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CH=03 Assignment.

Q.1 (a) RE = $1^*(01)^*0^*$

$\Rightarrow L = \{\epsilon, 1, 0, 01, 10, 100, 101, 110, 1000, 1010, 1100, 1101 \ldots \infty\}$

$\Rightarrow L'$ ~~RE~~ = $\{0,1\}^*$

$\Rightarrow L' = \{\epsilon, 0, 1, 00, 01, 10, 11, 010, 011, 100, 101, 110, 111, 000, 001, \ldots \ldots \infty\}$

$\Rightarrow$ string of min. length in $\{0,1\}^*$ not in the language $1^*(01)^*0^*$ is are

$\{001, 011\}$.

(b) $(0^* + 1^*)(0^* + 1^*)(0^* + 1^*)$.

$L = \{0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111, \ldots \infty\}$

$\Rightarrow$ string of min. length = $\{0101, 1010\}$.

(c) $0^*(100)^*1^*$

$L = \{0, 1, 01, 100, \ldots \infty\}$

$\rightarrow$ String of min. length = $\{10\}$

(d) $1^*(0+10)^*1^*$

$L = \{1, 0, 10, 11, 00, 01, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, \ldots \infty\}$

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→ string of min. length = {0110}.

Q.2 $r = 0^* + 1^*$

$s = 01^* + 10^* + 1^*0 + (0^*1)^*$

(a) $L = \{0, 1, 00, 11, 000, 111, \ldots \infty\}$
(based on r)

$L' = \{0, 1, 01, 10, 11, 100, \ldots \infty\}$
(based on s)

→ string corresponding to r but not to s = {00}

(b) → String corresponding to s but not to r = {10, 01}

(c) String corresponding to both r and s = {0}

(d) $t = \{0, 1\}^*$

$L_1 = \{0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111, \ldots \infty\}$

→ string in $\{0,1\}^*$ but not in r and s = {010}.

3.3 (a) $(H+S+HS+SH)^* = \{\varepsilon, H, S, HS, SH, HH, SS, \dots \infty\}$

→ simpler expression = $\cancel{H^* (HS)^* H^* S^* + S^* H^*}$. $(H+S+HS)^*$.

(b) $(H(H+S)^*)^+$

$= \{H, HH, HS, HHH, HHS, HSS, HSH, \dots \infty\}$

→ simpler expression = $H(H+S)^+$.

(c) $H(H^*H + H^*) + H^*$

→ simpler expression = H^*.

(d) $(H+\Lambda)^*$

→ simpler expression = H^*.

(e) $(H+S)^* HS (H+S)^* + S^* H^*$

$= (H, S, HS, HH, SH, SS, \dots \infty\}$

→ simpler expression = $(H+S)^*$

3.4 $(111^*)^* = (11+111)^*$

→ LHS = $(111^*)^* = \{11, 111, 1111, 11111, \dots \infty\}$

$\cancel{= (11 1^*)(111^*)^*}$ (~~we know that $L^* = \varepsilon + LL^*$~~)

→ RHS = $(11+111)^*$
$= \{11, 111, 1111, 11111, \dots \infty\}$

→ As we can see that any string generated

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by $(111^*)^*$ is accepted by $(11+111)^*$ as $(11+111)^*$ has an odd and even pair of 1's that can be used to generate all strings generated by $(111^*)^*$.

$\boxed{3.5}$

$$(aa^* bb^*)^* = \Lambda + a(a+b)^* b.$$

LHS = { ε, ~~aab~~ ab, aab, abb, ... ∞ }

RHS = { ε, ab, aab, abb, ... ∞ }

⇒ LHS = RHS as both languages contain ε and except for ε they start with a and end with b with any combination of "a"'s and "b"'s possible between them.

