CD = 0.021 + 0.045CL2$, and the maximum lift coefficient is 1.5. The power developed by the engine is 900 kW with a propulsive efficiency of 0.82. The structural limit load factor is 3.0. Determine the performance characteristics for (a) fastest sustained rate of turn, (b) sharpest sustained rate of turn, and (c) maximum load factor turn.

MATLAB code

MSTR (Maximum Sustained Turn rate) for PROP ACFT

```
W=45000;      % Weight of aircraft [N]
S=28;         % Wing area [m^2]
Cd0=0.021;    % Lift independent drag
k=0.045;
Clmax=1.5;    % Max Lift Coefficient
Psh=900000;   % shaft power [W]
np=0.82;      % Propeller efficiency
nlim=3;       % Limit load factor
d=1.225;      % Density [kg/m^3], considered at mean sea level (MSL)
g=9.81;       % Gravitational acceleration [m/s^2]


Pa=np*Psh;    % Power Available


V=20:0.1:120;


n=(S/W)*sqrt(((Pa*d.*V)./(2*k*S))-((d*d*Cd0.*V.^4)./(4*k)));       % Load factor
Vstall=sqrt((2*W.*n)./(d*Clmax*S));




%[maxval_n,idx] = fun(fun==0)     % Max load factor
%V_maxval_n = V(idx)         % Value corresponding to Max load factor


%X_int=fsolve(fun,0);
```

MSTR (Maximum Sustained Turn Rate)

```
fprintf('<strong> MSTR - Maximum Sustained Turn Rate </strong>')


Cl=(2*n.*W)./(d.*V.*V.*S);       % Coefficient of lift
w=g*sqrt(n.*n-1)./V;             % Rate of Turn


plot(V,w,'k',"LineWidth",1)
hold on
plot(Vstall,w,'r--',"LineWidth",1)
hold off
title('Turn rate v/s Velocity (PROP ACFT)')
xlabel('Velocity [m/s]')
ylabel('Turn rate [rad/s]')
legend('Turn rate','Stall velo.')
text(90,0.4,'Possible')
text(90,0.37,'flight')
text(90,0.34,'region')


fprintf('<strong> # Intersection of w v/s V and w v/s Vstall : </strong>')


b_line1 = polyfit(V,w,4);
y_line1 = polyval(b_line1,Vstall);
x_int1 = interp1((y_line1 - w),Vstall,0)
y_int1 = polyval(b_line1,x_int1)


fprintf('<strong> # Performance characteristics (MSTR) : </strong>')


n = (S/W)*sqrt(((Pa*d*x_int1)/(2*k*S))-((d*d*Cd0*x_int1^4)/(4*k)));


if n<nlim
    n_MSTR = n;
    V_MSTR = x_int1;
    fprintf('n_MSTR = %f (Load Factor corresponding to MSTR)\n', n_MSTR)
    fprintf('V_MSTR = %f m/s(Velocity corresponding to MSTR)\n', V_MSTR)
else
    n_MSTR = nlim;
    fprintf('n_MSTR = %f (Load Factor corresponding to MSTR)\n', n_MSTR)
    A1 = (d*d*Cd0)/(4*k);
    B1 = -(Pa*d)/(2*S*k);
    C1 = (n_MSTR*n_MSTR*W*W)/(S*S);
```

```
    L1 = roots([A1 0 0 B1 C1]);
    V_MSTR=real(L1(imag(L1) == 0))
    fprintf('V_MSTR = %f m/s (Velocity corresponding to MSTR)\n', V_MSTR)
end


w_MSTR=g*sqrt((n_MSTR^2)-1)/V_MSTR;
fprintf('w_MSTR = %f rad/s (Maximum Turn Rate)\n', w_MSTR)


R_MSTR=(V_MSTR^2)/(g*sqrt(n_MSTR^2-1));
fprintf('R_MSTR = %f m (Turn Radius corresponding to MSTR)\n', R_MSTR)


t_2pi_MSTR=2*pi/w_MSTR;
fprintf('t_2pi_MSTR = %f s (Time to complete one turn corresponding to MSTR)\n',
t_2pi_MSTR)


Cl_MSTR=(2*n_MSTR*W)/(S*d*V_MSTR^2);
fprintf('Cl_MSTR = %f (Lift coefficient corresponding to MSTR)\n', Cl_MSTR)
```

