4442 Assignment 2
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Question 1

(a)

p(Water = cool|Play = yes): 1/3
p(Water = cool|Play = no): 0

(b)

p(Play = yes|Water = warm): 2/3
p(Play = no|Water = warm): 1/3

(c)

p(Play = yes|Humid = high): 2/3
p(Play = yes|Humid = normal): 1

(d) *With Laplace Smoothing*

p(Water = cool|Play = yes): 2/5
p(Water = cool|Play = no): 1/3

Question 2

(a)

$k(x,z) = k_1(x,z)k_2(x,z)$ is a valid kernel because it is symmetric and positive semidefinite.

$$Proof\ that\ this\ kernel\ is\ symmetric:$$

$$\begin{aligned} k(x,z) &= k_1(x,z)k_2(x,z) \\ &= k_1(z,x)k_2(z,x) \\ &= k(z,x) \end{aligned}$$

$$Proof\ that\ \alpha^T K\alpha \geq 0\ (positive\ semidefinite):$$

$$\begin{aligned} \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j K(x_i, x_j) &= \sum_{ij=1}^{n} a_i a_j \left(\sum_{k=1}^{n} x_{ik}x_{jk}\right)\left(\sum_{m=1}^{n} x_{im}x_{jm}\right) \\ &= \sum_{km=1}^{n}\left(\sum_{i=1}^{n} a_i x_{ik}x_{im}\right)^2 \geq 0 \end{aligned}$$

(b)

$k(x,z) = f_1(x)f_1(z) + f_2(x)f_2(z)$ is a valid kernel because it is symmetric and positive semidefinite.

$$Proof\ that\ kernel\ is\ symmetric:$$

$$Let\ \phi(x) = [f_1(x), f_2(x)]^T and\ let\ \phi(z) = [f_1(z), f_2(z)]$$

$$Therefore,$$

$$\begin{aligned} k(x,z) &= [f_1(x) \quad f_2(x)] \begin{bmatrix} f_1(z) \\ f_2(z) \end{bmatrix} \\ &= f_1(x)f_1(z) + f_2(x)f_2(z) \end{aligned}$$

$$\begin{aligned} k(z,x) &= [f_1(z) \quad f_2(z)] \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} \\ &= f_1(z)f_1(x) + f_2(z)f_2(x) \\ &= f_1(x)f_1(z) + f_2(x)f_2(z) = k(x,z) \end{aligned}$$

$$Proof\ that\ \alpha^T K\alpha \ \geq 0\ (positive\ semidefinite):$$

$$\begin{aligned} \sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{n} a_i a_j K(x_i,x_j) &= \sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{n} a_i a_j\big(f(x_i)f(x_j) + \big(f(x_i)f(x_j)\big)\big) \\ &= \sum\nolimits_{i=1}^{n} a_i a_j\, f(x_i)f(x_j) + \sum\nolimits_{j=1}^{n} a_i a_j\, f(x_i)f(x_j)\ \geq 0 \end{aligned}$$

(c)

$k(x,z) = \frac{k_1(x,z)}{\sqrt{k_1(x,x)k_1(z,z)}}$ is a valid kernel because it is symmetric and positive semidefinite. This kernel is simply a normalized kernel.

$$Proof\ that\ kernel\ is\ symmetric:$$

$$\begin{gathered} Since \\ k_1(x,z) = \ k_1(z,x) \\ and\ k_1(x,x)k_1(z,z) = \ k_1(z,z)k_1(x,x), \\ the\ kernel\ is\ equivalent\ to: \end{gathered}$$

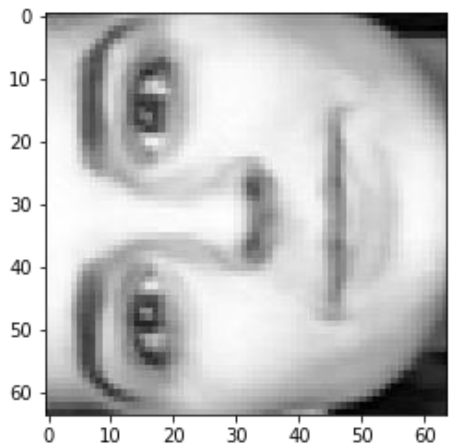
$$\frac{k_1(z,x)}{\sqrt{k_1(z,z)k_1(x,x)}} = k(z,x)$$

$$Proof\ that\ \alpha^T K\alpha \ \geq 0\ (positive\ semidefinite):$$

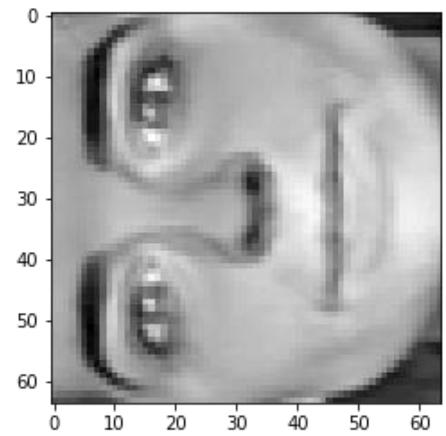
$$\begin{aligned} \sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{n} a_i a_j K(x_i,x_j) &= \sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{n} a_i a_j \left(\frac{x_i x_j}{\|x_i\|\|x_j\|}\right) \\ &= \sum\nolimits_{i=1}^{n} a_i \left(\frac{x_i}{\|x_i\|}\right) \geq 0 \end{aligned}$$

