*** Q-3, DIRECT MRAC ***

PROBLEM

3. Consider the dynamical system

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u}$$

With A and B constant matrices. A is unknown matrix, B is known matrix. Derive MRAC law to track the reference model

$$\dot{\boldsymbol{x}}_m = A_m \boldsymbol{x_m} + B_m \boldsymbol{r}(t)$$

Where $A_m \in \boldsymbol{R}^{3\times3}, A \in \boldsymbol{R}^{3\times3}, B \in \boldsymbol{R}^{3\times2}, B_m \in \boldsymbol{R}^{3\times2}, \boldsymbol{x} \in \boldsymbol{R}^{3\times1}, \boldsymbol{u} \in \boldsymbol{R}^{2\times1}$

$$A_m = -\begin{bmatrix} 4 & 2 & 2 \\ 2 & 5 & 3 \\ 2 & 3 & 3 \end{bmatrix}, B_m = \begin{bmatrix} 5 & 4 \\ 5 & 7 \\ 10 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 4 \end{bmatrix}$$

Simulate the system with any initial condition using the MRAC law by taking

$$\boldsymbol{r}(t) = \begin{bmatrix} \sin(t) + 0.5\cos(0.5t) \\ \sin(t) - 0.5\cos(0.5t) \end{bmatrix}$$

INITIAL CONDITIONS ASSUMED

```
X = [1   2   -2]'                 % Plant
Xm = [2   0.1   -2]'              % Reference model
Kx = [-1   0.2   2   2   -1   2]  % Parameter Estimated
Kr = [2   0.1   1   -2]           % Parameter Estimated
```

MATLAB code

>> main_Q3_AS_03.m

```
%% Question 3 (ASSIGNMENT-03)

close all
clear all
clc

% Initial conditions to start the Simulation
% y0(1:3) = x(3x1)
% y0(4:6) = xm(3x1)
% y0(7:12) = Kx(2x3)
% y0(13:16) = Kr(2x2)
% y0 = [x xm Kx Kr]
y0 = [1    2   -2   ...
      2    0.1  -2  ...
      -1    0.2    2    2   -1    2 ...
      2     0.1    1      -2];

[t,y] = ode45(@dy_dt_Q3, [0 20], y0);

plotting_Q3;
```

>> dy_dt_Q3.m

```
%% dydt function

function dy = dy_dt_Q3(t, y)

A = [1 2 1; 3 2 1; 8 5 3];
B = [1 2; 3 1; 2 4];
Am = -[4 2 2; 2 5 3; 2 3 3];
Bm = [5 4; 5 7; 10 8];

x = reshape(y(1:3), [3 1]);
xm = reshape(y(4:6), [3 1]);
Kx = reshape(y(7:12), [2 3]);
Kr = reshape(y(13:16), [2 2]);

r = [sin(t) + 0.5*cos(0.5*t);
    sin(t) - 0.5*cos(0.5*t)];

% Controller
u = Kx*x + Kr*r;

% Plant
dx = A*x + B*u;
```

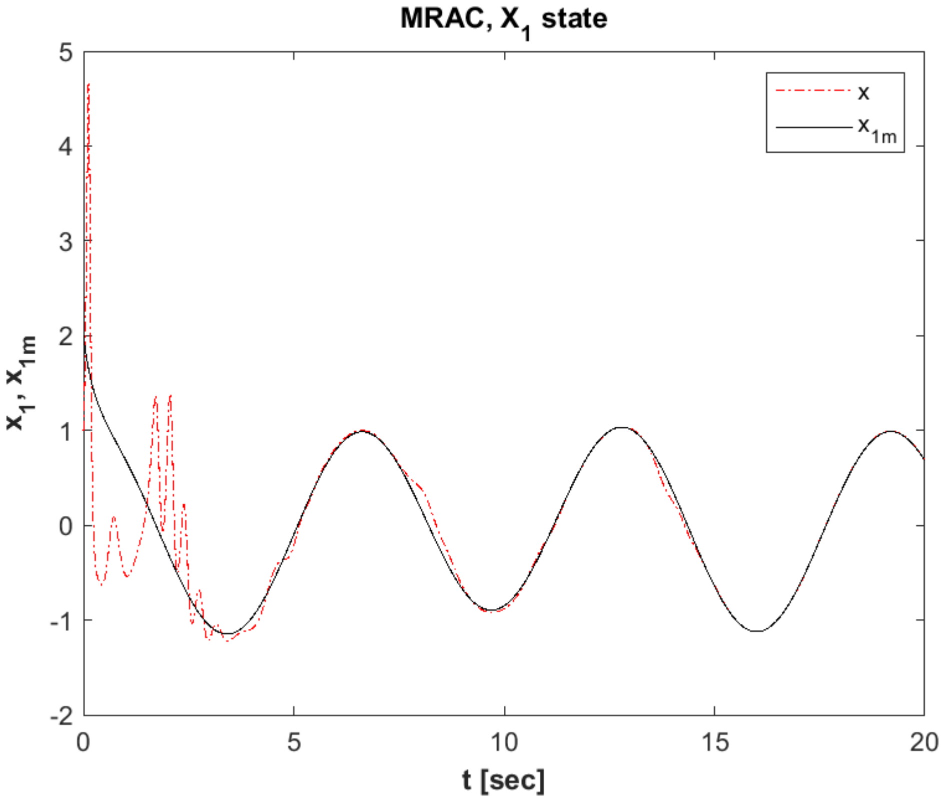
```
dxm = Am*xm + Bm*r;

% Adaptive Laws

gammax = eye(3);
gammar = eye(2);
P = lyap(Am', eye(3));          % Solution of Lyapunov function AX+XA'+Q=0 (Q=eye(3))
e = xm - x;

dKx = transpose(gammax * x * e' * P * B);
dKr = transpose(gammar * r * e' * P * B);

dKx = reshape(dKx, [6 1]);      % Conversion to get compatible with vector
dKr = reshape(dKr, [4 1]);

dy = [dx; dxm; dKx; dKr];

end
```

