

Quantitative Proficiency Test

Question 1

Consider the following partial differential equation:

$$2 \frac{\delta^2 u}{\delta x^2} + \frac{1}{2} \frac{\delta^2 u}{\delta y^2} = u$$

where $u = u(x, y)$ is the unknown function. Define the following functions:

$$u_1(x, y) = xy^2, u_2(x, y) = \sin(xy), \text{ and } u_3(x, y) = e^{\frac{1}{2}(x-2y)}$$

Which of these functions are solutions to the partial differential equation?

- Only u_3
- None of the functions
- u_2 and u_3
- All the functions

 Time left:
42:46

QUESTIONS

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50
51	52	53	54	55
56	57	58	59	60

Question 2

Which of the following is the general solution for the differential equation:

$$y'' + y' - 6y = 0$$

where A, B, and C are arbitrary constants?

- $Ae^{2x} + Bxe^{-3x} = y$
- $Ae^{2x} + Be^{-3x} = y$
- $Ax^2 + Bx - 6C = y$
- $Ae^{-3x} + Bx + 6C = y$

Question 3

What is the inverse of:

$$\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

- $\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$
- $\begin{bmatrix} 5 & 2 \\ 2 & -1 \end{bmatrix}$
- doesn't exist

Question 4

Which Python data structure is constructed using the code below?

```
a = (1, 2, 3)
```

- List
- Dictionary
- Tuple
- Set

Question 5

For each $n = 1, 2, 3, \dots$, define

$$f(X) = \frac{n^2 x^3}{n}$$

$f_n(x) = \frac{1}{1+2n^2x^2}$

Then the function f defined by

$$f(x) = \lim_{n \rightarrow \infty} f_n(x)$$

exists for each $x \in \mathbb{R}$ and is equal to:

- $f(x) = x/2$
- $f(x) = x^2$
- $f(x) = x$
- $f(x) = 0$



Question 6

What is the probability of getting the sum of two 6 sided dice as a prime number?

- $\frac{15}{36}$
- $\frac{1}{2}$
- $\frac{1}{6}$
- $\frac{14}{36}$



Question 7

Find a particular solution to the differential equation:

$$y' = 3x^2 + 2x + 1 \text{ at } (2, 3)$$

- $y = x^3 + x^2 + x + 3$
- $y = x^3 + x^2 + x - 11$
- $y = x^3 + x^2 + x + 0$
- $y = x^3 + x^2 + x - 15$



Question 8

In how many ways can you rearrange the letters in the word "QUANT"?

- 6
- 120
- 1
- 24



Question 9

What is the output of this program?

```
dict1 = {"key": "A", "key2": 2}
dict2 = {"key2": 2, "key": "A"}
print(dict1 == dict2)
```

- False
- The program gives an error
- True True
- True



Question 10

What is the output of this program?

```
set1 = {'stock', 'bond', 'option'}
print (set[-1])
```

- 'stock', 'bond', 'option'

- 'stock', 'bond'
 'option'
 The program gives an error



Question 11 Which of the following is the definition of a mode.

- The value which appears most often in a sample set.
 The central number in an ordered sample set.
 None of these.
 The average value of a sample set.



Question 12 If $f : (-\pi/6, \pi/6) \rightarrow \mathbb{R}$ is defined by
 $f(x) = 2^x \tan(3x)$ for every $x \in (-\pi/6, \pi/6)$,
then $f'(0)$ is:

- $\ln 2$
 2π
 3
 0



Question 13 How many combinations are there of choosing 2 items from 12?

