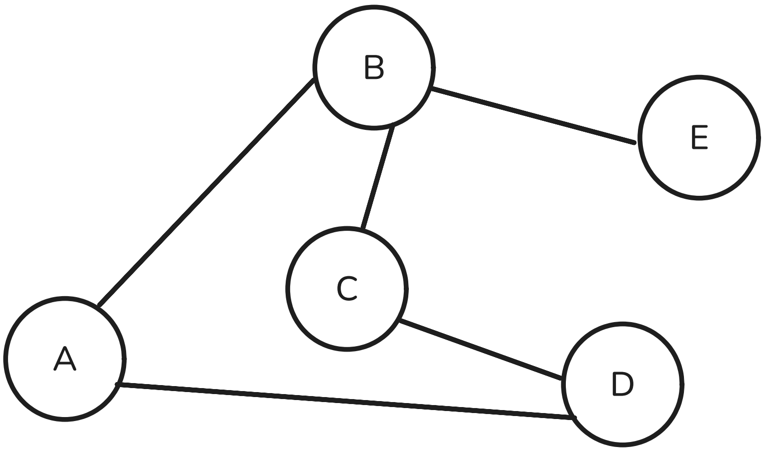
1. Representation of Networks

We can represent the same network in 4 ways:

1. **Graph Representation**



2. **Adjacency Matrix**

$$\begin{bmatrix} Node & A & B & C & D & E \\ \\ A & 0 & 1 & 0 & 1 & 0 \\ B & 1 & 0 & 1 & 0 & 1 \\ C & 0 & 1 & 0 & 1 & 0 \\ D & 1 & 0 & 1 & 0 & 0 \\ E & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

3. **Edge List**

$$(A, B)\ (A, D)\ (B, C)\ (B, E)\ (C, D)$$

4. **Adjacency List**

$$\begin{aligned} &A \rightarrow B, D \\ &B \rightarrow A, C, E \\ &C \rightarrow B, D \\ &D \rightarrow A, C \\ &E \rightarrow B \end{aligned}$$

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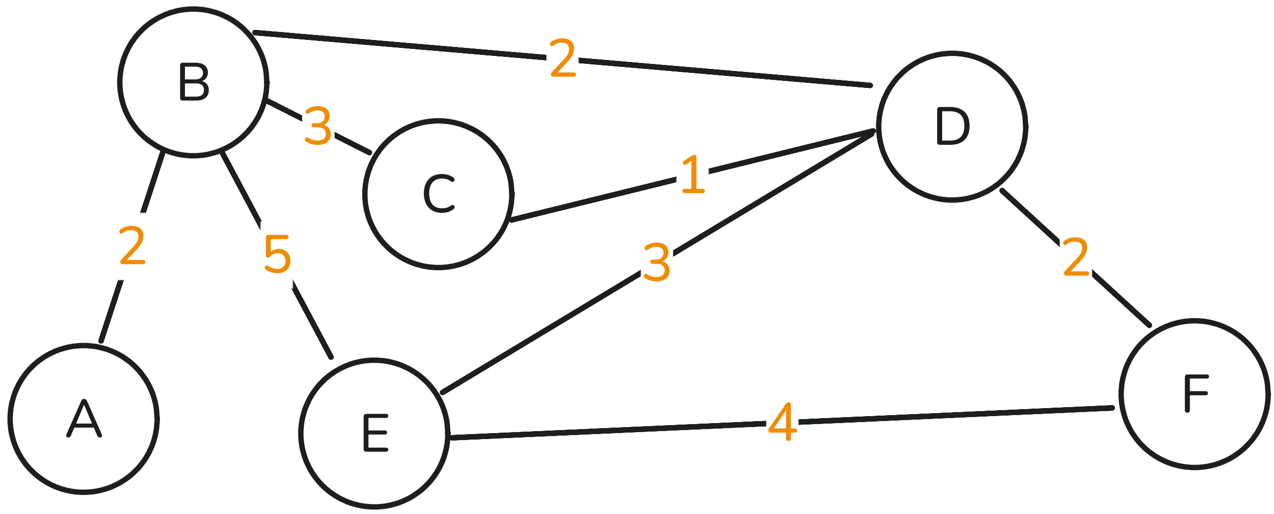
2. Degree and Strength Centrality

Find the "center-most" (central) node of this network:



We can create this table by seeing which node to connected to which other nodes.