$\boxed{3.9}$ (a) { 00, 001, 100, 1001, 1100, 11001, 11100, ... ∞ }

$$R.E = 1^* 0 1^* 0 (1^*)^*.$$

(b) { 00, 000, 001, 100, 1000, 1001, 1100, 11000, 11001, 11100, 111000, 111100, 100111, 100100, ... ∞ }

$$RE = (1^* 0)^* 1^* 0 (1^* 0)^*.$$

(c) { 00, 10, 11, 100, 110, ... ∞ }

$$RE = (0+1)^* 0.$$

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(d) $\{00, 11, 0011, 1100, \dots \infty\}$

$RE = (00 + 11)(0+1)^*(00+11)$

(e) $\{01, 10, 101, 010, 011, 110, 111, \dots \infty\}$

$RE = \cancel{(0+1)\,1\,(0+1)}$
$= (0+1)(10+11)^* + \Lambda.$

(f) $\{\cancel{00}\,00, 100, 1100, 10000, 1001, \dots \infty\}$

$RE = 0^*(10^*10^*)^*.$

(g)

(h) $\cancel{(0+1)11}$ $\{011, 111, 011011, 1111, 1011, 11011, \dots \infty\}$

$RE = \cancel{(\Lambda + 1)(1)^*(0(11)^*(}$

$RE = 1^*\,0(11)^*\,1^*$

(i) $\{11, 010, 11010, 001001, 1011, \dots \infty\}$

$RE = (1^*11 + 001001)$

~~L^R denotes reverse of language~~ L

3.15 (a) $(a(a+a^* aa)+aaa)^*$

$\Rightarrow sh(a^*) = sh(a)+1$
$= 1$

$\Rightarrow sh(\cancel{a^*}aa) = sh(a+a)$
$= \max(sh(a), sh(a))$
$= 0$

$\Rightarrow sh(a^* aa) = sh(a^* + aa)$
$= \max(sh(a^*), sh(aa))$
$= 1$

$\Rightarrow sh(a+a^* aa) = \max(sh(a), sh(a^* aa))$
$= 1$

$\Rightarrow sh(a.(a+a^* aa)) = \max(sh(a), sh(a+a^* aa))$
$= 1$

$\Rightarrow sh(aaa) = \max(sh(a), sh(aa))$
$= 0$

$\Rightarrow sh(a(a+a^* aa)+aaa) = 1$

$\Rightarrow sh((a(a+a^* aa)+aaa)^*) = 1+1$
$= \boxed{2}$

(b) $(((a+a^* aa)aa)^* + aaaaaa^*)^*$

$\Rightarrow \boxed{sh(a)=0}$ $\Rightarrow sh(aa) = 0$

$\Rightarrow sh(a)^* = 0+1$

$\boxed{sh(a^*) = 1}$ $\Rightarrow sh(aaaa) = 0$

$\rightarrow sh(a^* aa) = 1$ $\Rightarrow sh(aaaaa) = 0$

$\Rightarrow sh(a+a^* aa) = 1$ $\Rightarrow sh(aaaaaa^*) = 1.$

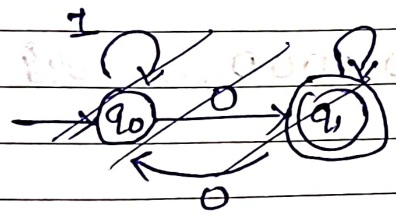
$\Rightarrow sh((a + a^*aa)\, aa) = 1$

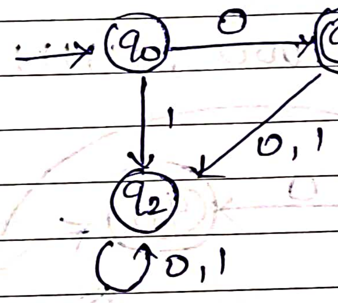
$\Rightarrow sh\{((a + a^*aa)\, aa)\}^* = 2$

$\Rightarrow sh(\underbrace{((a+a^*aa)\, aa)^* + aaaaa^*}_{p}) = 2$

$\Rightarrow \boxed{sh(p)^* = 3}$

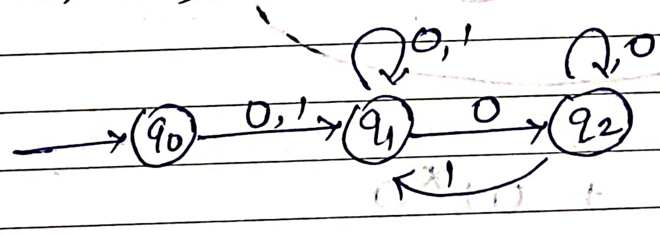
3.18 Let α be "~~0101~~". "0" $\Rightarrow$ States required = 3.