Question 3

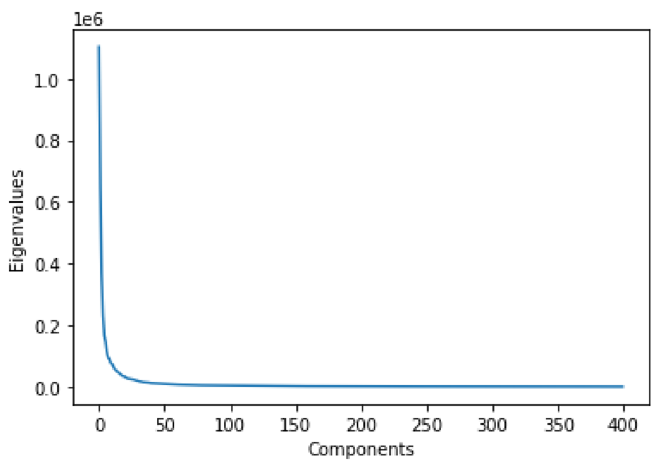
(a) *Display the 100th image:*



(b) *Display the 100th image without mean:*



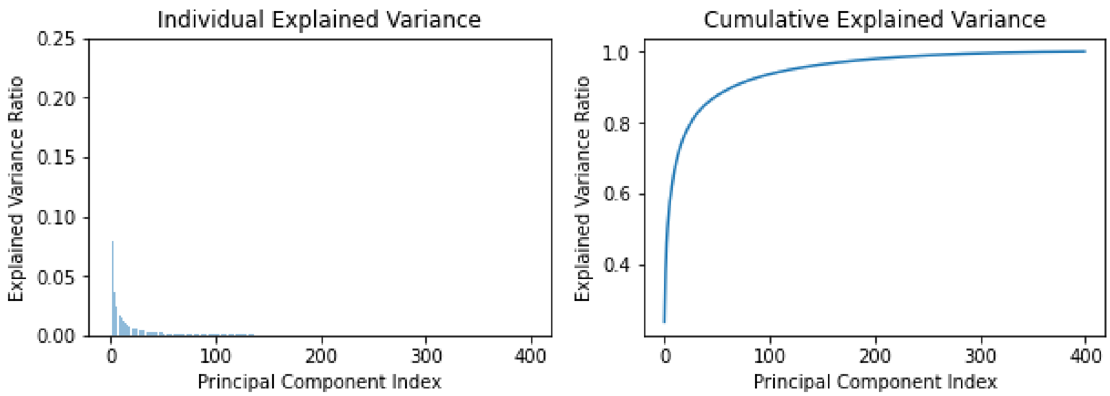
(c) *Plot the eigenvalues:*



(d)

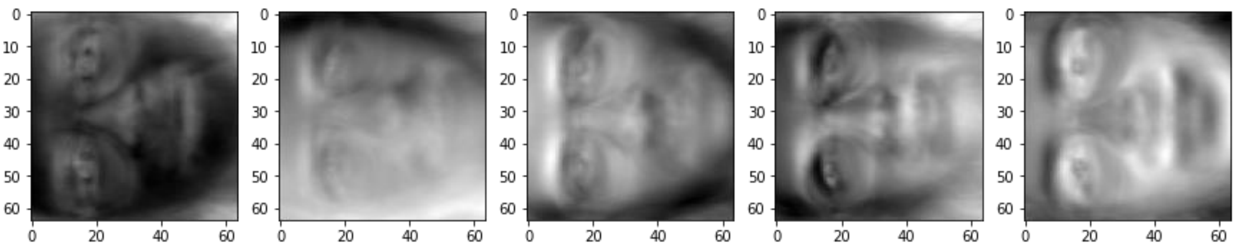
The final eigenvalue is 0 due to dimensionality reduction. This results in the final component having no more dimensions to remove, meaning that all variances have been accounted for and the only possible eigenvalue at this point is 0.

(e)



Looking at the cumulative explained variance graph, we can see that the first 150 components account for most of the variance (about 90%). Therefore, the dimensionality of data we want to keep is 150 because these are accountable for a sufficient amount of variance.

(f)



(g)