Formula Sheet EE2M11

MaybE_Tree

2022-09-07

Principal Argument

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

Triangle Inequality

$$\begin{cases} |z_1 \pm z_2| \leq |z_1| + |z_2| \\ |z_1 \pm z_2| \geq |z_1| - |z_2| \end{cases}$$

Limits to Infinity

$$\begin{cases} \lim\limits_{z \to z_0} f(z) = \infty \iff \lim\limits_{z \to z_0} \dfrac{1}{f(z)} = 0 \\ \lim\limits_{z \to \infty} f(z) = L \iff \lim\limits_{z \to 0} f\left(\dfrac{1}{z}\right) = L \end{cases}$$

L must be finite, maybe??

Cauchy-Riemann

CR1	u	v
x	$\dfrac{du}{dx}$	$\dfrac{dv}{dx}$
y	$\dfrac{du}{dy}$	$\dfrac{dv}{dy}$

For $f(x,y) = u(x+y) + iv(x,y)$

Harmonic Check

$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0 \implies \frac{\iint\limits_D f(z)}{\operatorname{Area}(D)} = f(z_c)$$

For $f(x,y) = u(x+y) + iv(x,y)$ *Around a circular domain D with centerpointn* z_c

Exponential

$$e^z = e^x(\cos y + i \sin y)$$

Power

$$z^w = e^{w \log(z)}$$

Log

$$\operatorname{Log}(z) = \ln|z| + i \arg(z)$$

$-\pi < Arg(z) < \pi$

Trig Functions

$$\begin{cases} \sin z = \dfrac{1}{2i}(e^{iz} - e^{-iz}) \\ \cos z = \dfrac{1}{2}(e^{iz} + e^{-iz}) \\ \sinh z = \dfrac{1}{2}(e^z - z^{-z}) = -i \sin(iz) \\ \cosh z = \dfrac{1}{2}(e^z + e^{-z}) = \cos(iz) \end{cases}$$

Range of sin *and* cos *is not longer* ± 1 .

Zakarian Identities

$$\begin{cases} \sin z = \dfrac{1}{2i}(e^{iz} - e^{-iz}) = \\ \quad = \sin x \cosh y + i \cos x \sinh y \\ \cos z = \dfrac{1}{2}(e^{iz} + e^{-iz}) = \\ \quad = \cos x \cosh y - i \sin x \sinh y \end{cases}$$

Highschool Identities	$\begin{cases}\sin(z+w)=\sin z\cos w+\cos z\sin w\\ \cos(z+w)=\cos z\cos w-\sin z\sin w\end{cases}$	
Cauchy's First Formula	$2\pi i f(z_0)=\int\limits_C \frac{f(z)}{(z-z_0)}$	
Cauchy's Second Formula	$\frac{2\pi i}{n!}f^n(z_0)=\int\limits_C \frac{f(z)}{(z-z_0)^{n+1}}$	*For* $n=0,1,2,...$
Parama	$\int\limits_C f(z)dz=\int\limits_a^b f(z(t))z'(t)dt$	

	0	$1/6\pi$	$1/4\pi$	$1/3\pi$	$1/2\pi$
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
tan	0	$\sqrt{3}/3$	1	$\sqrt{3}$	???