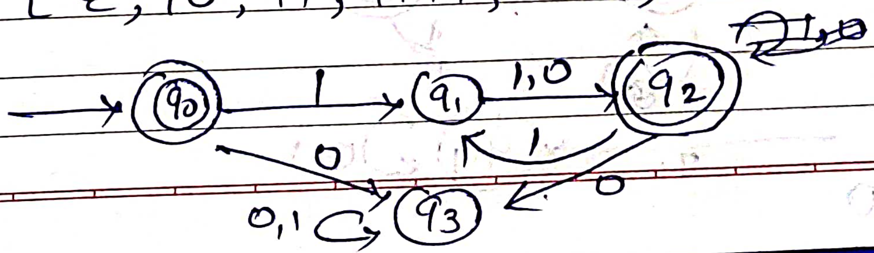
3.20 (a) $(0+1)^*0$

$L = \{00, 10, 000, 010, 100, 1\cancel{0}0, \ldots \infty\}$



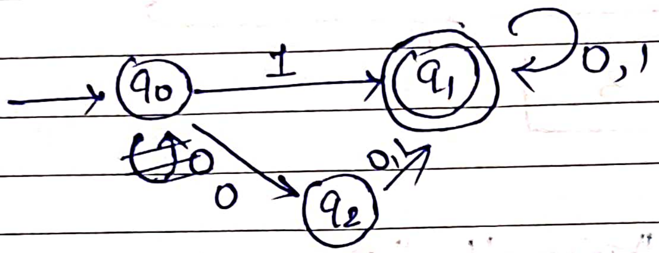
(b) $(11+10)^*$

$L = \{\varepsilon, 10, 11, 1111, 1110, \ldots \infty\}$



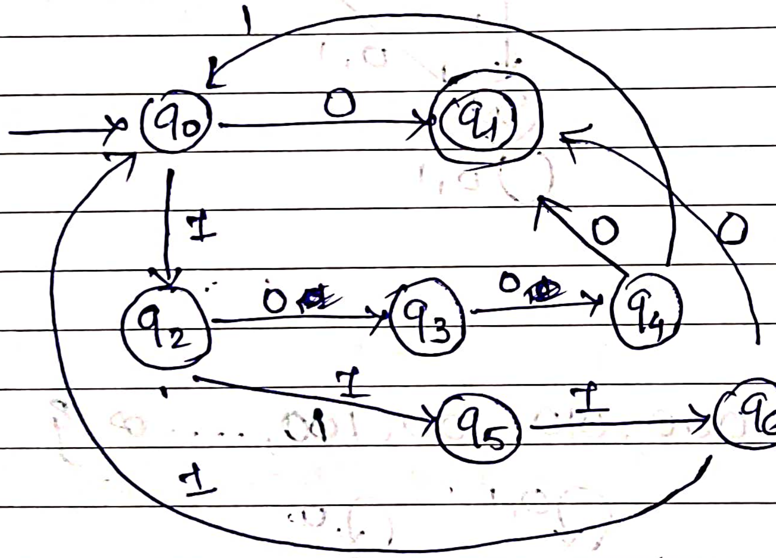
(c) $(0+1)^*(1+00)(0+1)^*$

$L = \{1, 00, 01, 11, 000, 100, 010, 011, 0000, 0001, 110, 111, 1000, 1001, \ldots \infty\}$.



