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Assignment – Parameter Estimation

Q1 $X_1, X_2, \ldots, X_n$ is a random sample from a Normal Population with mean $= \theta_1$ and variance $= \theta_2$

Pdf of normal distribution

$$f(x_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$\mu = \theta_1, \ \sigma^2 = \theta_2$

$$\therefore f(x_i) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

Likelihood function, $L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i)$

$$L(\theta_1,\theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}} = (2\pi)^{-n/2} \theta_2^{-n/2} e^{-\frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2}$$

$$\log L(\theta_1,\theta_2) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(\theta_2) - \sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{2\theta_2}$$

differentiating wrt θ_1

$$\frac{\partial \log L(\theta_1,\theta_2)}{\partial \theta_1} = \frac{-2}{2\theta_2}\sum (x_i-\theta_1)(-1)$$

Setting to 0 and multiplying by θ_2

$$\sum x_i - n\theta_1 = 0$$

$$\theta_1 = \hat{\mu} = \frac{\sum x_i}{n}$$

$$\theta_{1\,MLE} = \frac{\sum x_i}{n} = \bar{x}$$

$$\frac{\partial \log L(\theta_1,\theta_2)}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\sum (x_i-\theta_1)^2$$

Multiplying by $2\theta_2^2$ after setting to 0

$$-n\theta_2 + \Sigma (x_i - \theta_1)^2 = 0$$

$$\theta_{2MLE} = \hat{\sigma}^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

Q2 Pdf of binomial distribution

$$B(x, n, p) = {}^nC_x \, p^x (1-p)^{n-x}$$

$n = m$, $p = \theta$

$$\therefore f(x_i) = {}^mC_{x_i} \, \theta^{x_i} (1-\theta)^{m-x_i}$$

Likelihood function

$$L(m, \theta) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} {}^mC_{x_i} \, \theta^{x_i} (1-\theta)^{m-x_i}$$

$$L(m, \theta) = \left(\prod_{i=1}^{n} {}^mC_{x_i}\right) \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{mn - \sum_{i=1}^{n} x_i}$$

$$\frac{\partial \log(m, \theta)}{\partial \theta} = \left(\sum_{i=1}^{n} x_i\right) \times \frac{1}{\theta} + \frac{1}{1-\theta} \left(\sum_{i=1}^{n} x_i - mn\right)$$

setting to 0

$$\frac{1}{1-\theta}\left(mn - \sum_{i=1}^{n} x_i\right) - \frac{1}{\theta}\left(\sum_{i=1}^{n} x_i\right) = 0$$

$$\frac{mn}{\Sigma x_i} - 1 = \frac{1-\theta}{\theta}$$

$$\frac{mn}{\Sigma x_i} = \frac{1}{\theta}$$

$$\therefore \theta_{MLE} = \frac{\bar{X}}{m}$$