*** Q-2, Sliding Mode Control ***

PROBLEM

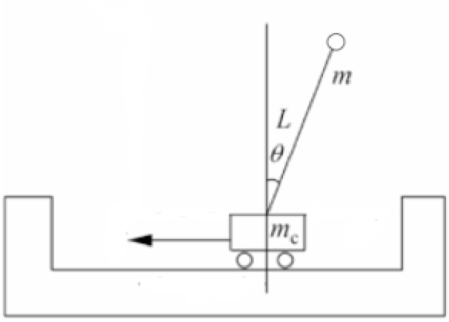
Consider the inverted pendulum dynamics

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = f(x) + g(x)u + \delta$$

$$f(x) = \frac{g\sin(x_1) - \dfrac{mlx_2^2\cos(x_1)\sin(x_1)}{m_c + m}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2 x_1}{m_c + m}\right)}$$

$$g(x) = \frac{\cos x_1 / (m_c + m)}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2 x_1}{m_c + m}\right)}$$



$x_1 = \theta$, $x_2 = \dot{\theta}$ are the angular position and velocity of the pendulum. u is the control input and δ is unknown bounded disturbance.

(a) Take $s = x_2 + cx_1$, $V = \frac{1}{2}s^2$, Design a sliding mode controller to render $\dot{V} \le -kV^{(1+\alpha)/2}$ by considering bounds of disturbance δ as $-0.3 \le \delta \le 0.1$ Where, $0 < \alpha < 1$ (δ should not be used in the controller's expression only bounds of disturbance can be used)

(b) Simulate the system using the controller by taking initial conditions as $x_1(0) = \pi/30$ and $x_2(0) = 0$ and $\delta = -0.1 + 0.2\sin(0.5t)$. Take $m_c = 1$, $m = 0.1$, $l = 0.5$, $g = 9.81$. Use saturation function instead of signum function to reduce chattering.

MATLAB code

>> main_smc.m

```
close all;
clear all;
clc

initial_conditions;

tf = 80;

[t,y] = ode45(@(t,y) dy_dt(mc,m,l,g,a,c,k1,k2,t,y), [0  tf], y0);

u = y(:,1)*0;
for i = 1:length(y(:,1:2))
    u(i) = control(mc,m,l,g,a,c,k1,k2,y(i,:)')';
end

plotting;
```

>> initial_conditions.m

```
%% Initial conditions

y0 = [pi/30; 0];
mc = 1;
m = 0.1;
l = 0.5;
g = 9.81;
a = 0.7;
c = 0.2;
k1 = 1;
k2 = 100;
```

>> dy_dt.m

```
%% dy_dt

function dy = dy_dt(mc,m,l,g,a,c,k1,k2,t,y)

x1 = y(1);
x2 = y(2);

denom = l*(4/3 - m*cos(x1)^2/(mc+m));       % denominator

fx = (g*sin(x1) - m*l*x2*x2*cos(x1)*sin(x1)/(mc+m))/denom;
gx = cos(x1)/(mc+m)/denom;
```

```
u = control(mc,m,l,g,a,c,k1,k2,y);
d = -0.1 + 0.2*sin(0.5*t);

dx1 = x2;
dx2 = fx +gx*u +d;

dy = [dx1; dx2];
end
```

>> control.m

```
%% control

function u = control(mc,m,l,g,a,c,k1,k2,y)

x1 = y(1);
x2 = y(2);

denom = l*(4/3 - m*cos(x1)^2/(mc+m));
fx = (g*sin(x1)-m*l*x2*x2*cos(x1)*sin(x1)/(mc+m))/denom;
gx = cos(x1)/(mc+m)/denom;

s = x2 + c*x1;
u = -(1/gx)*(k1*abs(s)^a*sat(s) + k2*sat(s) + fx + c*x2);

end
```

>> sat.m

```
%% Saturation function

function y = sat(x)

if x > 1
   y = 1;
elseif x < -1
   y = -1;
else
   y = x;
end

end
```

>> plotting.m

```
%% plot

figure('Name','Question-02 (Assignment-02)','NumberTitle','off')

subplot(3,1,1);
plot(t,y(:,1),'r');

ylabel('\theta','FontSize',13);
grid on

subplot(3,1,2);
plot(t,y(:,2),'r');
ylabel('$\dot{\theta}$', 'Interpreter','latex','FontSize',13)
grid on

subplot(3,1,3);
plot(t,u,'r');
ylabel('u','FontSize',13);
xlabel('t','FontSize',13);
grid on

sgt = sgtitle('**Question 2** Control of inverted pendulum by Sliding Mode Controller')
sgt.FontSize = 20;
```

OUTPUT PLOT

****Question 2** Control of inverted pendulum by Sliding Mode Controller**