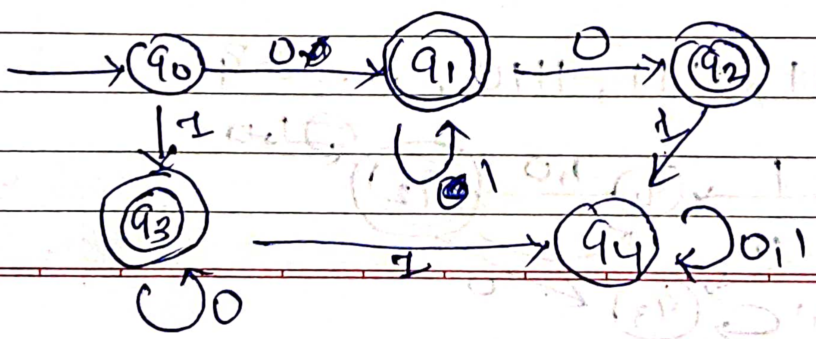
(d) $(111+100)^*0$

$L = \{0, 1110, 1000, 111110, 1000000, \ldots \infty\}$



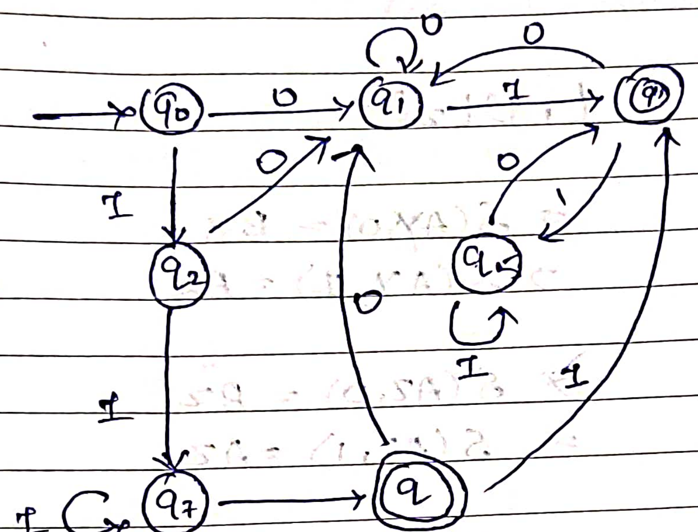
(e) $0+10^*+01^*0$

$L = \{0, 10, 00, 1, 010, 100, 0110, 1000, \ldots \infty\}$

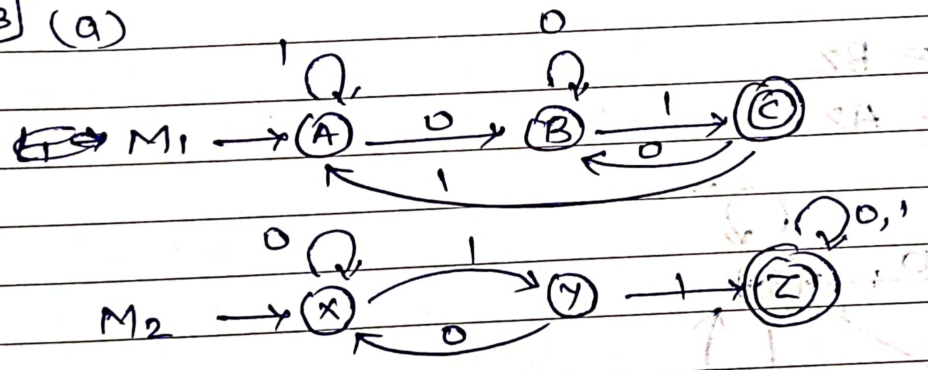


(f) $(0+1)^* \; (01 + 110)$

$L = \{01, 110, 001, 0110, 101, 1110, \ldots \infty\}$



3.33 (a)



$L_1 \cup L_2$:- $A = \{AX, AY, AZ, BX, BY, BZ, CX, CY, CZ\}$

$\Rightarrow \delta(AX, 0) = \delta(\delta(A,0), \delta(X,0))$
$= (BX)$

$\Rightarrow \delta(BY, 1) = CZ$ $\qquad \Rightarrow \delta(BZ, 1) = CZ$ $\qquad \Rightarrow \delta(CX, 1) = AY$

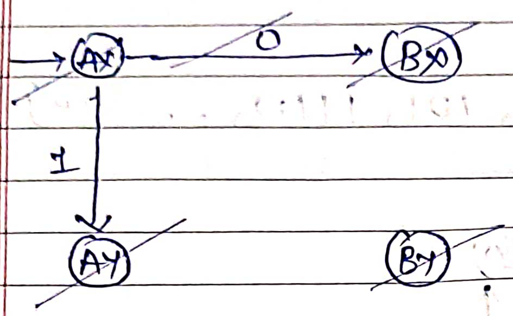
$\Rightarrow \delta(AY, 0) = BX$ $\qquad \Rightarrow \delta(BZ, 0) = BZ$ $\qquad \Rightarrow \delta(AX, 1) = AY$