Node	is connected to	Degree	Strength	$d \times (s/d)^{\alpha}$
A	B (2)	1	2	$1 * (2/1)^{0.5} = 1.414$
B	A (2), C (3), D (2), E (5)	4	2+3+2+5=12	$4 * (12/4)^{0.5} = 6.928$
C	B (3), D (1)	2	3+1=4	$2 * (4/2)^{0.5} = 2.828$
D	B (2), C (1), E (3), F (2)	4	2+1+3+2=8	$4 * (8/4)^{0.5} = 5.656$
E	B (5), D (3), F (4)	3	5+3+4=17	$3 * (17/3)^{0.5} = 7.098$
F	D (2), E (4)	2	2+4=6	$2 * (6/2)^{0.5} = 3.464$

Now, according to **Degree Centrality**, B and D are the center-most since they have the highest degree but, according to **Strength Centrality**, E is the center-most since it has the highest strength.

We can instead use Opsahl's Degree and Weight Centrality

- If we put $\alpha = 0$ our result completely depends on Degree.
- If we put $a = 1$ our result completely depends on Strength.
- For a neutral result we can put $\alpha = 0.5$.

In column 5, we got E having the highest value (7.098) so according to Opsahl's formula E is the center-most.

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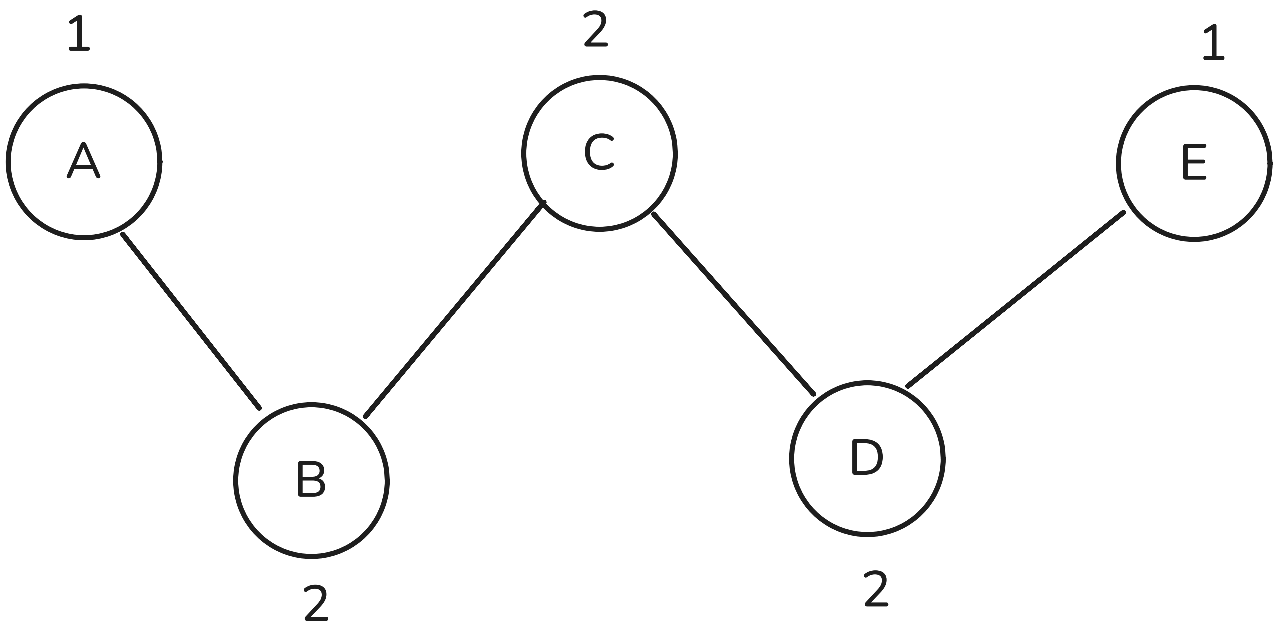
3. Betweenness and Closeness Centrality