>> plotting_Q3.m

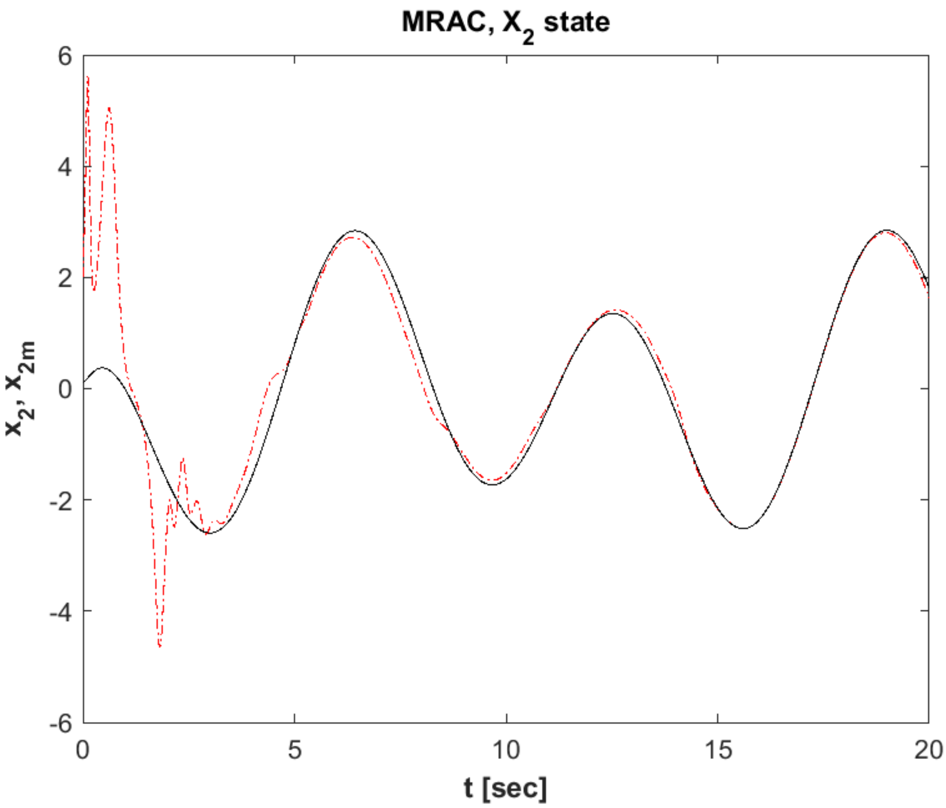
```
%% Plotting Q3

figure(1);
plot(t,y(:,1),'-.r');
hold on
plot(t,y(:,4),'k');
hold off
xlabel('t [sec]', 'FontWeight','bold');
ylabel('x_1, x_1_m', 'FontWeight','bold');
legend('x','x_1_m');
title('MRAC, X_1 state', 'FontWeight','bold')

figure(2);
plot(t,y(:,2),'-.r');
hold on
plot(t,y(:,5),'k');
hold off
xlabel('t [sec]', 'FontWeight','bold');
ylabel('x_2, x_2_m', 'FontWeight','bold');
title('MRAC, X_2 state', 'FontWeight','bold')

figure(3);
plot(t,y(:,3),'-.r');
hold on
plot(t,y(:,6),'k');
hold off
xlabel('t [sec]', 'FontWeight','bold');
ylabel('x_3, x_3_m', 'FontWeight','bold');
title('MRAC, X_3 state', 'FontWeight','bold')
```

OUTPUT PLOT

>> Plot 1



>> Plot 2



>> Plot 3