$\Rightarrow \delta(AZ, 0) = BZ$ $\qquad \Rightarrow \delta(CX, 0) = BX$ $\qquad \Rightarrow \delta(AY, 1) = AY$

$\Rightarrow \delta(BX, 0) = BX$ $\qquad \Rightarrow \delta(CY, 0) = BX$ $\qquad \Rightarrow \delta(AZ, 1) = AZ$

$\Rightarrow \delta(BY, 0) = BY$ $\qquad \Rightarrow \delta(CZ, 0) = BZ$ $\qquad \Rightarrow \delta(BX, 1) = CY$

$\Rightarrow \delta(CY, 1) = AZ$ $\qquad \Rightarrow \delta(CZ, 1) = AZ.$



$L_1 \cup L_2$:-

$\Rightarrow \delta(AX, 0) = BX$
$\Rightarrow \delta(AX, 1) = AY$

$\Rightarrow \delta(BX, 0) = BX$
$\Rightarrow \delta(BX, 1) = CY$

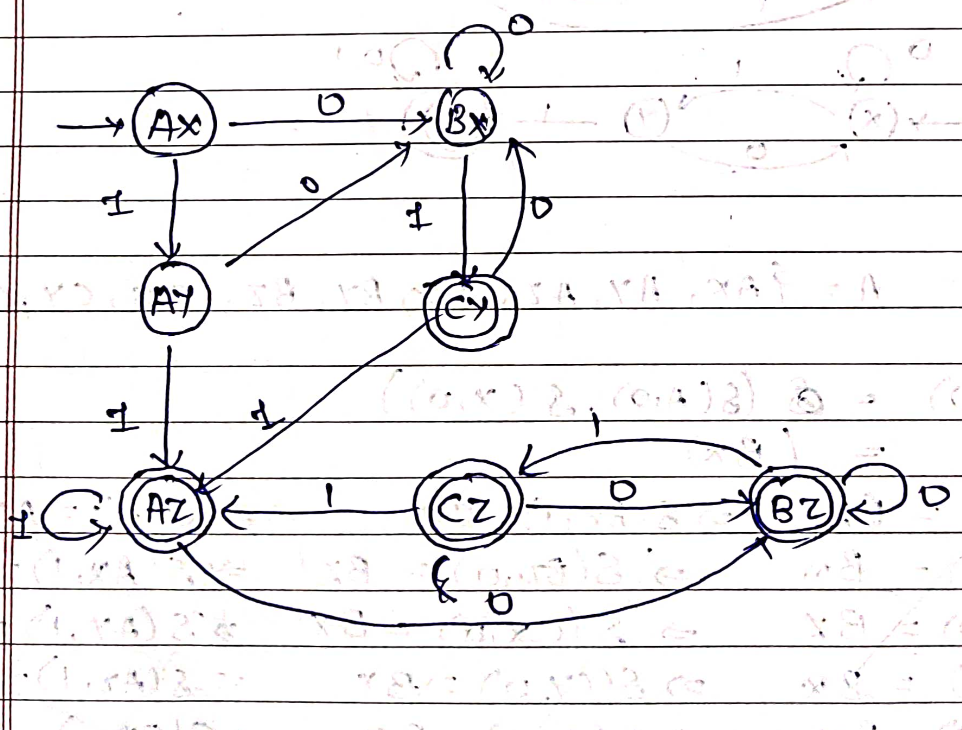
$\Rightarrow \delta(CY, 0) = BX$
$\Rightarrow \delta(CY, 1) = AZ$

