*** Q-2, INDIRECT MRAC ***

PROBLEM

Consider the dynamical system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = a_0 x_1 + a_1 \sin(2x_2) + a_2 \cos(x_2) + a_3 \tanh(x_1) + b_0 u$$

Where a_0, a_1, a_1, a_3, b are unknown constants with $|b_0| > 1$.

(a) Derive Indirect MRAC law to track a reference model

$$\dot{x}_{1m} = x_{2m}$$

$$\dot{x}_{2m} = -3x_{2m} - 4x_{1m} + r(t)$$

(b) Simulate the system by taking any initial condition and take $r(t) = 2\sin(t) + 2\cos(1.5t)$. For simulation use $a_3 = 3, a_2 = 2, a_1 = 1, a_0 = 1, b_0 = 2$.

INITIAL CONDITIONS ASSUMED

```
X = [1 2]'          % Plant
Xm = [-1 3]'        % Reference model
a0_h = 2;           % System parameters
a1_h = 2;
a2_h = 4
a3_h = 6
b0_h = 4
```

MATLAB code

>> main_Q2_AS_03.m

```
%% Question-02 Assignment-03

close all
clear all
clc

y0 = [1    2 ...
    -1    3 ...
     2   2   4    6    4];

[t,y] = ode45(@dy_dt_Q2, [0 40], y0);

plotting_Q2;
```

>> dy_dt_Q2.m

```
%% Question-02 Assignment-03 (function dy_dt)

function dy = dy_dt_Q2(t,y)

x1 = y(1);
x2 = y(2);

xm1 = y(3);
xm2 = y(4);

a0_h = y(5);
a1_h = y(6);
a2_h = y(7);
a3_h = y(8);

b0_h = y(9);

% Reference input
r = 2*sin(t) + 2*cos(1.5*t);

% Controller
u = (1/b0_h)*( -a1_h*sin(2*x2) - a2_h*cos(x2) - a3_h*tanh(x1) - 4*x1 - 3*x2 - a0_h*x1 ) +
(r/b0_h) ;

a0 = 1;
a1 = 1;
a2 = 2;
a3 = 3;
b0 = 2;
```

```
dx1 = x2;
dx2 = a0*x1 + a1*sin(2*x2) + a2*cos(x2) + a3*tanh(x1) + b0*u;

% Reference Model
xm1_dot = xm2;
xm2_dot = - 4*xm1 - 3*xm2 + r;

% Adaptive Laws
e1 = x1 - xm1;
e2 = x2 - xm2;

gamma_a0 = 10;
gamma_a1 = 10;
gamma_a2 = 10;
gamma_a3 = 10;
gamma_b0 = 10;

a0_h_dot = (1/gamma_a0) * (6*e1 + 10*e2) * x1 ;
a1_h_dot = (1/gamma_a1) * (6*e1 + 10*e2) * sin(2*x2) ;
a2_h_dot = (1/gamma_a2) * (6*e1 + 10*e2) * cos(x2) ;
a3_h_dot = (1/gamma_a3) * (6*e1 + 10*e2) * tanh(x1) ;
b0_h_dot = (1/gamma_b0) * (6*e1 + 10*e2) * u ;

dy = [dx1 dx2 xm1_dot xm2_dot a0_h_dot a1_h_dot a2_h_dot a3_h_dot b0_h_dot]';
end
```

