

# Machine Learning - Exercise 2: Regularized Logistic Regression

## Instructions

You will need to complete the following functions in this exercise:

- % plotData.m (same as the last in-class exercise)
- % sigmoid.m (same as the last in-class exercise)
- % predict.m (same as the last in-class exercise)
- % costFunctionReg.m

```
%% Initialization
clear ; close all; clc

%% Load Data
% The first two columns contains the X values and the third column
% contains the label (y).

data = load('ex2data2.txt');
X = data(:, [1, 2]); y = data(:, 3);

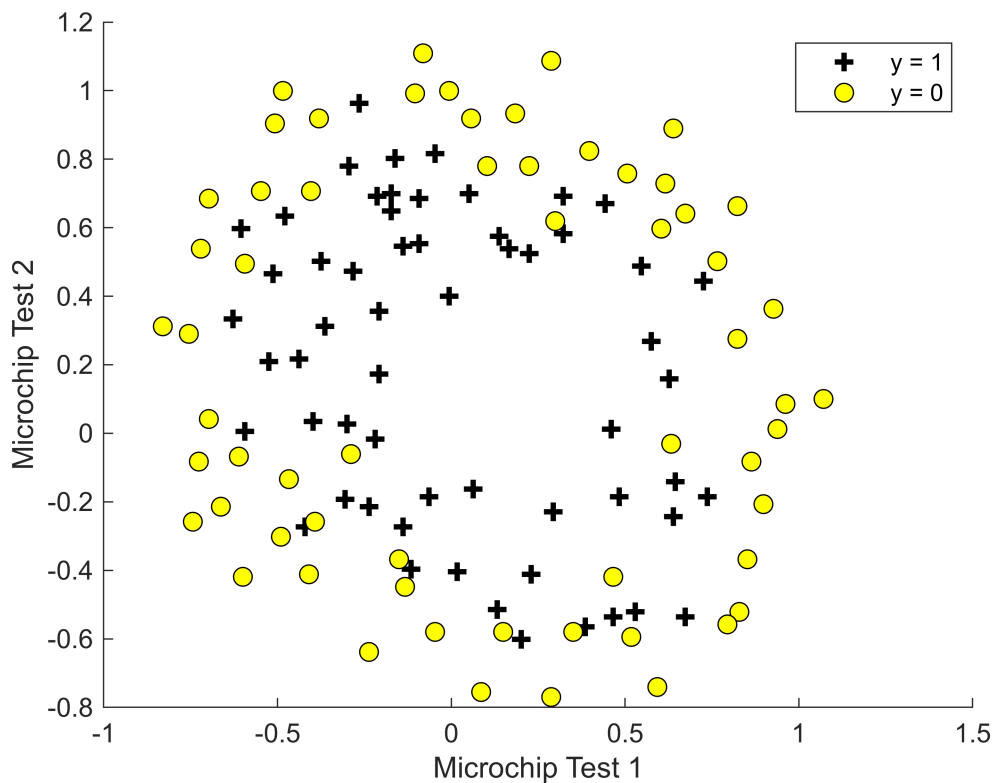
plotData(X, y);

% Put some labels
hold on;

% Labels and Legend
xlabel('Microchip Test 1')
ylabel('Microchip Test 2')

% Specified in plot order

legend('y = 1', 'y = 0')
hold off;
```



```
%% ===== Part 1: Regularized Logistic Regression =====
% In this part, you are given a dataset with data points that are not
% linearly separable. However, you would still like to use logistic
% regression to classify the data points.
%
% To do so, you introduce more features to use -- in particular, you add
% polynomial features to our data matrix (similar to polynomial
% regression).
%
% Add Polynomial Features

% Note that mapFeature also adds a column of ones for us, so the intercept
% term is handled
X = mapFeature(X(:,1), X(:,2));

% Initialize fitting parameters
initial_theta = zeros(size(X, 2), 1);

% Set regularization parameter lambda to 1
lambda = 1;

% Compute and display initial cost and gradient for regularized logistic
% regression
```

```
[cost, grad] = costFunctionReg(initial_theta, X, y, lambda);
```

```
fprintf('Cost at initial theta (zeros): %f\n', cost);
```

