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COURSE :- AE-686A (Helicopter Theory)

HOMEWORK-5

Question-01

* Steps of calculation :-

→ Blade rotor plane is divided in 360, and the forces and moments are calculated at each location of azimuth (ψ) with difference of 1°.

→ To calculate forces & Moments, blade is divided into 20 parts, and the numerical Integration is done using gaussian quadrature

→ β, β^*, β^{**} values are obtained from Newmark's algorithm (Homework-04).

→ Uniform inflow is considered

Formulas to calculate Loads :

* Velocities in non-dimensional form

$$u_T = r + \mu \sin\psi$$

$$u_p = \lambda \cos\beta + r\beta^* + \mu \cdot \cos\psi \cdot \sin\beta$$

$$u_R = \mu \cos\psi$$

* velocities in dimensional form

$$U_T = \Omega y + \mu \Omega R \cdot \sin\psi$$

$$U_p = \lambda \Omega R \cos\beta + y\dot{\beta} + \mu \Omega R \cdot \cos\psi \cdot \sin\beta$$

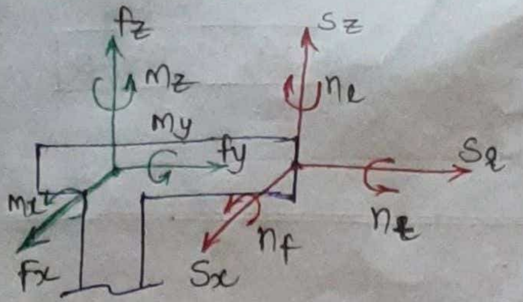
$$U_R = \mu \Omega R \cdot \cos\psi$$

* Forces on section dr:

$$dF_z = \frac{1}{2} \rho c\, C_{l_\alpha} \cdot U_T^2 \left(\theta - \frac{U_P}{U_T}\right) \cdot dr$$

$$dF_x = \frac{1}{2} \rho c \cdot C_{l_\alpha} \cdot \left(U_P U_T \theta - U_P^2 + \frac{C_d}{C_{l_\alpha}} \cdot U_T^2\right) dr$$

$$dF_r = -\frac{1}{2} \rho c \cdot C_{l_\alpha} \cdot \beta \left(U_T^2 \cdot \theta - U_P U_T\right) \cdot dr$$



* Root shear loads (Rotating frame)

$$S_x = \int_e^R \left[\frac{dF_x}{dr} \right] dr$$

$$S_z = \int_e^R \left[\frac{dF_z}{dr} - m(r-e)\ddot{\beta} \right] dr$$

$$S_r = \int_e^R \left[m\Omega^2 r - \beta \cdot \frac{dF_r}{dr} \right] dr$$

* Hub loads & Moments (Rotating frame)

$f_x = S_x$

$f_y = S_r$

$f_z = S_z$

$m_x = n_f + e S_z$

$m_y = n_t$

$m_z = -n_l - e S_x$

* Root bending loads (Rotating frame)

$$n_f = \int_e^R (r-e)\,dF_z - \int_e^R (m\,dr)\,\Omega^2 r (r-e)\beta - \int_e^R m(r-e)^2 \ddot{\beta}\,dr$$

$$= K_\beta(\beta - \beta_p) = \left(\nu_\beta^2 - 1 - \frac{3}{2}\frac{e}{R}\right) I_\beta \Omega^2 (\beta - \beta_p)$$

$$n_l = \int_e^R \left[(r-e) \cdot \frac{dF_x}{dr} \right] dr$$

$$n_t = \int_e^R \left[\frac{dM_x}{dr} \right] dr$$

- Here, dM_x is about elastic axis, which is considered at quarter-chord.
  Hence, $n_t = 0$ (twisting moment)
  $M_x = 0$

* Hub loads & moments (Fixed frame)

Rotor Drag : $H = F_x = \sum_{m=1}^{N_b} \left[f_y^{(m)} \cdot \cos(\psi_m) + f_x^{(m)} \cdot \sin(\psi_M) \right]$

Rotor side Force : $Y = F_y = \sum_{m=1}^{N_b} \left[f_y^{(m)} \cdot \sin(\psi_M) - f_x^{(m)} \cdot \cos(\psi_M) \right]$

Thrust : $T = F_x = \sum_{m=1}^{N_b} \left[f_z \right]$

Rotor roll Moment : $M_x = \sum_{m=1}^{N_b} \left[m_x \cdot \sin(\psi_m) + m_y \cdot \cos(\psi_M) \right]$

Rotor pitch Moment : $M_y = \sum_{m=1}^{N_b} \left[-m_x \cdot \cos(\psi_M) + m_y \cdot \sin(\psi_M) \right]$

Rotor Torque : $\varphi = \sum_{m=1}^{N_b} \left[m_z \right]$

→ $\mathring{\beta} = \Omega \overset{*}{\beta}$

$\ddot{\beta} = \Omega^2 \overset{**}{\beta}$

→ $\theta_{(\psi)} = \theta_0 + \theta_{tw} \cdot r + \theta_{1s} \cdot \sin\psi + \theta_{1c} \cdot \cos\psi$

→ $\beta(\psi) = \beta_0 + \beta_{1c} \cdot \cos\psi + \beta_{1s} \cdot \sin\psi$

OBSERVATIONS :-

(1) Plot 1 (Shear forces at hub- Rotating Frame)

→ f_z (force in z-direction) will be dependent on periodic flap dynamics. (The force varies highly over one revolution)

- f_z depends upon lift, lift depend on azimuth (ψ) location because it has different relative velocity and hence different forces.

