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A Report on Nonlinear Control of BLDC using Sliding
Mode Control

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CERTIFICATE



This is to certify that the TIP report entitled “**Nonlinear Control of BLDC using Sliding Mode Control**”, has been done by **Mayank Sharma** under my guidance and supervision & has been submitted for term work evaluation for “**Technical Internship Program**” in semester “**VI**” for the degree of Bachelor of Technology in Mechatronics of MPSTME, SVKM’s NMIMS (Deemed-to-be University), Mumbai, India.

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Date:

Place: Mumbai, India.

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ABSTRACT

In order to optimize the speed-control performance of the Brushless DC motor (BLDC) system with different disturbances and uncertainties, a nonlinear speed-control algorithm for the BLDC using sliding-mode control technique is developed in this paper. A sliding-mode control method based on one novel sliding-mode reaching law (SMRL) is presented. This SMRL can dynamically adapt to the variations of the controlled system, which allows chattering reduction on control input while maintaining high tracking performance of the controller. Simulation and experimental results both show the validity of the proposed control approach.

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1. INTRODUCTION

1.1 Purpose, Scope and Limitation

Now a day the demand of BLDC motor increases due to their high efficiency, higher torque and power density, lower cost, simpler structure, better controllability, and large torque to inertia ratio compared to brushless AC motors. So, they are used in many domestic and industrial applications ranging from servo to traction drives. A BLDC motor is an inside out DC commutator motor with the mechanical commutator replaced by an electronic switching converter. The most popular way to control a BLDC motor is via voltage –source current-controlled inverters. The inverter must supply a rectangular current waveform whose magnitude is proportional to the motor's shaft torque. The back-EMF waveform of a BLDC motor is trapezoidal shape due to the concentrated winding. In this paper BLDC with 120° conduction mode is proposed, that means only two-phase conduct at any instant of time. BLDC motor fed by two-phase conduction has higher power/weight and torque/current ratios [21]. Ideally a BLDC motor supplied with rectangular 120° elec. Phase currents produce a trapezoidal back-EMF waveform whose amplitude is constant over >120° elec. will result a ripple free torque. However, in a practical BLDC drive torque pulsation arise due to the deviation of back-EMF waveform from the ideal.

In THE BLDC motor control system, the classical proportional integral (PI) control technique is still popular due to its simple implementation [22]. However, in a practical PMSM system, there are large quantities of the disturbances and uncertainties, which may come internally or externally, e.g., unmodeled dynamics, parameter variation, friction force, and load disturbances. It will be very difficult to limit these disturbances rapidly if adopting linear control methods like PI control algorithm [20].

Therefore, many nonlinear control methods have been adopted to improve the control performances in systems with different disturbances and uncertainties, e.g., robust control [4], [5], sliding-mode control (SMC) [6], [7], [10], [16], adaptive control [8], backstepping control [9], predictive control [11], intelligent control [13], [14], and so on. In these nonlinear control methods, SMC method is well known for its invariant properties to certain internal parameter variations and external disturbances, which can guarantee perfect tracking performance despite parameters or model uncertainties.

In [17], the performance of a sliding mode controller was studied using a hybrid controller applied to induction motors via sampled closed representations. The results were very conclusive regarding the effectiveness of the sliding-mode approach. Neuron-fuzzy sliding-mode controller applied to induction machine can also be found in [15].

However, the robustness of SMC can only be guaranteed by the selection of large control gains, while the large gains will lead to the well-known chattering phenomenon, which can excite high-frequency dynamics. Thus, some approaches have been proposed to overcome the chattering, such as continuation control, high-order sliding-mode method [16], complementary sliding-mode method [18], and reaching law method [2],[3],[12], [19]. The reaching law approach deals directly with the reaching process since chattering is caused by the nonideal reaching at the end of the reaching phase. In [3], authors presented some reaching laws, which can restrain chattering by decreasing gain or making the discontinuous gain a function of sliding-mode surface. In [12], a novel exponential reaching law was presented to design the speed- and current-integrated controller. To suppress chattering problem, system variable was used in this reaching law. However, in the aforementioned reaching laws, the discontinuous gain rapidly decreases because of variation of the functions of the sliding surface, thus reducing the robustness of the controller near the sliding surface and increasing the reaching time.