- 660
 132
 66
 33



Question 14 Simplify:

$$\sin^2 x + \cos^2 x =$$

- 1
 None of these
 $\tan^2 x$
 $2 \sin(x) \cos(x)$



Question 15 If you have imported the math module using the following statement, how would you call the exponential function ($\exp(x)$)?

```
import math as mt
```

- $mt.\exp(1)$
 The exponential function has not been imported
 $math.\exp(1)$
 $\exp(1)$



Question 16 For what value of x are the following vectors NOT linearly independent?

$$\begin{bmatrix} x \\ -1 \end{bmatrix} \quad \begin{bmatrix} 12 \\ -1 \end{bmatrix}$$

[3] [-18]

- 0
- 2
- 2
- there is no such value



Question 17

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by
 $f((x, y)) = x^2 e^{-x-2y}$, for every $(x, y) \in \mathbb{R}^2$
The value of $f_x((0, 0)) + f_y((0, 0)) + f_{yy}((0, 0))$ is:

- 0
- 2
- Undefined
- 2



Question 18

What is the probability of drawing 4 cards of the same suit from a 52 card deck?

- $(\frac{12}{51})(\frac{11}{50})(\frac{10}{49})$
- $(\frac{3}{4})(\frac{3}{4})(\frac{3}{4})(\frac{3}{4})$
- $(\frac{1}{4})(\frac{1}{4})(\frac{1}{4})(\frac{1}{4})$
- $(\frac{13}{52})(\frac{12}{51})(\frac{11}{50})(\frac{10}{49})$



Question 19

Which of the following integrals cannot be evaluated?

- $\int_{-1}^1 \frac{x+1}{x-1} dx$
- $\int_0^2 \frac{x^2+2x-3}{x-1} dx$
- $\int_{-1}^1 \sin^{-1}(x) dx$
- $\int_{-\pi/2}^{\pi/2} \tan(x) dx$



Question 20

Which of the following is not a feature of Python?

- Object oriented
- Readable
- Interpreted
- Statically typed



Question 21

Which of the following numbers does this program not output?

```
x = 3
while x < 11:
    if x%2==0:
        x= x-1
    else:
        x=x*2
    print(x)
```

- 9
- 18
- 6



○ 4



Question 22

Which of the following functions is continuous on all x ?

- $y = \ln(x)$
- $y = \tan(x)$
- $y = |x|$
- $y = 1/x$

Question 23

Factor:

$$2x^3 - 10x^2 - 28x$$



- $x(2x - 4)(x + 7)$
- $2x(x + 2)(x - 7)$
- $(x^3 - 2)(x^2 - 10)(x - 28)$
- $(2x^2 + 4)(x - 7)$

Question 24

The value of the limit

$$\lim_{x \rightarrow \infty} \frac{2-x^3}{x^2+3x^3}$$

is:



- ∞
- 1
- 0
- $-1/3$

Question 25

V_2 is an eigenvector of A. Which of these statements is true?

- $A - \lambda_2 = 0$
- $A - V_2 = 0$
- $A - \lambda_2 V_2 = 0$
- $A + V_2 = 0$



Question 26

What is the output of this line of code:

```
2**3-3**2
```

- The program gives an error
- 0
- 25
- 1

Question 27

Given the function:

$$f(x, y) = x - y$$

with the constraint that $e^x = y$, at which point does the maximum of $f(x, y)$ occur?



- $(\infty, 0)$
- $(1, e)$
- $(0, 1)$
- $(-1, 1/e)$



Question 28

Out of the following copying statements, which are deep copies for a list?

- 1.** `y = x`
- 2.** `y = x[:]`
- 3.** `y = copy.deepcopy(x)`



- Statements 2 and 3
- All of the statements
- Statement 3
- Statements 1 and 3



Question 29

Given:

```
import pandas as pd
import numpy as np
data = {'Ticker': ['IBM', 'AAPL'], 'Price'
```

which of the following creates a data frame?

- `df = (np.DataFrame) data`
- `df = (pd.DataFrame) data`
- `df = pd.DataFrame(data)`
- `df = np.DataFrame(data)`



Question 30

The middle value of a data set equals the:

- Median
- Mean
- All of these
- Mode



Question 31

The Pearson correlation coefficient between 2 variables is:

- Close to 1 if they tend to move in the same direction.
- All of these
- Close to 0 if they move independently of each other.
- Close to -1 if they tend to move in the opposite direction.