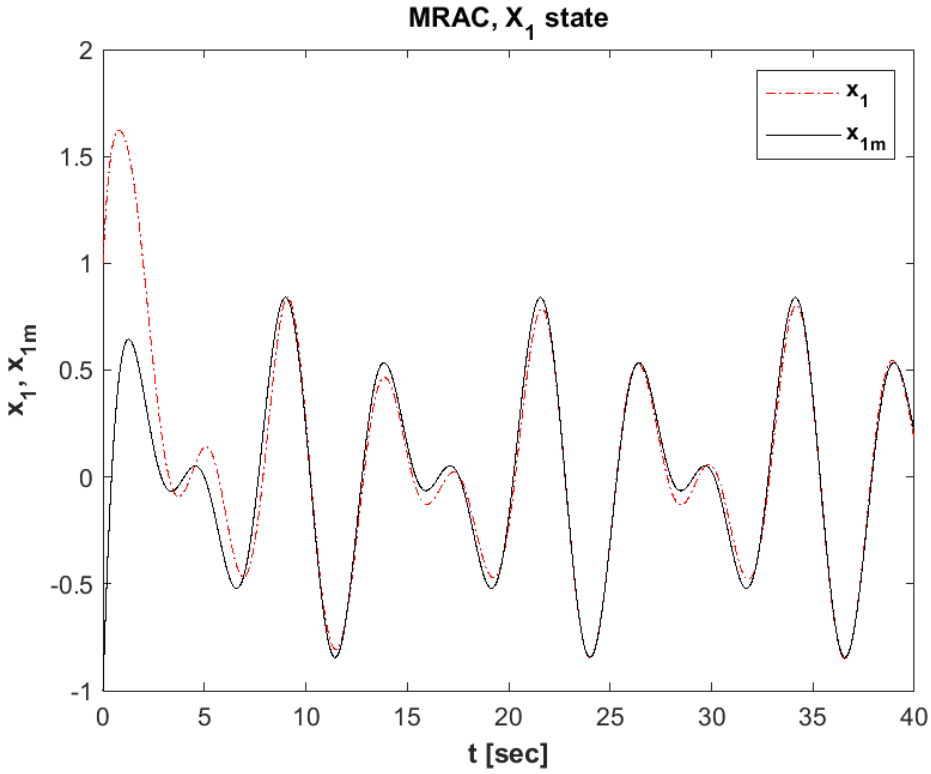
>> plotting.m

```
%% Question-02 Assignment-03 (Plots)

figure(1);
plot(t,y(:,1),'-.r')
hold on
plot(t,y(:,3),'k')
hold off
title('MRAC, X_1 state', 'FontWeight','bold')
xlabel('t [sec]', 'FontWeight','bold')
ylabel('x_1, x_1_m', 'FontWeight','bold')
legend('x_1','x_1_m', 'FontWeight','bold')

figure(2)
plot(t,y(:,2),'-.r')
hold on
plot(t,y(:,4),'k')
hold off
title('MRAC, X_2 state', 'FontWeight','bold')
xlabel('t [sec]', 'FontWeight','bold')
ylabel('x_2, x_2_m', 'FontWeight','bold')
legend('x_2','x_2_m', 'FontWeight','bold')
```

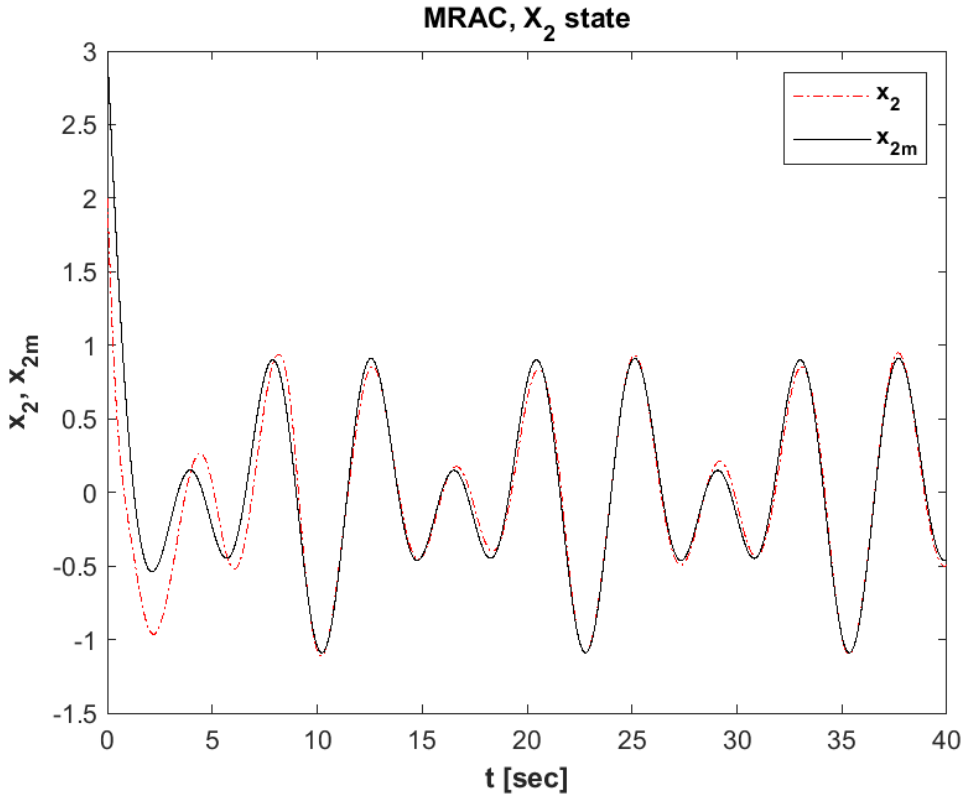
```
figure(3);
subplot(5,1,1);
plot(t,y(:,5),'k');
ylabel('$$\hat{a_0}$$','Interpreter','Latex', 'FontWeight','bold')
title('Estimation of Plant parameters', 'FontWeight','bold')

subplot(5,1,2);
plot(t,y(:,6),'k');
ylabel('$$\hat{a_1}$$','Interpreter','Latex', 'FontWeight','bold')

subplot(5,1,3);
plot(t,y(:,7),'k');
ylabel('$$\hat{a_2}$$','Interpreter','Latex', 'FontWeight','bold')

subplot(5,1,4);
plot(t,y(:,8),'k');
ylabel('$$\hat{a_3}$$','Interpreter','Latex', 'FontWeight','bold')

subplot(5,1,5);
plot(t,y(:,9),'k');
xlabel('t', 'FontWeight','bold');
ylabel('$$\hat{b_0}$$','Interpreter','Latex', 'FontWeight','bold')
```