In order to solve the aforementioned problems, a reaching law, which is based on the choice of an exponential term that adapts to the variations of the sliding-mode surface and system states, is proposed in [1]. This reaching law can deal with the chattering/reaching time dilemma. Based on this reaching law, a sliding-mode speed controller of BLDC is developed

2. Mathematical Modelling of BLDC Motor

Three phase star connected BLDC motor can be described by the following equations, to generate larger voltages, currents, and power levels, photovoltaic cells are electrically coupled in series and/or parallel circuits

$$V_{ab} = R_s(i_a - i_b) + L_s \frac{d}{dt}(i_a - i_b) + e_a - e_b \dots\dots (1)$$

$$V_{bc} = R_s(i_b - i_c) + L_s \frac{d}{dt}(i_b - i_c) + e_b - e_c \dots\dots (2)$$

$$V_{ca} = R_s(i_c - i_a) + L_s \frac{d}{dt}(i_c - i_a) + e_c - e_a \dots\dots (3)$$

The equation of motion can be expressed as:

$$T_e = T_L + B\omega_m + J \frac{d\omega_m}{dt} \dots\dots\dots (4)$$

Where v , i and e denote the phase-to-phase voltage, phase current and back-emf respectively in the three-phase a, b and c. R_s and L_s denote the line-to-line resistance and inductance of stator winding. T_e is the electromagnetic generated torque, T_L is the load torque, B is the friction coefficient, J is the polar moment of inertia and ω_m is the angular velocity of rotor [3],[4].

BLDC model in dq axis: -

$$\begin{aligned}T_e &= 1.5p\psi_a i_q \\T_e - T_L &= \frac{J}{p}\dot{\omega} + B\omega \\u_d &= ri_d - \omega Li_q + L\dot{i}_d \\u_q &= ri_q + \omega Li_d + \omega\psi_a + L\dot{i}_q \quad \dots (1)\end{aligned}$$

where u_d and u_q represent d and q axes stator voltages, respectively. i_d and i_q are d and q axes currents, respectively; L is stator inductance; r is stator resistance; T_e is electrical magnetic torque; T_L is load torque; p is number of pole pairs; ψ_a is flux linkage of permanent magnets; ω is electrical angular velocity; B is viscous friction coefficient; J is rotational inertia

3. Sliding Mode Controller

Now explaining the SMC used in [1]. In general, SMC design can be divided into two steps, the first step is to choose the sliding-mode surface, and the next step is to design the control input such that the system trajectory is forced toward the sliding-mode surface, which ensures the system to satisfy the sliding mode reaching condition that is expressed as follows:

$$s \cdot \dot{s} < 0 \quad \dots(2)$$

where s is the sliding-mode surface.

The following second-order nonlinear model is generally used to describe the SMC system adopting one reaching law method:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + g(x) + b(x)u \end{cases} \quad \dots(3)$$

where $x = [x_1, x_2]^T$ is system state, $g(x)$ represents the system disturbances, and $b(x)$ is not zero. The concrete steps include the following. First, the typical sliding-mode surface is chosen as follows:

$$s_1 = cx_1 + x_2 \quad \dots(4)$$

Such sliding-mode surface can guarantee the asymptotic stability of the sliding mode, and the asymptotic rate of convergence is in direct relation with the value of c .