Cost at initial theta (zeros): 0.693147

```
fprintf('Expected cost (approx): 0.693\n');
```

Expected cost (approx): 0.693

```
fprintf('Gradient at initial theta (zeros) - first five values only:\n');
```

Gradient at initial theta (zeros) - first five values only:

```
fprintf(' %f \n', grad(1:5));
```

0.008475  
0.018788  
0.000078  
0.050345  
0.011501

```
fprintf('Expected gradients (approx) - first five values only:\n');
```

Expected gradients (approx) - first five values only:

```
fprintf(' 0.0085\n 0.0188\n 0.0001\n 0.0503\n 0.0115\n');
```

0.0085  
0.0188  
0.0001  
0.0503  
0.0115

```
fprintf('\nProgram paused. Press enter to continue.\n');
```

Program paused. Press enter to continue.

```
pause;
```

```
% Compute and display cost and gradient
```

```
% with all-ones theta and lambda = 10
```

```
test_theta = ones(size(X,2),1);
```

```
[cost, grad] = costFunctionReg(test_theta, X, y, 10);
```

```
fprintf('\nCost at test theta (with lambda = 10): %f\n', cost);
```

Cost at test theta (with lambda = 10): 3.206882

```
fprintf('Expected cost (approx): 3.16\n');
```

Expected cost (approx): 3.16

```
fprintf('Gradient at test theta - first five values only:\n');
```

Gradient at test theta - first five values only:

```
fprintf(' %f \n', grad(1:5));
```

```
0.430791
0.161352
0.194796
0.226863
0.092186
```

```
fprintf('Expected gradients (approx) - first five values only:\n');
```

Expected gradients (approx) - first five values only:

```
fprintf(' 0.3460\n 0.1614\n 0.1948\n 0.2269\n 0.0922\n');
```

```
0.3460
0.1614
0.1948
0.2269
0.0922
```

```
fprintf('\nProgram paused. Press enter to continue.\n');
```

Program paused. Press enter to continue.

```
pause;
```

```
%% ===== Part 2: Regularization and Accuracies =====
```

```
% Optional Exercise:
```

```
% In this part, you will get to try different values of lambda and
```

```
% see how regularization affects the decision boundart
```

```
%
```

```
% Try the following values of lambda (0, 1, 10, 100).
```

```
%
```

```
% How does the decision boundary change when you vary lambda? How does
```

```
% the training set accuracy vary?
```

```
%
```

```
% Initialize fitting parameters
```

```
initial_theta = zeros(size(X, 2), 1);
```

```
% Set regularization parameter lambda to 1 (you should vary this)
```

```
lambda = 1;
```

```
% Set Options
```

```
options = optimset('GradObj', 'on', 'MaxIter', 400);
```

```
% Optimize
```

```
[theta, J, exit_flag] = ...
```

```
    fminunc(@(t)(costFunctionReg(t, X, y, lambda)), initial_theta, options);
```

Local minimum found.