Find the center-most node of this network:



If we use **Degree Centrality** for this network then it says that B, C, D are center-most... but we can visually tell that **C** is more central than B and D. Hence, we'll use Betweenness and Closeness Centralities instead:

Node	possible shortest paths	paths that go through	Betweenness
A	6 (BC, BD, BE, CD, CE, DE)	0	0
B	6 (AC, AD, AE, CD, CE, DE)	3 (AC, AD, AE)	$3/6$
C	6 (AB, AD, AE, BD, BE, DE)	4 (AD, AE, BD, BE)	$4/6$
D	6 (AB, AC, AE, BC, BE, CE)	3 (AE, BE, CE)	$3/6$
E	6 (AB, AC, AD, BC, BD, CD)	0	0

Node	distances to other nodes	sum	Closeness
A	AB (1), AC (2), AD (3), AE (4)	1+2+3+4 = 12	$1/12$
B	BA (1), BC (1), BD (2), BE (3)	1+1+2+3 = 10	$1/10$
C	CA (2), CB (2), CD (2), CE (2)	2+2+2+2 = 8	$1/8$
D	DA (3), DB (2), DC (1), DE (1)	3+2+1+1 = 10	$1/10$
E	EA (4), EB (3), EC (2), ED (1)	4+3+2+1 = 12	$1/12$

Now we just pick the node with the highest number which is **C**, by both Betweenness (4/6) and Closeness (1/8)

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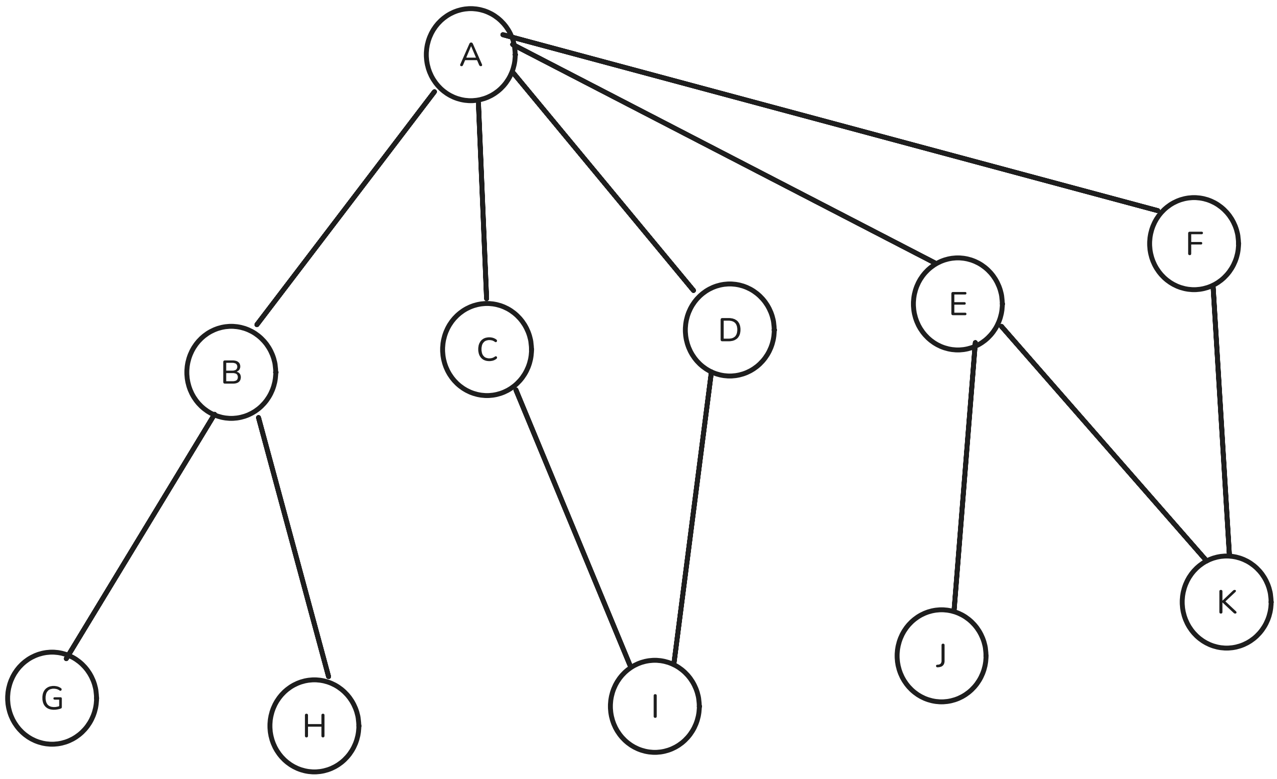
4. One-Mode and Two-Mode Degree

