

# Matrix Factorization

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5 November 2025

Let the system of equation be

$$AX = B$$

We factorize  $A$  into  $LU$  which are lower and upper triangular matrices respectively. If the principal minors of  $A_{n \times n}$  are non singular, then this factorization is possible as well as unique.

$$L = \begin{bmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & 0 \\ l_{31} & l_{32} & l_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & l_{n3} & \cdots & l_{nn} \end{bmatrix}$$

and

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & u_{22} & u_{23} & \cdots & u_{2n} \\ 0 & 0 & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & u_{nn} \end{bmatrix}$$

$AX = B$  becomes  $LUX = B$

Let  $UX = Z$  So That  $LZ = B$ . Now  $Z$  can be found by the following

$$\left\{ \sum_{j=1}^n l_{ij} z_j = b_j \right\}_{i=1}^n$$

Then we use  $UX = Z$  as

$$\left\{ \sum_{j=1}^n u_{ij} x_j = z_j \right\}_{i=1}^n$$

When  $u_{ii} = 1 \forall i$ , then this method is called Crout's decomposition method.

When  $l_{ii} = 1 \forall i$ , its called Dolittle Method.

Particularly, when  $u_{ii} = l_{ii} = 1 \forall i$ , its Cholesky's decomposition method.

It may be noted that for a triangular matrix, its determinant is the product of its diagonal elements.