STR (Sharpest Turn Radius)

```
fprintf('<strong> STR - Sharpest Turn Radius </strong>')


V=20:0.1:120;


n=(S/W)*sqrt(((Pa*d.*V)./(2*k*S))-((d*d*Cd0.*V.^4)./(4*k)));       % Load factor
Vstall=sqrt((2*W.*n)./(d*Clmax*S));


R=(V.*V)./(g*sqrt(n.*n-1));     % Turn Radius


plot(V,R,'k',"LineWidth",1)
hold on
plot(Vstall,R,'r--',"LineWidth",1)
hold off


title('Turn Radius v/s Velocity (PROP ACFT)')
xlabel('Velocity [m/s]')
ylabel('Turn Radius [m]')
legend('Turn Radius','Stall velo.')
text(85,200,'Possible')
text(85,140,'flight')
text(85,80,'region')
```

```
fprintf('<strong> # Intersection of R v/s V and R v/s Vstall : </strong>')


b_line2 = polyfit(V,R,4);
y_line2 = polyval(b_line2,Vstall);
x_int2 = interp1((y_line2 - R),Vstall,0)
y_int2 = polyval(b_line2,x_int2)


fprintf('<strong> # Performance characteristics (STR) : </strong>')


n = (S/W)*sqrt(((Pa*d*x_int2)/(2*k*S))-((d*d*Cd0*x_int2^4)/(4*k)));


if n<nlim
    n_STR = n;
    V_STR = x_int2;
    fprintf('n_STR = %f (Load Factor corresponding to STR)\n', n_STR)
    fprintf('V_STR = %f m/s(Velocity corresponding to STR)\n', V_STR)
else
    n_STR = nlim;
    fprintf('n_STR = %f (Load Factor corresponding to STR)\n', n_STR)
    A1 = (d*d*Cd0)/(4*k);
    B1 = -(Pa*d)/(2*S*k);
    C1 = (n_STR*n_STR*W*W)/(S*S);
    L1 = roots([A1 0 0 B1 C1]);
    V_STR=real(L1(imag(L1) == 0))
    fprintf('V_STR = %f m/s (Velocity corresponding to STR)\n', V_STR)
end


w_STR=g*sqrt((n_STR^2)-1)/V_STR;
fprintf('w_STR = %f rad/s (Maximum Turn Rate)\n', w_STR)


R_STR=(V_STR^2)/(g*sqrt(n_STR^2-1));
fprintf('R_STR = %f m (Turn Radius corresponding to STR)\n', R_STR)


t_2pi_STR=2*pi/w_STR;
fprintf('t_2pi_STR = %f s (Time to complete one turn corresponding to STR)\n',
t_2pi_STR)


Cl_STR=(2*n_STR*W)/(S*d*V_STR^2);
fprintf('Cl_STR = %f (Lift coefficient corresponding to STR)\n', Cl_STR)
```

MLT (Maximum Load Factor Turn)

```
fprintf('<strong> MLT - Maximum Load factor Turn </strong>')


V=20:0.1:120;
n=(S/W)*sqrt(((Pa*d.*V)./(2*k*S))-((d*d*Cd0.*V.^4)./(4*k)));       % Load factor


plot(V,n,'k',"LineWidth",1)
hold on
plot(Vstall,n,'r--',"LineWidth",1)
hold off
title('Turn Radius v/s Velocity (PROP ACFT)')
xlabel('Velocity [m/s]')
ylabel('Load Factor (n)')
legend({'Load Factor','Stall velo.'},'Location','southeast')
text(75,2.75,'Possible')
text(75,2.65,'flight')
text(75,2.55,'region')


fprintf('<strong> # Intersection of n v/s V and n v/s Vstall : </strong>')


b_line3 = polyfit(V,n,4);
y_line3 = polyval(b_line3,Vstall);
x_int3 = interp1((y_line3-n),Vstall,0)
y_int3 = polyval(b_line3,x_int3)
```