Next, the control input u should be designed in such a way that the sliding-mode reaching condition (inequality (2)) is met. Thus, equal reaching law is typically chosen as follows:

$$\dot{s}_1 = -k_1 \cdot \text{sgn}(s_1) \quad \dots(5)$$

On substitution we get the control input as:

$$u = -b^{-1}(x)[cx_2 + f(x) + g(x) + k_1 \cdot \text{sign}(s_1)] \quad \dots(6)$$

Here, it can be found that the discontinuous term $-b^{-1}(x)k_1 \cdot \text{sign}(s_1)$ is contained in the control input, which leads to the occurrence of chattering. And the chattering level is up to the value of k_1 directly. The time required to reach sliding-mode surface can be derived by integrating (5) with respect to time as follows:

$$t_1 = \frac{|s(0)|}{k_1} \quad \dots(7)$$

It can be observed that this reaching time can also be regulated by the value of k_1 directly. If the value of k_1 is increased, a faster reaching time and a good robustness can be obtained, but the chattering level on the control input also increases. Thus, in order to solve this dilemma, a novel reaching law is proposed in [1].

SMRL is realized based on the choice of an exponential term that adapts to the variations of the sliding-mode surface and system states. This reaching law is given in [1] is as follows:

$$\begin{aligned} \dot{s} &= -eq(x_1, s) \cdot \text{sign}(s), eq(x_1, s) \\ &= \frac{k}{\left[\varepsilon + \left(1 + \frac{1}{|x_1|} - \varepsilon \right) \cdot e^{-\delta|s|} \right]} \end{aligned}$$

where $k > 0$, $\delta > 0$, and $0 < \varepsilon < 1$. x_1 is the system state. In this novel reaching law, it can be found that if $|s|$ increases, the $eq(x_1, s)$ converges to the value of k/ε that is greater than the value of k . This indicates that a faster reaching time can be obtained. On the other hand, if $|s|$ decreases, denominator term of the $eq(x_1, s)$ approaches $1 + 1/|x_1|$, then the $eq(x_1, s)$ converges to $k|x_1|/(1 + |x_1|)$, in which system state $|x_1|$ gradually decreases to zero under the control input designed in the next section. This indicates that when the system

trajectory approaches the sliding-mode surface, the $eq(x1 .s)$ gradually decreases to zero to suppress the chattering. Thus, the controller designed by proposed reaching law can dynamically adapt to the variations of the sliding-mode surface and system states $|x1|$ by making $eq(x1 .s)$ vary between k/ε and zero.

3.1 Speed Controller Design Based on the Proposed Reaching Law

Speed-control algorithms should keep the actual speed track of the speed reference ω_{ref} accurately under the occurrence of disturbances. To achieve this control objective, the tracking error is defined as $e = \omega_{ref} - \omega$. Then, according to aforementioned sliding-mode design method, the following sliding-mode surface is chosen:

$$S = e = \omega_{ref} - \omega$$

which is called linear sliding-mode surface. Taking the time derivative of the sliding-mode surface yields

$$\dot{S} = \dot{\omega}_{ref} - \dot{\omega}$$

Moreover, according to the (1), the dynamic equation of the motor can be expressed as follows, with the parameters variations taken into accounts:

$$\begin{aligned}\dot{\omega} &= ai_q - bT_L - c\omega \\ &= a_n i_q - b_n T_L - c_n \omega + \Delta a i_q - \Delta b T_L - \Delta c \omega \\ &= a_n i_q - c_n \omega + r(t)\end{aligned}$$

where $a = a_n + \Delta a = 3p2\psi a/2J$, $b = b_n + \Delta b = p/J$, and

$c = c_n + \Delta c = B/J$. a_n , b_n , and c_n are nominal parameter.

Δa , Δb , and Δc are parameter variations. In the above equation,

$r(t) = \Delta a i_q - \Delta c \omega - b T_L$ represents the lumped disturbances including internal parameter variation, friction force, and external load disturbances, which is assumed to be bounded.

$$|r(t)| \leq 1$$

where l is the upper bound of the lumped disturbances.

Furthermore, substituting and the reaching law yields.

$$\dot{S} = \dot{\omega}_{ref} + c_n \omega - r(t) - a_n i_q$$

$$= -eq(x_1, S) \cdot \text{sgn}(S)$$

Therefore, the control input i_q is designed as follows:

$$i_q^* = a_n^{-1} \{ \dot{\omega}_{ref} + c_n \omega + [l + eq(x_1, S)] \cdot \text{sgn}(S) \}.$$

It can be found that upper bound l has an important effect on the control performance. However, it is difficult to select upper bound in practical application, because the lumped disturbances are difficult to know the exact value and measure. Though some methods, such as error control and trial, can be used to select upper bound, these approaches are time consuming and cannot provide enough robustness.

