A Gallery of Examples for Additive Regular Functions - Working Notes

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Abstract. As part of our effort to build a library for cost register automata with focus on additive regular functions, we maintain the following list of examples.

1 Introduction

2 Preliminaries

2.1 Definitions

Cost register automata are defined in [2]. Our focus here is on *additive cost register automata.*

Definition 1 (Additive CRAs (ACRA)).
An ACRA *is a tuple* $\mathcal{A} = (\Sigma, Q, X, \delta, \mu, q_0, \eta_0, F, \nu)$ *where* Σ *is a finite non-empty set of input letters,* Q *is a finite non-empty set of states,* X *is a finite set of registers,* $\delta : Q \times \Sigma \to Q$ *is the state transition function,* $\mu : Q \times \Sigma \times X \to X \times \mathbb{Z}$ *is the register update function,* q_0 *is the initial state,* $\eta_0 : X \to \mathbb{Z}$ *is the initial registers value map,* $F \subseteq Q$ *is the set of final states, and* $\nu : F \to X \times \mathbb{Z}$ *is the output function.*

We often assume all registers start with initial value 0 and remove η_0 from the description of an ACRA. The configuration of $\mathcal{A}$ is a pair (q, η) where $q \in Q$ is the current state and $\eta : X \to \mathbb{Z}$ maps each register to its value. For a letter $\sigma \in \Sigma$, the σ-successor of a configuration (q, η) is the configuration (q', η') such that $\delta(q, \sigma) = q'$ and for each register $x \in X$ if $\mu(q, \sigma, x) = (y, c)$ then $\eta'(x) = \eta(y) + c$. The successor notation is extended from letters to words in the usual manner. We use $\Delta(q, \eta, w)$ to denote the w-successor of (q, η). The ACRA $\mathcal{A}$ implements a function $[\![\mathcal{A}]\!] : \Sigma^* \to \mathbb{Z}_\perp$ defined as follows. If $\Delta(q_0, \eta_0, w) = (q_f, \eta_f)$, $q_f \in F$ and $\nu(q_f) = (x, c)$ then $[\![\mathcal{A}]\!](w) = \eta_f(x) + c$. Otherwise $[\![\mathcal{A}]\!](w) = \perp$.

If the update function of register x depends only on x for every $x \in X$, we say that $\mathcal{A}$ is a *simple* ACRA (ASCRA). If $|X| = k$ we say that $\mathcal{A}$ is a k-ACRA (or k-ASCRA if it is also simple). We use k-$\mathbb{ACRA}$ and k-$\mathbb{ASCRA}$ to denote the class of functions $f : \Sigma^* \to \mathbb{Z}$ that can be implemented by a k-ACRA and k-ASCRA, resp. We use $\mathbb{ACRA}$ and $\mathbb{ASCRA}$ for the union of the classes k-$\mathbb{ACRA}$ and k-$\mathbb{ASCRA}$ for $k \in \mathbb{N}$, resp.

Known facts: [1]

- The class $\mathbb{ACRA}$ is equivalent to *unambiguous weighted automata*, and is therefore strictly sandwiched between *weighted automata* and *deterministic weighted automata* in expressiveness.
- Deterministic weighted automata are 1-$\mathbb{ACRA}$ and 1-$\mathbb{ASCRA}$.
- The class $\mathbb{ACRA}$ is as expressive as the class of functions implemented by cost register automata with binary addition (i.e. when registers can be added to each other as in $x = y+z+c$) conditioned the updates are copyless.

3 Examples

In the examples below we use the following convetions:

- the notation above the split line of a state is the state name,
- the notation below the split line of a state q specifies the output for that state, i.e. $\nu(q)$,
- all registers start with initial value 0, unless specified otherwise,
- if the update of a register x is not specified on an edge transition then the update is $x := x$.