MLT (Direct method)

```
fprintf('<strong> # Performance characteristics (MLT) : </strong>')


V_n_max=(Pa/(2*S*d*Cd0))^(1/3);
fprintf('V_n_max = %f m/s (Velocity corresponding to MLT)\n', V_n_max)


n_max=(S/W)*sqrt(((Pa*d)/(2*S*k))*V_n_max-((d*d*Cd0)/(4*k))*V_n_max^4);
fprintf('n_max = %f (Maximum Load Factor corresponding to MLT)\n', n_max)


R_n_max=(V_n_max^2)/(g*sqrt(n_max*n_max-1));
fprintf('R_n_max = %f m (Turn Radius corresponding to MLT)\n', R_n_max)
```

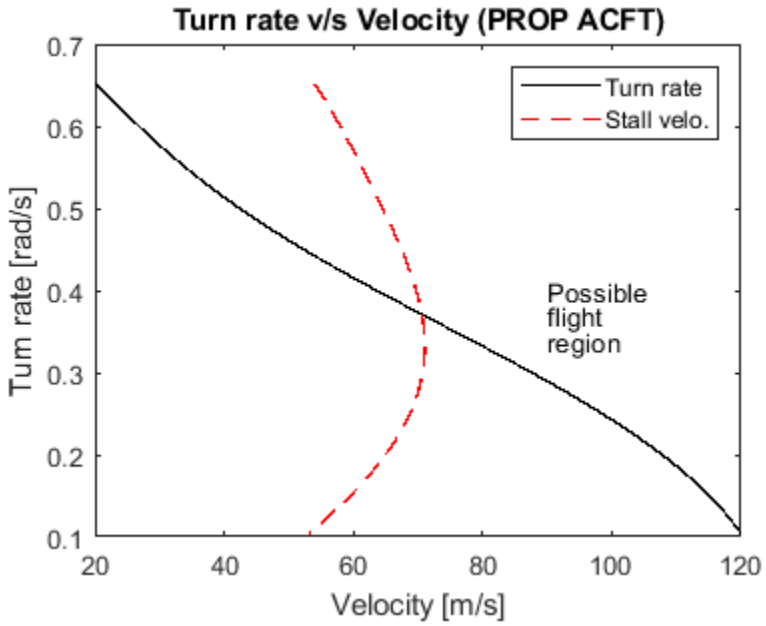
```
w_n_max=V_n_max/R_n_max;
fprintf('w_n_max = %f rad/s (Velocity corresponding to MLT)\n', w_n_max)


t_2pi_n_max=2*pi/w_n_max;
fprintf('t_2pi_n_max = %f s (Time to complete one turn corresponding to MLT)\n',
t_2pi_n_max)


Cl_n_max=(2*n_max*W)/(S*d*V_n_max^2);
fprintf('Cl_n_max = %f (Lift coefficient corresponding to Max n Turn)\n',
Cl_n_max)
```

OUTPUT

MSTR - Maximum Sustained Turn Rate



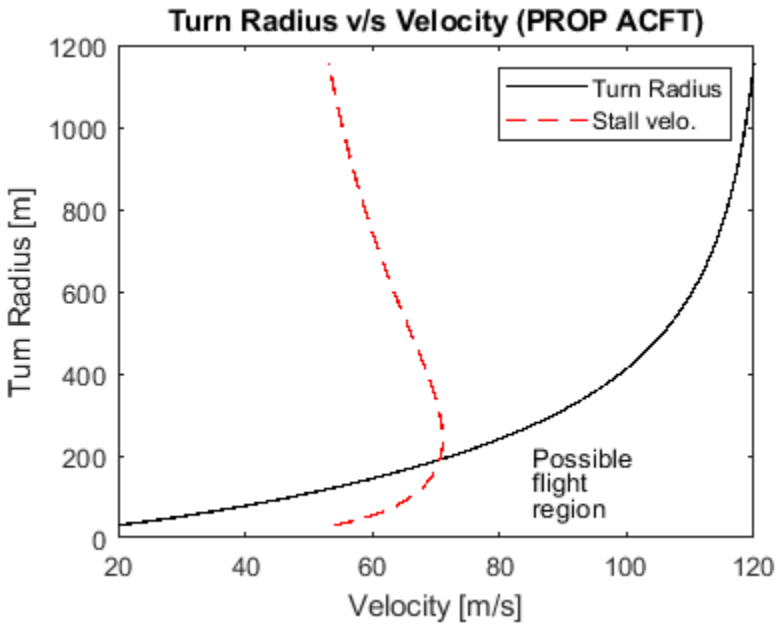
 # Intersection of w v/s V and w v/s Vstall :

```
x_int1 = 70.6176
y_int1 = 0.3703
```