The Lyapunov function $V = S^2/2$ is chosen, and the following relations can be obtained:

$$\begin{aligned} \dot{V} &= S \cdot \dot{S} = S[\dot{\omega}_{ref} + c_n \omega - r(t) - a_n i_q] \\ &= S[-eq(x_1, S) \cdot \text{sgn}(S)] \\ &= -|S| eq(x_1, S) \leq 0. \end{aligned}$$

This can guarantee that the designed control system is stable and any tracking error trajectory will converge to zero in a finite time.

4. Control Architecture

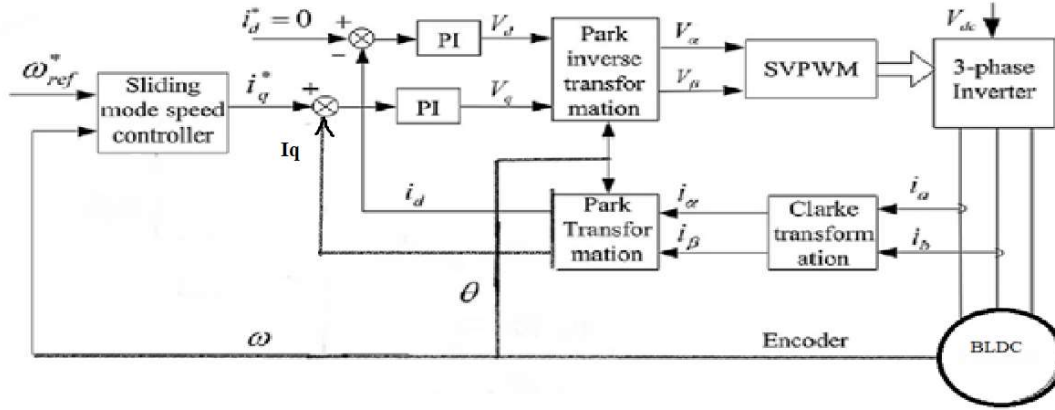


Figure 1: Control Scheme of the proposed system

The proposed control scheme in [1] is modified such that the model is suitable for a BLDC system. The blocks such as [Clarke transform](#), [Park Transform](#), and [Inverse Park](#) transforms are the blocks used to convert the parameters like voltages and currents from one frame of references to other. Using the mathematical equations mentioned in the previous sections this control scheme is designed.

The user gives reference speed and power to the inverter and the motor. The sliding mode control block gives the estimated control input which is compared with the current i_q calculated from the Park Transformation block. The difference is then fed to a PI controller and then to Inverse Park transformation Block which then gives output Voltages as V_α & V_β which then goes to the inverter and the inverter has received the logic to commutate the BLDC. The commutation logic is same as any other BLDC commutation which is detecting the rotor angle and then the BLDC is commutated accordingly. To measure the rotor angle we have used a encoder which is fed to the Park transform blocks and Inverse Park Transform blocks.

Position (θ)	Sector	Switching Sequence (AA' BB' CC')		
		AA'	BB'	CC'
(-30°, 30°]	1	00	10	01
(30°, 90°]	2	01	10	00
(90°, 150°]	3	01	00	10
(150°, 210°]	4	00	01	10
(210°, 270°]	5	10	01	00
(270°, 330°]	6	10	00	01

Figure 2: Inverter Switching Sequence

We can take a look at the Setup of a BLDC with a Inverter and Hall sensor explaining the BLDC commutation with respect to the inverter logic levels and the *Figure 3* is cited from MATHWORKS.