OUTPUT PLOT

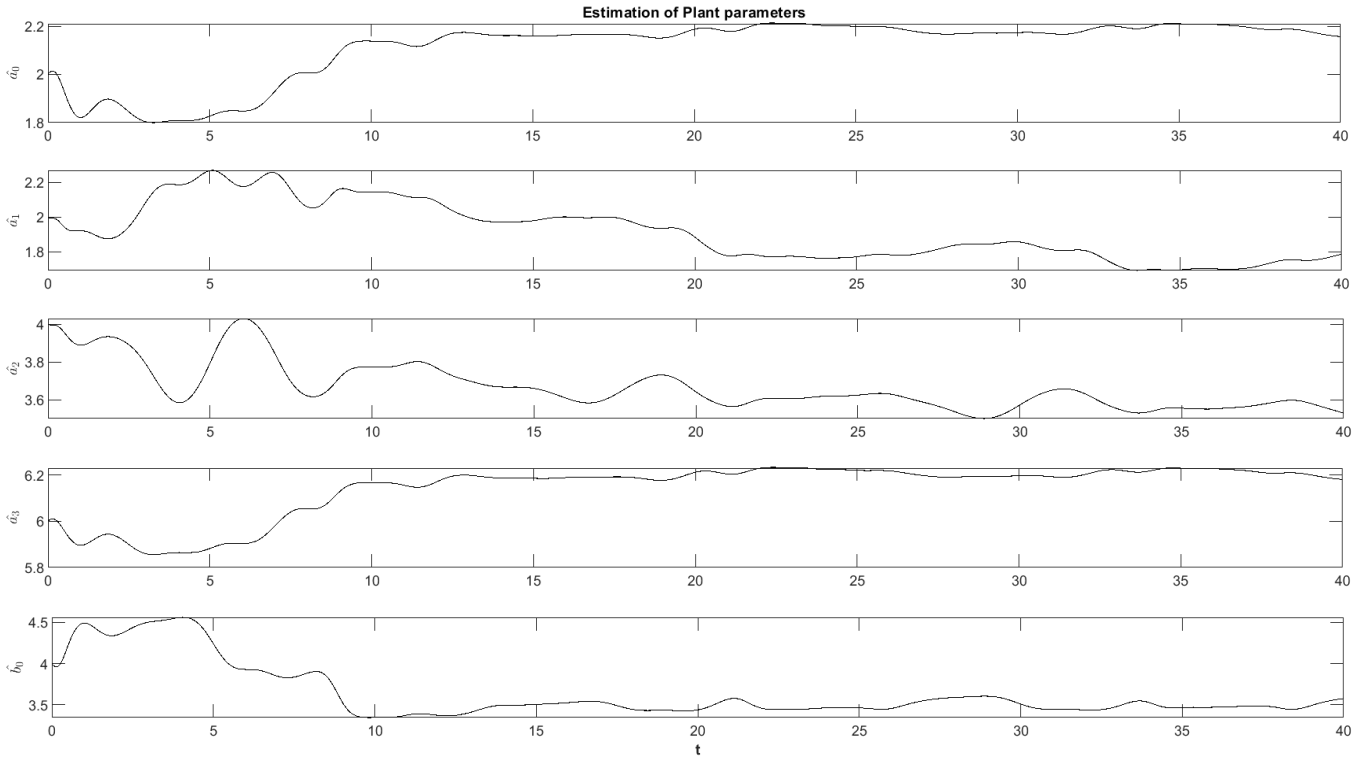
>> Plot 1



>> Plot 2



>> Plot 3


Estimation of Plant parameters
$\hat{a}_0$
$\hat{a}_1$
$\hat{a}_2$
$\hat{a}_3$
$\hat{b}_0$
t