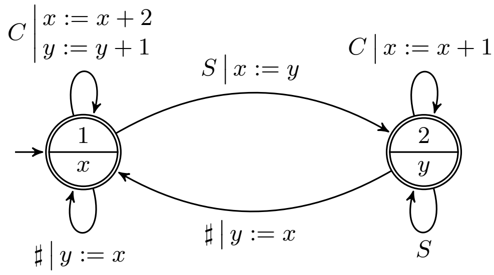

Fig. 1. An example of a 2-ACRA. Captures the story of a coffee house where a cup of coffee (denoted by letter C) costs \$2 but if you fill in a survey (denoted by letter S) you get a discount, and pay \$1 for every cup of coffee purchased in this month. The letter $\sharp$ indicates it is the end of the month.

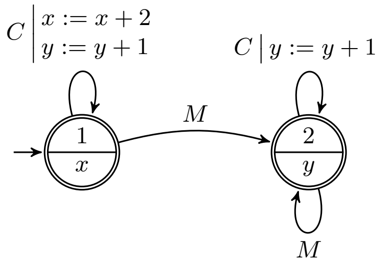

Fig. 2. An example of a 2-ASCRA. It captures the story of a coffee house where a cup of coffee (denoted by letter C) costs \$2 but if you apply for membership (denoted by letter M) you get a discount, and pay \$1 for every cup of coffee purchased.

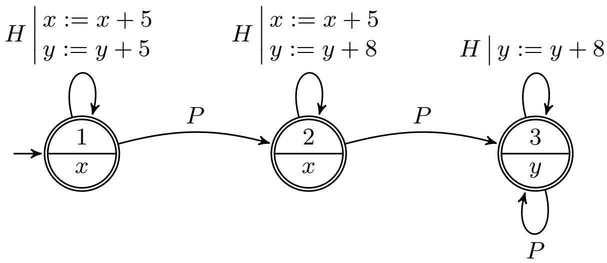

Fig. 3. An example of a 2-ASCRA. It captures the story of a business where an employee is payed \$5 an hour (denoted by letter H) but if he is promoted (denoted by letter P) twice then his salary per hour is \$8 , starting from the first promotion.

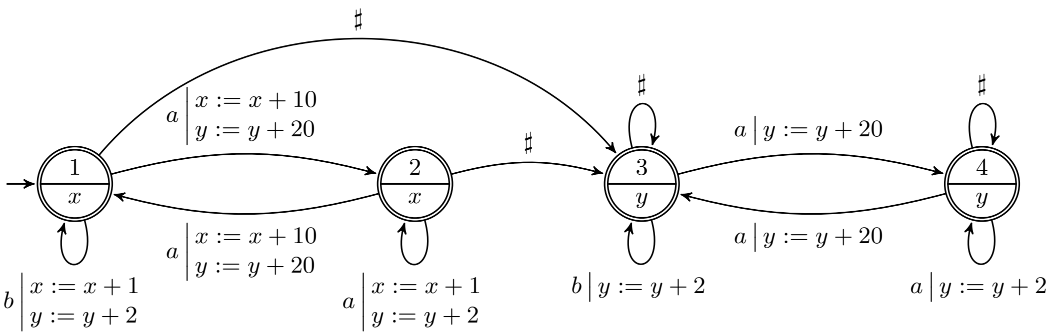

Fig. 4. An example of a 2-ASCRA. Roughly, for $\sigma \in \{a, b\}$ awards the first σ in a block of σ's with 20, and every consequent σ with 1 or 2 if no $\sharp$ was seen, otherwise consequent σ's are awarded 2.

References

1. R. Alur and M. Raghothaman. Decision problems for additive regular functions. pages 37–48, 2013.
2. Rajeev Alur, Loris D'Antoni, Jyotirmoy V. Deshmukh, Mukund Raghothaman, and Yifei Yuan. Regular functions and cost register automata. In *28th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2013, New Orleans, LA, USA, June 25-28, 2013*, pages 13–22, 2013.