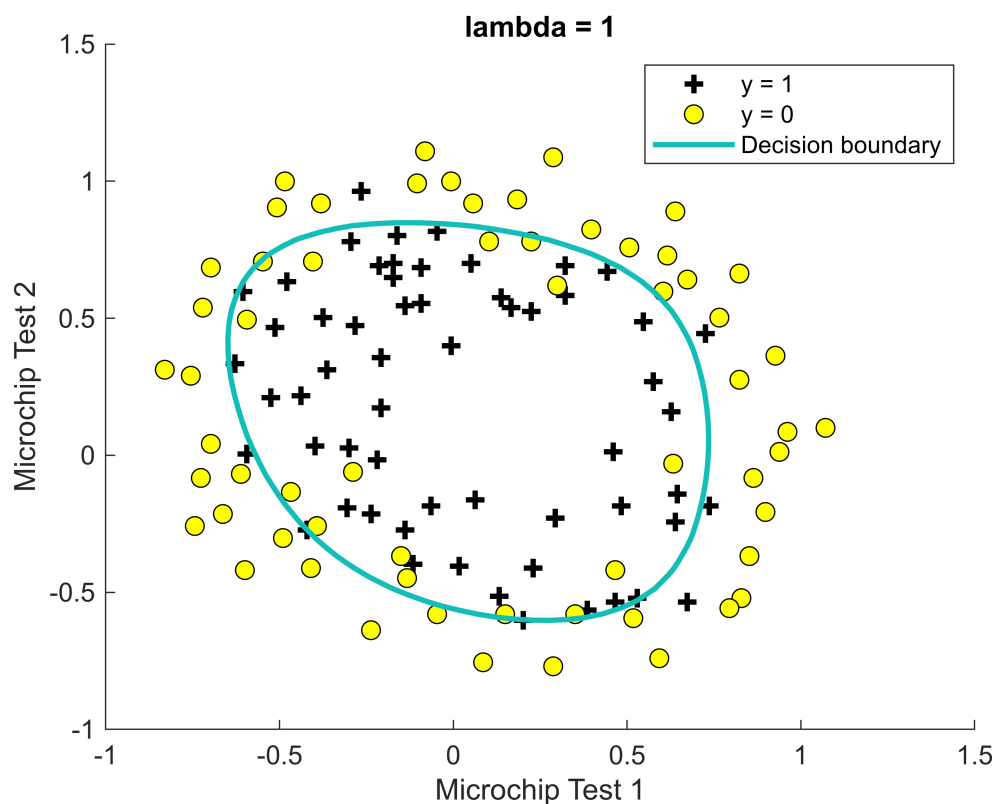
Optimization completed because the size of the gradient is less than the value of the optimality tolerance.

<stopping criteria details>

```
% Plot Boundary
plotDecisionBoundary(theta, X, y);
hold on;
title(sprintf('lambda = %g', lambda))

% Labels and Legend
xlabel('Microchip Test 1')
ylabel('Microchip Test 2')

legend('y = 1', 'y = 0', 'Decision boundary')
hold off;
```



```
% Compute accuracy on our training set
p = predict(theta, X);

fprintf('Train Accuracy: %f\n', mean(double(p == y)) * 100);
```

Train Accuracy: 82.203390

```
fprintf('Expected accuracy (with lambda = 1): 83.1 (approx)\n');
```

Expected accuracy (with lambda = 1): 83.1 (approx)

```

function plotData(X, y)
%PLOTDATA Plots the data points X and y into a new figure
% PLOTDATA(x,y) plots the data points with + for the positive examples
% and o for the negative examples. X is assumed to be a Mx2 matrix.

% Create New Figure
figure; hold on;

% ===== YOUR CODE HERE =====
% Instructions: Plot the positive and negative examples on a
%               2D plot, using the option 'k+' for the positive
%               examples and 'ko' for the negative examples.
%
% Find Indices of Positive and Negative Examples
pos = find(y);
neg = find(y==0);

% Plot Examples
plot(X(pos, 1), X(pos, 2), 'k+', 'LineWidth', 2, ...
     'MarkerSize', 7);
plot(X(neg, 1), X(neg, 2), 'ko', 'MarkerFaceColor', 'y', ...
     'MarkerSize', 7);

% =====

hold off;

end

```

sigmoid/logistic function  $g(z) = \frac{1}{1 + e^{-z}}$ .

```

function g = sigmoid(z)
%SIGMOID Compute sigmoid function
% g = SIGMOID(z) computes the sigmoid of z.

% You need to return the following variables correctly
g = zeros(size(z));

```

```
% ===== YOUR CODE HERE =====
% Instructions: Compute the sigmoid of each value of z (z can be a matrix,
%               vector or scalar).

g = 1./(1+exp(-z));

% =====

end
```