$\Rightarrow \delta(CZ, 0) = BZ$
$\Rightarrow \delta(CZ, 1) = AZ$

$\Rightarrow \delta(A\cancel{X}Y, 0) = BX$
$\Rightarrow \delta(AY, 1) = AZ$

$\Rightarrow \delta(AZ, 0) = BZ$
$\Rightarrow \delta(AZ, 1) = AZ$

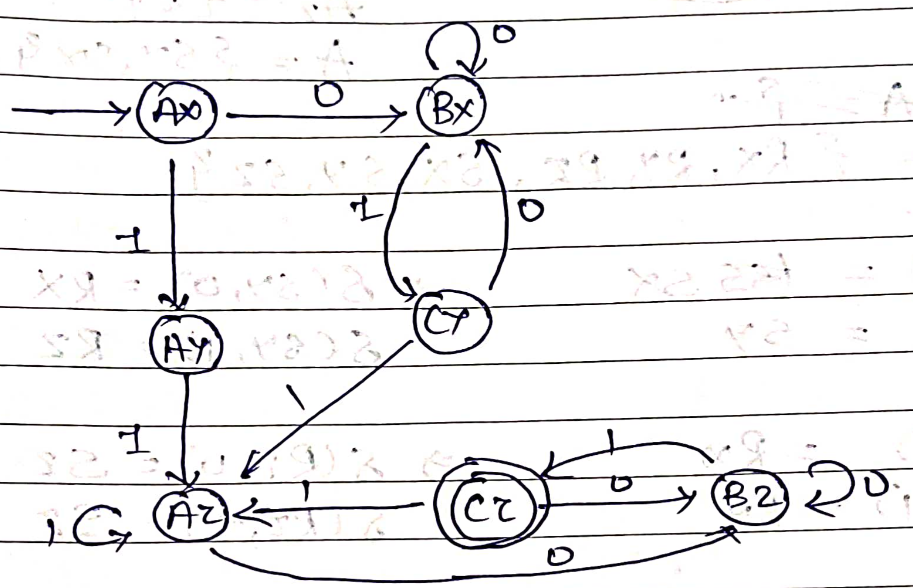
$\Rightarrow \delta(BZ, 0) = BZ$
$\Rightarrow \delta(BZ, 1) = CZ$



