Higher Engineering Mathematics

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Contents

Chapter 1

Solution of Equations

1.1 Introduction

The expression $f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$
where $\{a_i\}_{i=0}^n$ are constants and $a_0 \neq 0$ and $n \in \mathbb{Z}^+$, is called a *polynomial in x of degree n.* The polynomial $f(x) = 0$ is called an *algebraic equation of degree n.* If $f(x)$ contains some other function like trigonometric, logarithmic, exponential, etc. then $f(x) = 0$ is said to be a *transcendental equation*

$$\text{The value of } x \text{ for which} f(x) = 0, \tag{1.1}$$

is called its *root or solution.* Geometrically, a root of (1.1) is that value of x where the graph of $y = f(x)$ crosses the x-axis. We often come across problems in deflection of beams, electrical circuit and mechanical vibration which depend upon the *solution* of these equations. Hence, they are of great importance in the field of Applied as well as Pure Mathematics.

1.2 General Properties

1. If α is a *root of the polynomial equation* $f(x) = 0$, then the polynomial $f(x)$ is *exactly divisible by* $x - \alpha$ and conversely.
   For example: $x = 3$ is a solution of $x^4 - 6x^2 - 8x - 3 = 0$ since it satisfies the equation.
   $\therefore$ $(x - 3)$ is a factor of $x^4 - 6x^2 - 8x - 3 = 0$

2. Every *algebraic equation of degree n has n roots* (*real or imaginary*).
   Conversely, if $\{\alpha_i\}_{i=1}^n = \{\alpha_1, \alpha_2, \alpha_3 \ldots \alpha_n\}$ are the roots of *nth degree polynomial* $f(x) = 0$, then

   $$f(x) = A\prod_{i=1}^{n}(x - \alpha_i) = A(x - \alpha_1)(x - \alpha_2)(x - \alpha_3)\cdots(x - \alpha_n)$$

   where A is a constant.

   Obs1. $\boxed{\text{a polynomial of } n \text{ degree has more than } n\text{, then it must be identically zero}}$

3. **Intermediate Value Property**: If $f(a)$ and $f(b)$ have different signs, then the equation $f(x) = 0$ must have an *odd number of roots* between $x = a$ and $x = b$.

   Similarly, if $f(a)$ and $f(b)$ have different signs, then the equation $f(x) = 0$ must have an *even number of roots* between $x = a$ and $x = b$