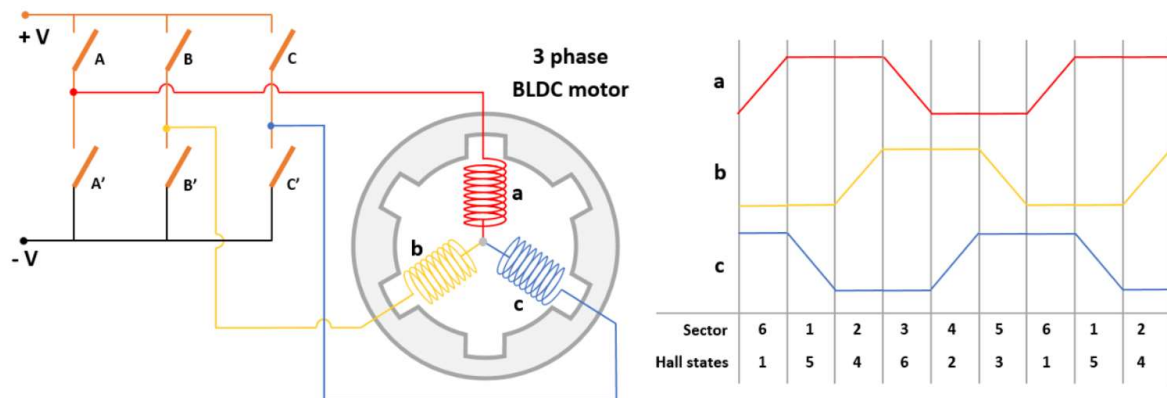


Figure 3: Commutation Sequence and Graph

5. Simulink Model of Proposed Idea

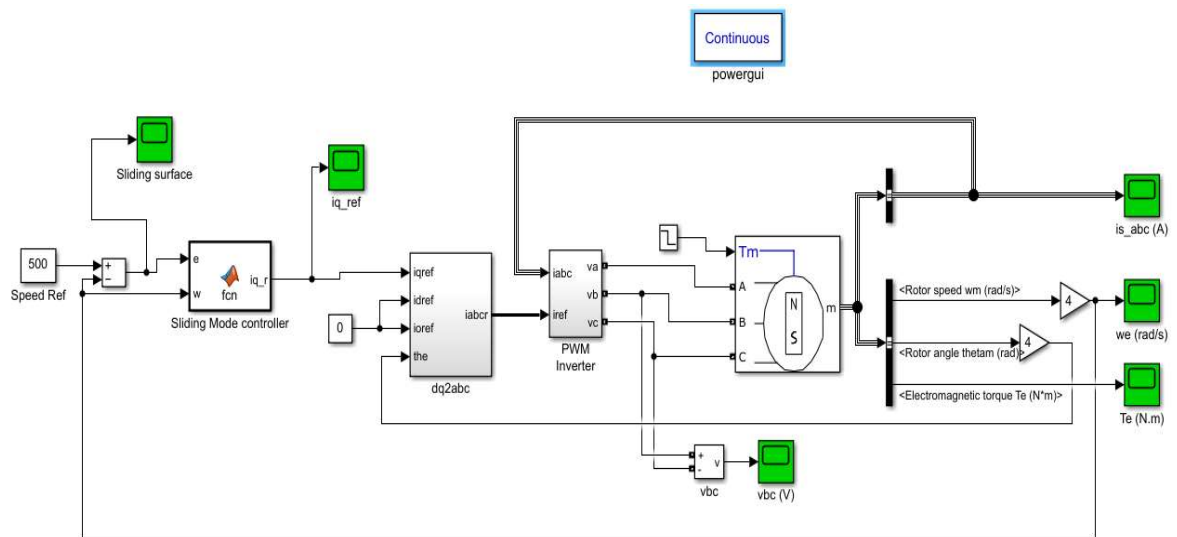


Figure 4: Simulink Model

5.1.1 The Sliding Mode Control Block:-

- Contains a MATLAB FUNCTION shown below in *Figure 5*. The equations are as per discussed in the section earlier.