→ f_y (force in y-direction) is very high because of presence of centrifugal term ($m\Omega^2 r$) in ~~Eq. formula~~ integration.

→ f_x (force in x-direction) is minimum, because drag is very small force compared to other lift force.

(2) Plot 2 (Bending moments at hub- Rotating Frame)

→ My - Moment w.r.t. y axis of blade is zero, because elastic axis coincide with quarter chord axis. (Hence, $M_y = m_t = 0$).

(3) ~~Plot-3~~ Plot-3 (Non-dimentional vertical shear forces - Rotating Frame)

→ All blades produce same magnitude of shear-force in z direction, however they are shifted by 90° (4-bladed rotor).

(4) Plot 4, 5 (Forces & Moments at hub - Fixed Frame)

→ $F_z(\psi) = 4a_0 + 4a_4 \cdot \sin 4\psi$

- Hub only transfers $p \cdot N_b$ /rev frequency. (Here, $p=4$). hence, it acts as a filter which removes all other frequencies except $p \cdot N_b$ /rev.
- Here, fixed frame hub forces & Moments have 4 /rev frequency.

⇒ The data for 1 blade is calculated and other blade data is obtained by phase shifting.

Blade-1	0°	90°	180°	270°	360°
Blade-2	270°	0°	90°	180°	270°
Blade-3	180°	270°	0°	90°	180°
Blade-4	90°	180°	270°	0°	90°

[Question-02]

Steps for calculation :-

⇒ Residual equations

(1) Vertical Force equilibrium :

$$R_1 = W - T\cdot\cos(\alpha_s)\cdot\cos(\phi_s) + Y\cdot\sin(\phi_s) - H\cdot\sin(\alpha_s)\cdot\cos(\phi_s) + Y_F\cos(\alpha_s)\cdot\sin(\phi_s) + D\cdot\sin(\theta_{FP}) = 0$$

(2) Longitudinal equilibrium :

$$R_2 = D\cos(\theta_{FP}) + H\cdot\cos(\alpha_s) - T\cdot\sin(\alpha_s) = 0$$

(3) Lateral ~~eq~~ force equilibrium :

$$R_3 = (Y+Y_F)\cdot\cos(\phi_s) + T\cdot\cos(\alpha_s)\cdot\sin(\phi_s) + H\cdot\sin(\alpha_s)\cdot\sin(\phi_s) = 0$$

(4) Pitch Moment equilibrium :

$$R_4 = M_y + M_{y_F} - W(x_{cg}\cdot\cos\alpha_s - h\sin\phi_s) - D(x_{cg}\cdot\sin\alpha_s + h\cdot\cos\alpha_s) = 0$$

(5) Roll Moment equilibrium :

$$R_5 = M_x + M_{x_F} + Y_F\cdot h + W(h\cdot\sin\phi_s - y_{cg}\cdot\cos\phi_s) = 0$$

(6) Yaw moment / Torque equilibrium

$$R_6 = Q - Y_F\, l_T = 0$$

(we have not considered this, we assumed this force is satisfied).

(7) Inflow equation

$$R_7 = \lambda - \mu\cdot\tan\alpha - \frac{C_T}{2\sqrt{\mu^2+\lambda^2}} = 0$$

⇒ Jacobian

$$J = \begin{bmatrix} \frac{\partial R_1}{\partial\theta_1} & \frac{\partial R_1}{\partial\theta_2} & \cdots & \frac{\partial R_1}{\partial\theta_6} \\ \frac{\partial R_2}{\partial\theta_1} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial R_6}{\partial\theta_1} & \frac{\partial R_6}{\partial\theta_2} & \cdots & \frac{\partial R_6}{\partial\theta_6} \end{bmatrix}$$