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m [-y^{(i)} \log(h_{\theta}(x^{(i)})) - (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)}))] + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

$$\frac{\partial J(\theta)}{\partial \theta_0} = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)} \quad \text{for } j = 0$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \left( \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)} \right) + \frac{\lambda}{m} \theta_j \quad \text{for } j \geq 1$$

```
function [J, grad] = costFunctionReg(theta, X, y, lambda)
%COSTFUNCTIONREG Compute cost and gradient for logistic regression with
regularization
% J = COSTFUNCTIONREG(theta, X, y, lambda) computes the cost of using
% theta as the parameter for regularized logistic regression and the
% gradient of the cost w.r.t. to the parameters.

% Initialize some useful values
m = length(y); % number of training examples

% You need to return the following variables correctly
J = 0;
grad = zeros(size(theta));

% ===== YOUR CODE HERE =====
% Instructions: Compute the cost of a particular choice of theta.
%               You should set J to the cost.
%               Compute the partial derivatives and set grad to the partial
%               derivatives of the cost w.r.t. each parameter in theta

h = sigmoid(X*theta);
J = 1/m * (-y'*log(h)-(1-y)'*log(1-h)) + lambda/(2*m)*(theta'*theta);
for j = 1:size(theta)
    grad(j) = 1/m*(h-y)'*X(:, j) + lambda/m*theta(j);
```

```
end
```

```
% =====
```

```
end
```

$h_{\theta}(x) \geq 0.5$ , predict 1, otherwise, predict 0

```
function p = predict(theta, X)
%PREDICT Predict whether the label is 0 or 1 using learned logistic
%regression parameters theta

m = size(X, 1); % Number of training examples

% You need to return the following variables correctly
p = zeros(m, 1);

% ===== YOUR CODE HERE =====
% Instructions: Complete the following code to make predictions using
%               your learned logistic regression parameters.
%               You should set p to a vector of 0's and 1's
%

h_theta = sigmoid(X*theta);

p(h_theta >= 0.5) = 1;

% =====

end
```

Appendix [DO NOT CHANGE THESE ]

```
function plotDecisionBoundary(theta, X, y)
%PLOTDECISIONBOUNDARY Plots the data points X and y into a new figure with
%the decision boundary defined by theta
% PLOTDECISIONBOUNDARY(theta, X,y) plots the data points with + for the
% positive examples and o for the negative examples. X is assumed to be
% a either
% 1) Mx3 matrix, where the first column is an all-ones column for the
%    intercept.
% 2) MxN, N>3 matrix, where the first column is all-ones
```



```

% Plot Data
plotData(X(:,2:3), y);
hold on

if size(X, 2) <= 3
    % Only need 2 points to define a line, so choose two endpoints
    plot_x = [min(X(:,2))-2, max(X(:,2))+2];

    % Calculate the decision boundary line
    plot_y = (-1./theta(3)).*(theta(2).*plot_x + theta(1));

    % Plot, and adjust axes for better viewing
    plot(plot_x, plot_y)

    % Legend, specific for the exercise
    legend('Admitted', 'Not admitted', 'Decision Boundary')
    axis([30, 100, 30, 100])
else
    % Here is the grid range
    u = linspace(-1, 1.5, 50);
    v = linspace(-1, 1.5, 50);

    z = zeros(length(u), length(v));
    % Evaluate z = theta*x over the grid
    for i = 1:length(u)
        for j = 1:length(v)
            z(i,j) = mapFeature(u(i), v(j))*theta;
        end
    end
    z = z'; % important to transpose z before calling contour

    % Plot z = 0
    % Notice you need to specify the range [0, 0]
    contour(u, v, z, [0, 0], 'LineWidth', 2)
end
hold off

end

function out = mapFeature(X1, X2)
% MAPFEATURE Feature mapping function to polynomial features
%
% MAPFEATURE(X1, X2) maps the two input features
% to quadratic features used in the regularization exercise.
%
% Returns a new feature array with more features, comprising of
% X1, X2, X1.^2, X2.^2, X1*X2, X1*X2.^2, etc..
%

```

```
% Inputs X1, X2 must be the same size
%

degree = 6;
out = ones(size(X1(:,1)));
for i = 1:degree
    for j = 0:i
        out(:, end+1) = (X1.^(i-j)).*(X2.^j);
    end
end

end
```