The screenshot shows the MATLAB editor interface. The title bar reads "Block slidingmode_control/Sliding Mode Controller". The editor has two tabs: "EDITOR" and "VIEW". The "EDITOR" tab is active, showing a script with the following code:

```

1  function iq_r = fcn(e,w)
2
3
4      k=20;
5      del=10;
6      ep=0.1;
7      p=8;
8      J= 0.44e-3;
9      S1=0.107;
10     B=0;
11     an=3*p^2*S1/(2*J);
12     bn=p/J;
13     cn=B/J;
14     eq_s = (k)/(ep*(1+1/(abs(e)-ep)))*exp(-del*abs(e));
15     iq_r= an^(-1)*(cn*w+(110000+eq_s)*sign(e));
16
17     if iq_r<-30
18         iq_r=-30;
19     end
20     if iq_r>30
21         iq_r=30;
22     end
23 end

```

The "VIEW" tab contains various toolbars for file operations (New, Open, Save, Find Files, Compare, Print), navigation (Go To, Find, NAVIGATE), editing (Insert, Comment, Indent, EDIT), and breakpoints (Breakpoints, BREAKPOINTS).

Figure 5: MATLAB Function

5.1.2 Motor Parameters: -

The Parameters are as Mentioned in the *Figure 6*

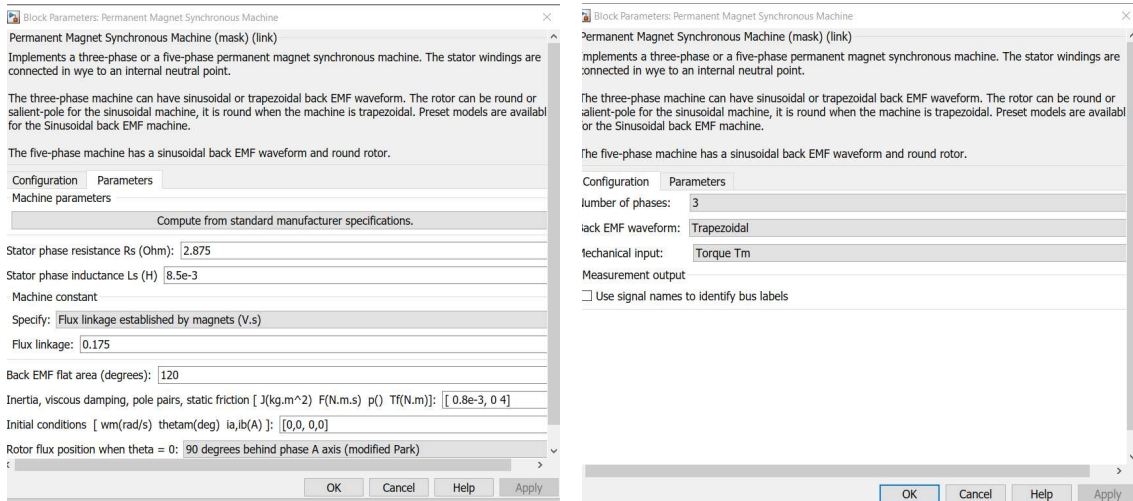


Figure 6: Motor Parameters

5.1.3 Power Block:-

The powergui block allows you to solve your circuit using one of the following methods:

- Simulink's variable-step solver is used in Continuous.
- Discretization of the electrical system to solve a problem in discrete time steps.
- The phasor solution can be continuous or discrete.

In addition, the powergui block provides access to tools for analyzing steady-state and simulation findings, as well as advanced parameter design.

To simulate any Simulink model including Simscape™ Electrical™ Specialized Power Systems blocks, you'll require the powergui block. It keeps track of the comparable Simulink circuit that embodies the model's state-space equations.



Figure 7: Power Block

When one powergui block is used in a model:

- To get the best performance, put the powergui block in the top-level diagram. Make sure the block has the name powergui on it.

6. Simulation Results

The simulation results obtained were very much like the literature review. The Sample time selected was $T=0.6s$.