⇒ $L_1 \cap L_2$:- $q_0 = \{q_1, q_2\}$, $q_1 = \{A\}$
$q_2 = \{P\}$

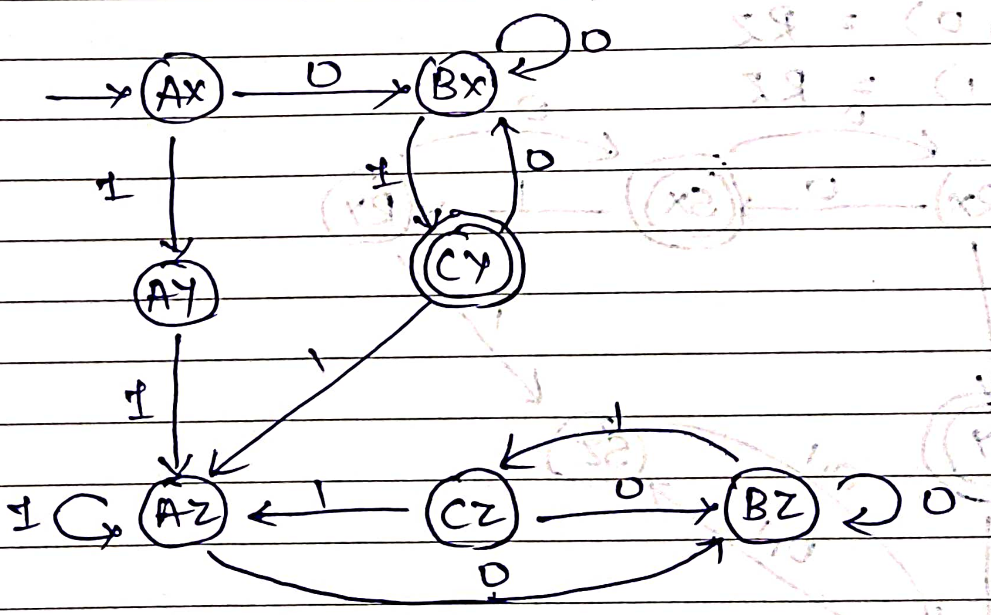
$q_0 = \{AP\}$

Accepting states $A = \{CZ\}$



⇒ $L_1 - L_2$: $A_1 = \{\cancel{A, B}, C\}$, $A_2 = \{\cancel{X, Y}, Z\}$

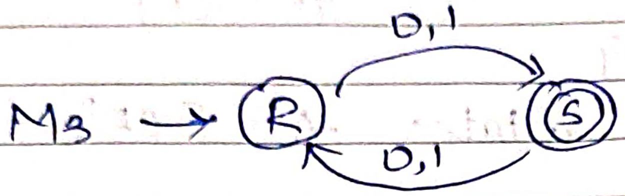
$A = \{CY \cancel{Z}\}$



→ $L_3 - L_2$:-

$M_3 = \{R, S\}$

$M_2 = \{x, y, z\}$



$A_3 = \{S\}$

$A_2 = \{z\}$

$A = \{Sy, Sx\}$

~~$A = \{$~~

$L_3 \times L_2 = \{Rx, Ry, Rz, Sx, Sy, Sz\}$

$\delta(Rx, 0) = \cancel{ISS}\ Sx$

$\delta(Rx, 1) = Sy$

⇒ $\delta(Sy, 0) = Rx$

$\delta(Sy, 1) = Rz$

$\delta(Sx, 0) = Rx$

$\delta(Sx, 1) = Ry$

⇒ $\delta(Rz, 0) = Sz$

$\delta(Rz, 1) = Sz$

~~$\delta(Rx, 0) =$~~

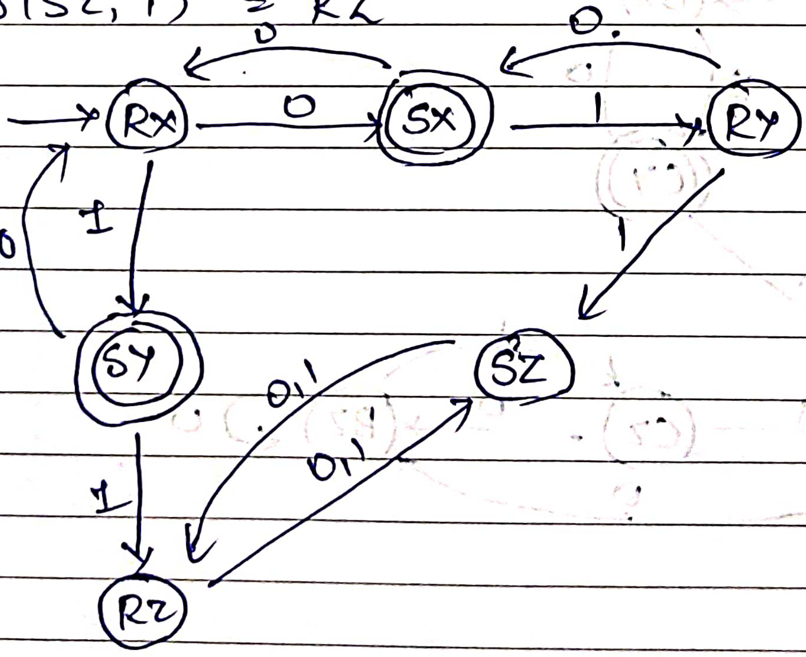
~~$\delta(Rx, 1) =$~~

⇒ $\delta(Ry, 0) = Sx$

⇒ $\delta(Ry, 1) = Sz$

⇒ $\delta(Sz, 0) = Rz$

$\delta(Sz, 1) = Rz$



→ $L_1 \cap L_3$:- $M_1 = \{A, B, C\}$, $M_3 = \{R, S\}$

$M_1 \times M_3 = \{AR, AS, BR, BS, CR, CS\}$ $q_0 = \{AR\}$

$\delta(AR, 0) = BS$ $\delta(AS, 0) = BR$
$\delta(AR, 1) = AS$ $\delta(AS, 1) = AR$

$\delta(BS, 0) = BR$ $\delta(\cancel{A}CR, 0) = BS$
$\delta(BS, 1) = CR$ $\delta(CR, 1) = AS$

$\delta(BR, 0) = BS$ $\delta(\cancel{B}CS, 0) = BR$
$\delta(BR, 1) = CS$ $\delta(CS, 1) = AR$