 # Performance characteristics (MSTR) :

```
n_MSTR = 2.850039 (Load Factor corresponding to MSTR)
V_MSTR = 70.617584 m/s(Velocity corresponding to MSTR)
w_MSTR = 0.370748 rad/s (Maximum Turn Rate)
R_MSTR = 190.473153 m (Turn Radius corresponding to MSTR)
t_2pi_MSTR = 16.947311 s (Time to complete one turn corresponding to MSTR)
Cl_MSTR = 1.499593 (Lift coefficient corresponding to MSTR)
```

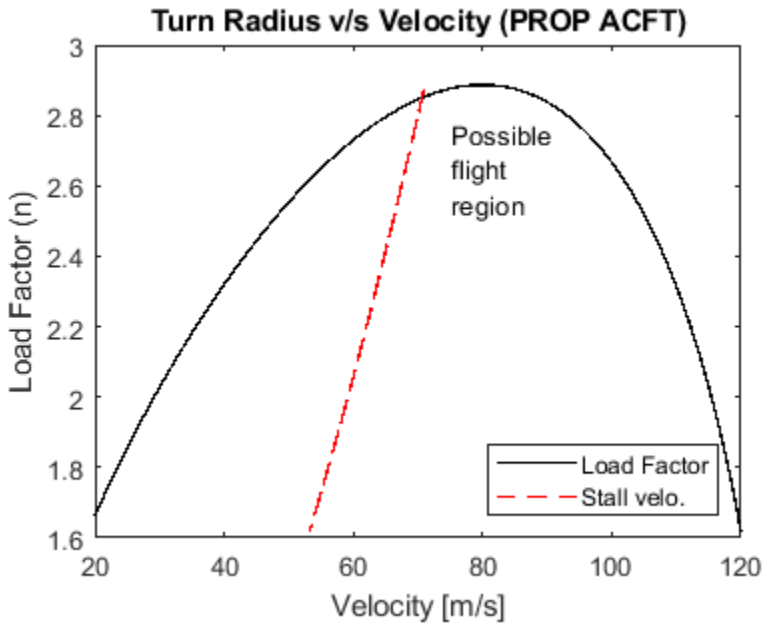
STR - Sharpest Turn Radius



 # Intersection of R v/s V and R v/s Vstall :
x_int2 = 70.7392
y_int2 = 197.8166

 # Performance characteristics (STR) :
n_STR = 2.850959 (Load Factor corresponding to STR)
V_STR = 70.739157 m/s(Velocity corresponding to STR)
w_STR = 0.370247 rad/s (Maximum Turn Rate)
R_STR = 191.059171 m (Turn Radius corresponding to STR)
t_2pi_STR = 16.970236 s (Time to complete one turn corresponding to STR)
Cl_STR = 1.494926 (Lift coefficient corresponding to STR)

MLT - Maximum Load factor Turn



 # Intersection of n v/s V and n v/s Vstall :

```
x_int3 = 70.5599
y_int3 = 2.8462
```

 # Performance characteristics (MLT) :

```
V_n_max = 80.014922 m/s (Velocity corresponding to MLT)
n_max = 2.887072 (Maximum Load Factor corresponding to MLT)
R_n_max = 240.972484 m (Turn Radius corresponding to MLT)
w_n_max = 0.332050 rad/s (Velocity corresponding to MLT)
t_2pi_n_max = 18.922405 s (Time to complete one turn corresponding to MLT)
Cl_n_max = 1.183216 (Lift coefficient corresponding to Max n Turn)
```