Question 32

$$1 + x + x^2 + x^3 + \dots =$$

- $\frac{1}{1-x^2}$
- $\frac{1}{1+x^2}$
- $\frac{1}{1-x}$
- $\frac{1}{1+x}$

Question 33

Which of the statements below is the derivative of $f(x)$ at $x = a$?

- $\lim_{x \rightarrow 0} \frac{f(x) - f(a)}{x - a}$
- $\lim_{x \rightarrow a} \frac{f(x) + f(a)}{x - a}$
- $\lim_{x \rightarrow 0} \frac{f(x) + f(a)}{x - a}$
- $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

Question 34

What does the following Python code return?

```
def func(x,y):
    if (x == 0):
        return y
    else:
        return func(x-1, x-y)

func(4,7)
```

- 5
- 4
- 5
- 4

Question 35

What is the sum of:

$$\begin{bmatrix} 5 \\ 0 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ 6 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 9 \\ 4 \end{bmatrix}$$

- 1054
- $\begin{bmatrix} 561 \\ 69 \\ 424 \end{bmatrix}$
- $\begin{bmatrix} 12 \\ 15 \\ 10 \end{bmatrix}$
- $\begin{bmatrix} 30 \\ 0 \\ 32 \end{bmatrix}$

Question 36

Python can be run:

- Using an editor such as IDLE
- All of these
- At a command line
- In a Jupyter Notebook

Question 37

Which of the following is the inverse function of:

$$\sin(x^3 + 1)$$

- $\sin^{-1}(3\sqrt[3]{x} - 1)$
- $3\sqrt[3]{\sin^{-1}(x)} - 1$
- $3\sqrt[4]{\sin^{-1}(x) - 1}$

$\sin^{-1}(3\sqrt{x-1})$

Question 38 What is the Taylor series for $f(x) = 10x^2 - 5x + 11$ around $x = 1$?

- $16 + 15(x-1) + 20(x-1)^2$
- $16 + 11(x-1) + 20(x-1)^2$
- $16 + 11(x-1) + 10(x-1)^2$
- $16 + 15(x-1) + 10(x-1)^2$



Question 39 Which of the following statements is true of a sample?

- A sample contains all the members of a specified group.
- A population is a subset of a sample.
- A measurable quality of a sample is a statistic.
- Reports of a sample are true without a margin of error.



Question 40 Let $x_0 \in \mathbb{R}$ be the value of $x \in \mathbb{R}$ that maximizes the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

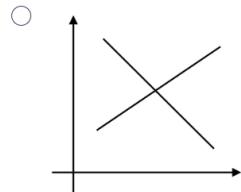
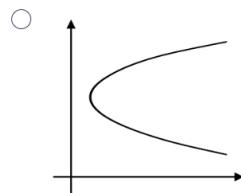
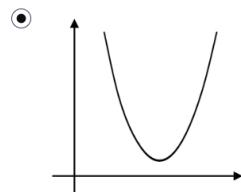
$f(x) = e^{-(x^2+1)}$ for every $x \in \mathbb{R}$
and $y_0 = f(x_0)$ be the value of this maximum.

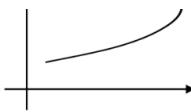
Then $x_0 + y_0$ is:

- 1
- $2e^{-1}$
- 2
- e^{-1}



Question 41 Which of these is a function:





Question 42

Find the solution to the following differential equation:

$$y'' + 2y' + y = 0, \quad y(0) = 2, \quad y'(0) = 10$$

- $y(t) = e^{-t}$
- $y(t) = 2e^{-t}$
- $y(t) = 2$
- $y(t) = 2e^{-t} + 12te^{-t}$



Question 43

The value of the integral

$$\int_0^{\sqrt{\pi/2}} x \cos(x^2) dx$$

is:

- $\sqrt{\pi}$
- $\sqrt{\pi} \cos \pi$
- $\frac{1}{2\sqrt{2}}$
- $\frac{1}{2}$



Question 44

What is the determinant of:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 6 & 5 \\ 9 & 7 & 8 \end{bmatrix}$$

- +39
- +91
- 39
- 91



Question 45

If a student has a z-score of 0.5 on a test where the average was 60 and the standard deviation was 10, what was the student's test score?

