PROBLEM STATEMENT

Program a numerical solution to the combined blade-element / momentum theory (BEMT). All integrations are to be performed numerically using either of two: (1) six-point Gaussian Quadrature, in this, the blade must first be discretized into a series of small elements of equal span, typically, 10 elements must be used to ensure adequate resolution of the inflow and spanwise loading, or (2) using trapezoidal or rectangular rule (in this atleast 100-200 points would have to be used). Please describe all the steps of your calculation, including the details of the algorithms used and methods of implementation.

Q1. Relevant data pertaining to a helicopter are given below:

Density of air, ρ 1.225 kg/m³

Number of blades, Nb 4

Blade radius, R 6 m
Blade chord, c 0.4 m
Profile Drag coefficient, C_{do} 0.01
Lift curve slope, C_{la} 5.73
Rotor angular rate, Ω 10 π rad/sec
Hover C_T 0.006
Root cutout 20% of R

Consider blade twist for four different configurations: (i) ideal twist with θ_{tip} , (ii) θ_{tw} of 0_{\circ} ,

- (iii) θ_{tw} of -15° (linear), and (iv) ideal twist with θ_{tip} and ideal taper (assume $c_{tip} = 0.5c$). Assuming non-uniform inflow (BEMT without tip loss), evaluate the following and show each item in one figure for the helicopter in hovering flight:
- (a) Find out the reference pitch input θ_{tip} or θ_{75} for hovering flight and plot the variation of pitch angle with non-dimensional radial location (all 4 twist cases in one figure). In a separate plot, compare the numerical solution obtained for ideal twist with the closed form exact formula derived in class.
- (b) Variation of angle of attack with non-dimensional radial location (all 4 twist cases in one figure).
- (c) Variation of induced inflow with non-dimensional radial location (all 4 twist cases in one figure).
- (d) Variation of thrust distribution (i.e. dCT/dr), total torque (CQ) distribution, induced torque distribution (CQ_l) and profile torque (CQ_p) with non-dimensional radial location (all 4 twist cases in one figure each).

Report your observations for each of the figures. Make sure you use legends and different line types to allow identification of each graph.

Q2. For the above problem, implement Prandtl's tip-loss function using the numerical technique discussed in class. For the \no blade" twist case, find out the reference pitch input θ 75 needed for hovering flight. Compare the result with that from Q1. Show the effect of the tip-loss on the induced inflow, thrust distribution, and total torque distribution with non-dimensional radial location on separate plots. Discuss your results.