Splines

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Spline interpolation is very powerful and widely used method for interpolation and has many applications. It interpolates a function between given set of points by means of piecewise smooth polynomials.

§1 Linear Spline

for given values $y(x_i) = y_i$ for $i = 0, 1, 2, \cdots, n$. We create linear functions for every subinterval. This is literally a graph made by joining the points with straight lines. The function is continous but not the slope or curvature

§2 Quadratic spline

If the given set of values $y(x_i) = y_i$ for $i = 1, 2, 3, \cdots, n$ has $p(x)$ as the quadratic spline function, then $p(x)$ must obey the following:

1. $y_i = p(x_i) \forall i$
2. $p(x)$ is a quadratic for every subinterval, except possibly the first or last subinterval
3. $p(x)$ and $p'(x)$ are continuous in (a, b)

§2.1 Disadvantage

1. There might be a straight line connecting the first(or last) subinterval.
2. Curvature is not guaranteed to be continuous, Hence the graph may not be pleasant to the eyes):

§3 Cubic spline

If the given set of values $y(x_i) = y_i$ for $i = 1, 2, 3, \cdots, n$ has $p(x)$ as the cubic spline function, then $p(x)$ must obey the following:

1. $y_i = p(x_i) \forall i$
2. $p(x)$ is a cubic polynomial for every subinterval
3. $p(x)$, $p'(x)$ and $p''(x)$ are continuous in (a, b)

$p''(x)$ is also written as M_x

§3.1 Natural Cubic Spline

if $p''(x_0) = p''(x) = 0$, then the subic spline $p(x)$ is said to be a natural cubic spline. For natural cubic splines, we have an explicit formula

Say the value set $y(x_i) = y_i$ is to be interpolated for $i = 0, 1, 2, \cdots, n$ using a natural cubic spline. We will have n subintervals, and each subinterval will have a cubic function associated to it. We define the spline $p_i(x)$ for the interval $[x_{i-1}, x_i]$ for $i = 1, 2, 3, \cdots, n$. then

$$\begin{aligned} p_i(x) = & \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})M_i\right] \\ & + \frac{1}{h}(x_i - x)\left(y_{i-1} - \frac{h^2}{6}M_{i-1}\right) \\ & + \frac{1}{h}(x - x_{i-1})\left(y_i - \frac{h^2}{6}M_i\right) \end{aligned}$$

for $1 \leq i \leq n$, where

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}(p_{i+1} - 2p_i + p_{i-1})$$

for $1 \leq i < n$
This is only applicable if the difference in given x values is constant.