- 65
- 55
- 70.5
- 60

Question 46

For the equation:

$$xy^2 - 2xy + x^2y = 12$$

what is the slope of the tangent line at $(x, y) = (1, 4)$

- $-22/11$
- $-16/11$
- $-16/7$
- $-22/7$



Question 47

Consider the following statements concerning a positive integer n :

1. if n is a multiple of 4, then n^2 is a multiple of 2
2. if n^2 is a multiple of 5, then n is a multiple of 5
3. if n^2 is a multiple of 10, then n is a multiple of 5

Which of the statements is true?

- Statements 1 and 3
- None of the statements
- Statements 2 and 3
- All of the statements



Question 48

You flip a coin. If you get heads you win \$5, if you get tails you lose \$3. What is the expected payoff of one flip?

- 1
- 1
- 2
- 2



Question 49

The Central Limit Theorem states that the distribution of:

- The sum of a large number of independent, identically distributed variables will be approximately normal
- The product of a large number of independent identically distributed variables will be approximately normal
- The sum of a large number of dependent, identically distributed variables will be approximately normal
- The product of a large number of dependent, identically distributed variables will be approximately normal



Question 50

Find x :

$$4^{\log_4(x/10)} = 16$$

- 40
- 1.6
- 120
- 160



Question 51

What is the output of this program?

```
es = [2, 3, 5, 7, 11, 13, 17, 19]
[i for i in primes[1:]] + [j for j in primes
t(x)]
```

- The program gives an error
- [2, 3, 5, 7, 11, 13, 17, 19]
- [3, 5, 7, 11, 13, 17, 19, 2, 3, 7, 11, 13, 17]
- [5, 8, 12, 18, 24, 30, 36]



Question 52

Express as summation:

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64}$$

- $\sum_{n=0}^6 (-1)^n 2^{-n}$
- $\sum_{n=1}^6 (-1)^n 2^{-n}$
- $\sum_{n=0}^6 (-1)^n 2^n$
- $\sum_{n=1}^6 (-1)^n 2^n$



Question 53

Let $f : \mathfrak{R} \rightarrow \mathfrak{R}$ and $g : \mathfrak{R} \rightarrow [0, \infty)$ be defined by

$$f(x) = g(x) = x^2 \text{ for every } x \in \mathfrak{R}$$

1. f is one-to-one
2. f is onto
3. g is one-to-one
4. g is onto

Which of the statements are true?

- Statements 2 and 4
- Statement 4
- All of the statements
- None of the statements



Question 54

What is the probability of getting a prime number when rolling a 6 sided die?

- Cannot be determined
- $\frac{4}{6}$
- $\frac{2}{6}$
- $\frac{3}{6}$



Question 55

Evaluate:

$$\int \frac{1}{2(x-1)^2} dx$$

- $-\frac{1}{x} + C$
- $\frac{1}{2(x-1)} + C$
- $-\frac{1}{2(x-1)} + C$
- $\frac{1}{2x} + C$



Question 56

A type I error is:

- Rejecting the null hypothesis when it is true.
- Rejecting the alternative hypothesis when it is false.
- Rejecting the alternative hypothesis when it is true.
- Rejecting the null hypothesis when it is false.



Question 57

What is the output of the following program?

```
print("Hello World"[:-1])
```

- diroW olleH
- d
- Hello world

-  Help now
 The program gives an error

Question 58

If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 3 & 2 & 1 \end{bmatrix}$$

rank A=

- 2
 1
 4
 3



Question 59

If $P(A) = 0.4$, $P(A|B) = 0.2$, and $P(B) = 0.6$, then find:

$$P(B|A)$$

- 0.6
 0.2
 0.3
 0.4



Question 60

If $f(x)$ is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)dt$ then which of the following is equivalent to:

$$\int_a^b f(x)dx$$

- $\frac{F(b)-F(a)}{b-a}$
 $\frac{d}{dx} F(b) - \frac{d}{dx} F(a)$
 $F(b) - F(a)$
 $b - a$



SUBMIT

