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Course :- AE-686A (Helicopter Theory)

$\boxed{\text{HOMEWORK-4}}$

[Question-01]

* **Steps of calculation :-**

Step 1 :- Rotor is divided into $n=20$ segments.
& one calculation is done at 5° difference in azimuth for 5 revolutions.

Step 2 :- Numerical integration to find $\overline{M_\beta}$.

$$\overline{M_\beta} = \frac{1}{2}\int_0^1 r\left[(r+\mu\sin\psi)^2\cdot\theta - (\lambda + r\overset{*}{\beta} + \mu\beta\cos\psi)(r+\mu\sin\psi)\right]dr \quad (1)$$

where, $\theta = \theta_{75} + \theta_{tw}(r-0.75) + \theta_{1s}\sin\psi + \theta_{1c}\cdot\cos\psi$

- The numerical integration is performed using six point Gaussian quadrature formula

Step 3 :- Calculate exact value of $\overline{M_\beta}$ from Analytical calculation

$$\cancel{\overline{M_\beta} = \frac{1}{2}\int_0^1 r\{(r+\mu\sin\psi}$$

$$\overline{M_\beta} = \left(\frac{1}{8} + \frac{\mu}{3}\sin\psi + \frac{\mu^2}{4}\sin^2\psi\right)(\theta_0 + \theta_{1c}\cdot\cos\psi + \theta_{1s}\cdot\sin\psi) - \lambda\left(\frac{1}{6} + \frac{\mu}{4}\sin\psi\right)$$
$$+ \theta_{tw}\left(\frac{1}{10} + \frac{\mu^2}{6}\sin^2\psi + \frac{\mu}{4}\sin\psi\right)$$
$$- \overset{*}{\beta}\left(\frac{1}{8} + \frac{\mu}{6}\sin\psi\right) - \mu\beta\cos\psi\left(\frac{1}{6} + \frac{\mu}{4}\sin\psi\right) \quad (2)$$

* Comments :-

→ The ~~rotor~~ blade starts revolution from 0° (from tail) in ~~and~~ counterclockwise direction.

→ There are two drops in flap moment (non-dimention $\bar{M}_B$) for every revation. i.e 0 to 2π OR 0° to 360°.

→ The complete revolution can be divided into two parts (in terms of aerodynamics)

① Advancing side (0 to π)

- The moment drop is much deeper as compared to the retreating side.
- This could be because of the fact that the advancing side generates lift by directly facing incoming free-stream (higher relative velocity) and hence, higher ~~drop in~~ lift and in turn higher drop in aerodynamic moment.

② Retreating side (π to 2π)

- The moment drop is less compared to the advacing side.
- Because retreating blade moves in the same direction of free-stream. which will generate lesser aerodynamic force and in turn lesser drop in moment.

[Question-02]

*** Steps of calculation :-**

Step 1 :- Solve governing equation of motion for flap dynamics using Harmonic balance approach

$$\overset{**}{\beta} + v_\beta^2 \cdot \beta = \gamma \bar{M}_\beta + \frac{\omega_{\beta 0}^2}{\Omega^2} \cdot \beta_p \quad \text{——— (3)}$$

Assume pre-cone angle $\beta_p = 0$

→ Coning angle:

$$\beta_0 = \frac{\gamma}{v_\beta^2}\left[\frac{\theta_0}{8}(1+\mu^2) + \frac{\theta_{tw}}{10}\left(1+\frac{5}{6}\mu^2\right) + \frac{\mu}{6}\theta_{1s} - \frac{\lambda}{6}\right] + \frac{\omega_{\beta 0}^2}{\Omega^2}\beta_p$$

→ Longitudinal flap (β_{1c}) & Lateral flap (β_{1s}) can be solved by linear solver:

$$A \cdot x = B$$

where, $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $x = \begin{bmatrix} \beta_{1c} \\ \beta_{1s} \end{bmatrix}$, $B = \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix}$

$$A_{11} = A_{22} = 1$$

$$A_{12} = \left(1+\frac{\mu^2}{2}\right)\left(\frac{\gamma}{8(v_\beta^2-1)}\right)$$

$$A_{21} = -\left(1-\frac{\mu^2}{2}\right)\left(\frac{\gamma}{8(v_\beta^2-1)}\right)$$

$$B_{11} = \left(\frac{\gamma}{v_\beta^2-1}\right)\left[\frac{\theta_{1c}}{8}\left(1+\frac{\mu^2}{2}\right) - \frac{\mu}{6}\beta_0\right]$$

$$B_{21} = \frac{\gamma}{(v_\beta^2-1)}\left[\frac{\theta_{1s}}{8}\left(1-\frac{\mu^2}{2}\right) + \frac{\mu}{3}\theta_0 - \frac{\mu}{4}\lambda + \frac{\mu^2}{4}\theta_{1s} + \frac{\mu \cdot \theta_{tw}}{4}\right]$$

- which can be solved by backslash operator in MATLAB

$$x = A \backslash B$$

Step 2 :- Application of Newmark's algorithm

→ Calculate $\bar{M_\beta}$ using equation-② (Exact solution)

→ Calculate $\ddot{\beta}$ from equation-③. (Flap equation)

$$\ddot{\beta} = \gamma \bar{M_\beta} - \nu_\beta^2 \cdot \beta$$

→ Calculate $\ddot{\beta}$, $\dot{\beta}$ and β for $(n+1)^{th}$ step using values of previous n^{th} step.

$$\ddot{\beta}_{n+1} = \frac{\gamma M_{\beta_n} - \nu_\beta^2 \left(\Delta t \cdot \dot{\beta}_n + \beta_n + \frac{\Delta t^2}{2} (1 - 2\beta_N) \cdot \ddot{\beta}_n \right)}{1 + \nu_\beta^2 \, \Delta t^2 \cdot \beta_N}$$

$$\dot{\beta}_{n+1} = \dot{\beta}_n + \Delta t \left[(1 - \gamma_N) \ddot{\beta}_n + \gamma_N \cdot \ddot{\beta}_{n+1} \right]$$

$$\beta_{n+1} = \beta_n + \Delta t \, \dot{\beta}_n + \frac{\Delta t^2}{2} \left[(1 - 2\beta_N) \ddot{\beta}_n + 2\beta_N \cdot \ddot{\beta}_{n+1} \right]$$

where, $\beta_N = \frac{1}{4}$ & $\gamma_N = \frac{1}{2}$

Δt = time step (difference in ψ taken)

→ calculate overall $\beta(\psi)$

$$\beta = \beta_0 + \underbrace{\beta_{1c} \cdot \cos\psi}_{\text{Longitudinal flap}} + \underbrace{\beta_{1s} \cdot \sin\psi}_{\text{Lateral flap}}$$

* **Comments** :-

→ We can see ~~curve for~~ curve for β v/s ψ for one revolut as well as five revolutions.

→ For $\beta = 0$ (initial), initial response ~~which~~ is transient, and ~~gives steo~~ gives steady-state response after 3-4 revolution

→ The steady state-solution gives good match with harmo balance solution (It can be achieved in first revolution itsl if initial β is set to around $\beta \sim 0.18°$).

→ Here, transient & steady state both compared.