The *Figure 8* shows the graph of sliding surface which reaches zero after 0.32s, we designed our Sliding surface that should be around zero and the results seen in the *Figure 8* proves that the Sliding surface designed is appropriate and behaving as expected.

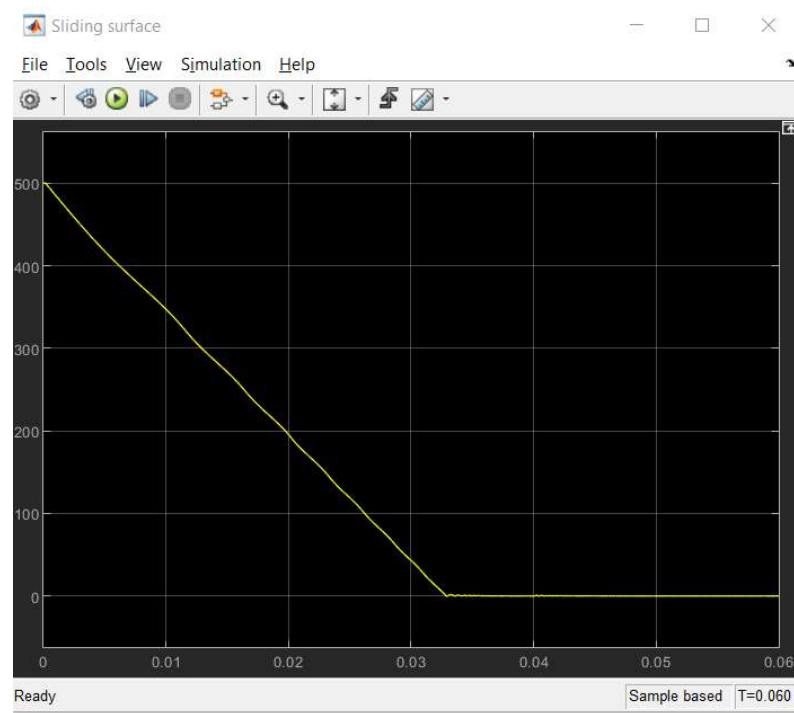


Figure 8: Graph of Sliding Surface

The reference speed give to the system is 500rpm and the Simulink model should reach speed of 500 rpm for our algorithm to work properly. The results shown in *Figure 9* prove that the Simulink model behaves normally and as expected to reach the speed of 500 rpm after 0.34s but it has ripples due to the chattering effect.