⇒ Finite difference

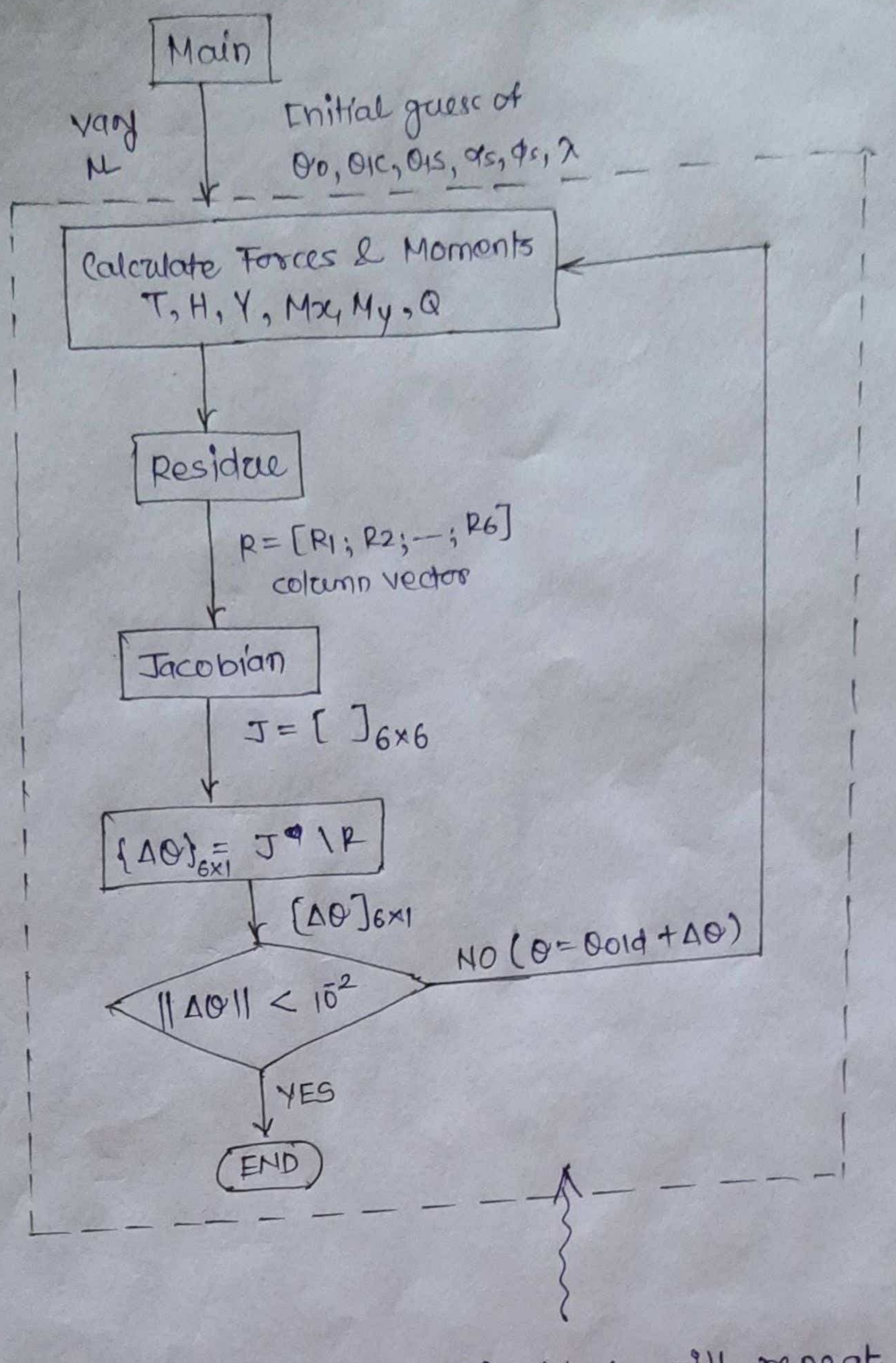
i^{th} residue, j^{th} input

$$\frac{\partial R_i}{\partial\theta_j} = \frac{R_i(\theta_j + \Delta\theta_j) - R_i(\theta_j)}{\Delta\theta_j}$$

⇒ Change in control input

$$\{\Delta\theta\} = [-J] \backslash R$$

$$\theta = \theta_0 + \Delta\theta$$



This block will repeat several times for each value of μ.

OBSERVATIONS :-

(1) Plot 1 (Control inputs : $\theta_0, \theta_{1c}, \theta_{1s}$ v/s μ)

θ_0 (collective pitch) → The RPM of rotor blade is almost constant.

- To increase forward speed, the thrust vector needs to be tilted which requires more lift/ thrust generation to support weight as well as forward speed component. which is achieved by changing cyclic variation (θ_{1s}) in longitudinal direction.
- Remains constant with μ.

θ_{1s} (Longitudinal cyclic) → θ_{1s} decreases with increase in μ, to balance out high pitch moments generated because of shaft tilting.

- It stabilizes forces & moments in forward flight.

θ_{1c} (lateral cyclic) → Remains almost constant with μ.

- There is no lateral movement at the time of trim. (No sideslip velocity).

(2) Plot 2 (Shaft angles : α_s, ϕ_s v/s μ)

α_s (Longitudinal shaft angle) → α_s increases with increase in μ (i.e. Advance ratio - Nondimensional forward speed)

- This is because forward speed is obtained by tilting the thrust vector in forward direction.

ϕ_s (Lateral shaft angle) → ϕ_s does not change with μ

- There is no sideward movements considered or no sideslip in velocity is considered. hence, ϕ_s remains constant.