*** Q-1, Back-stepping control ***

PROBLEM

The attitude kinematics and dynamics of a spacecraft are given by

$$\dot{\boldsymbol{\rho}} = 0.5\,[\boldsymbol{I}_{3\times3} + \boldsymbol{\rho}\boldsymbol{\rho}^T + S(\boldsymbol{\rho})]\boldsymbol{\omega}$$

$$J\dot{\boldsymbol{\omega}} + S(\boldsymbol{\omega})J\boldsymbol{\omega} = \boldsymbol{\tau}$$

Where $\boldsymbol{\rho} = [\rho_1, \rho_2, \rho_3]^T$ is the vector of Rodrigues parameters and $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T$ is the vector of angular velocities. $J = diag(J_1, J_2, J_3)$ is the diagonal positive definite inertia matrix. $\boldsymbol{I}_{3\times3}$ is the 3×3 identity matrix. $\boldsymbol{\tau} \in \boldsymbol{R}^3$ is the torque vector. $S(\cdot)$ is the skew-symmetric operator.

$$S(\boldsymbol{a}) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

(a) Design the control input $\boldsymbol{\tau}$ by Back-stepping method to stabilize the system to origin.
(Hint : Take $\boldsymbol{\omega} = \boldsymbol{z} + \boldsymbol{\alpha}$. Take $V_1 = \frac{1}{2}\boldsymbol{\rho}^T K\boldsymbol{\rho}$ and $V_2 = V_1 + \frac{1}{2}\boldsymbol{z}^T J\boldsymbol{z}$, where K is symmetric positive definite. Design $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$ to make $\dot{V}_2 \le 0$.)
(b) Choose any initial condition and simulate the system using this control law.

INITIAL CONDITIONS ASSUMED

```
J = diag([2 3 1]);            % Inertia matrix
K = diag([0.2 0.2 0.2]);

d0 = [0.1; 0.2; -0.2];        % Initial Rodrigues parameters
w0 = [0.3; 0.1; -0.4];        % Initial Angular velocities
```

MATLAB code

>> main_backstepping.m

```
%% Question-01 Assignment-02 (Backstepping control)

close all;
clear all;
clc
initial_conditions;
tf = 50;        % Integration limit (Time)

[t,y] = ode45(@(t,y) dy_dt(J,K,t,y), [0  tf], y0);

for i = 1:length(y(:,1:3))
    u(i,:) = control(J,K,y(i,1:3)',y(i,4:6)')';
end

plotting
```

>> initial_conditions.m

```
%% initial_conditions

J = diag([2 3 1]);          % Inertia matrix
K = diag([0.2 0.2 0.2]);

d0 = [0.1; 0.2; -0.2];      % Initial Rodrigues parameters
w0 = [0.3; 0.1; -0.4];      % Initial Angular velocities

y0 = [d0; w0]; % Combined matrix of Rodrigues parameters & Angular velocities
```

>> dy_dt.m

```
%%  dy/dt

function y_dot = dy_dt(J,K,~,y)

d = y(1:3);
w = y(4:6);

F = eye(3) + d*d' + S(d);
tau = control(J,K,d,w);

d_dot = 1/2*F*w;                          % Equation of d_dot
w_dot = pinv(J)*(tau - S(w)*J*w);         % Equation of w_dot
y_dot = [d_dot; w_dot];
end
```

>> control.m

```
%%  Control law

function tau = control(J,K,d,w)

F = eye(3) + d*d' + S(d);
tau = -0.5*F*d - 0.5*J*K*F*w + S(w)*J*w - K*(w+K*d);

end
```

>> S.m

```
%%  Skew-symmetric operator

function y = S(x)

y = [0 -x(3) x(2); x(3) 0 -x(1); -x(2) x(1) 0];

end
```

>> plotting.m

```
%% Plot

figure('Name','Question-01 (Assignment-02)','NumberTitle','off')

subplot(3,1,1);
plot(t,y(:,1),'r', t,y(:,2),'g', t,y(:,3),'b');
ylabel('\bf \rho');
legend('x','y','z','Location','Northeast');
grid on

subplot(3,1,2);
plot(t,y(:,4),'r', t,y(:,5),'g', t,y(:,6),'b');
ylabel('\bf \omega');
grid on

subplot(3,1,3);
plot(t,u(:,1),'r', t,u(:,2),'g', t,u(:,3),'b');
xlabel('\bf t [sec]');
ylabel('\bf \tau');
grid on

sgt = sgtitle('**Question 1** Control of spacecraft by back-stepping method')
sgt.FontSize = 20;
```

OUTPUT PLOT

****Question 1** Control of spacecraft by back-stepping method**