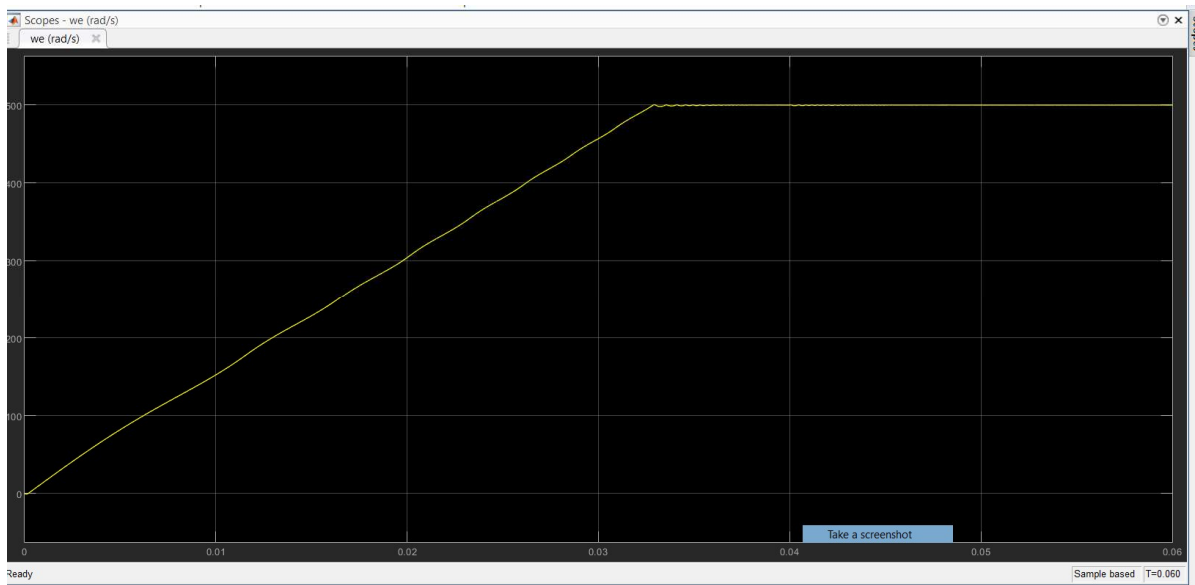


Figure 9 : Speed Measured

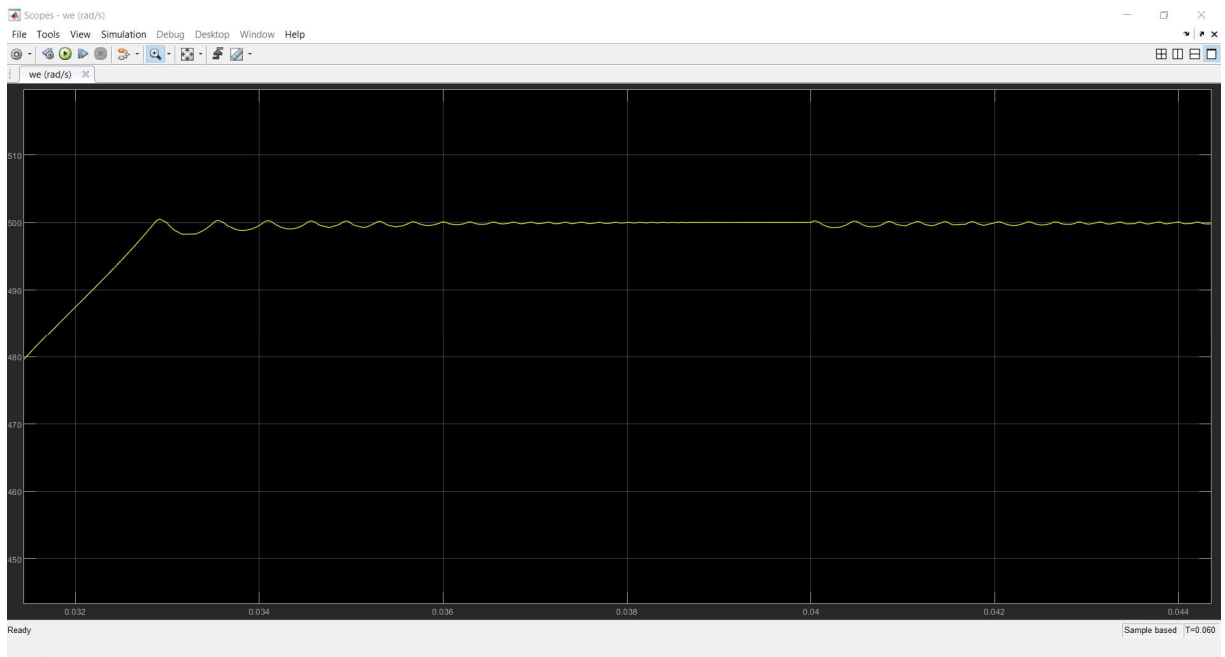


Figure 10: Ripples in the speed due to the Chattering effect of SMC

7. CONCLUSION

We learned about the Brushless Motor and how it works on the non-linear sliding mode control principles in this project.

The traditional sensored BLDC uses PI controllers but that have many errors. Sliding Mode Control (SMC) is a way of controlling the BLDC in a non-linear fashion and this concept of SMC can further be developed for the use of sensorless control of BLDC and PMSMs in future. The SMC method gives rise to chattering effect which can be clearly seen from the results.

We investigated the Simulink block in depth and were successful in becoming acquainted with the MATLAB platform.

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