We have this graph of a network



This network can be divided into 2 disjoint (no-overlapping) sets of nodes:

- $\text{Set } 1 = \{A, G, H, I, J, K\}$
- $\text{Set } 2 = \{B, C, D, E, F\}$

Hence, it is a Bipartite Graph. We can easily find the **Two-Mode Degree** of all nodes by counting their neighbors but to find the **One-Mode Degree** we would have to count all the neighbors of neighbors of a node.

Node	neighbours	2-Mode Degree	neighbours of neighbours	1-Mode Degree
A	B, C, D, E, F	5	G, H, I, J, K	5
B	A, G, H	3	C, D, E, F	4
C	A, I	2	B, D, E, F	4
D	A, I	2	B, C, E, F	4
E	A, J, K	3	B, C, D, F	4
F	A, K	2	B, C, D, E	4
G	B	1	A, H	2
H	B	1	A, G	2
I	C, D	2	A	1
J	E	1	A, K	2

Node	neighbours	2-Mode Degree	neighbours of neighbours	1-Mode Degree
K	E, F	2	A, J	2

A has the highest 2-mode as well as 1-mode degree.

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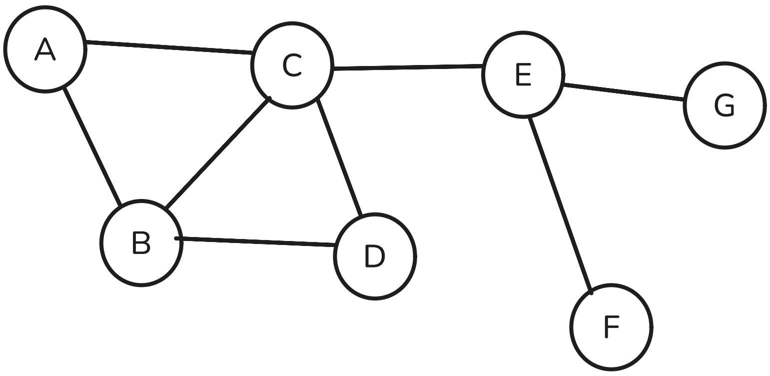
5. Global Clustering Co-efficient

We have this graph



We can calculate the **Global Clustering Co-Efficient** by dividing the total number of closed triplets (3-points that make a triangle) by the total number of open triplets (3-points that don't make a triangle) as well as closed.

- **Closed Triplets** = 6
    - Triangle 1: A→B←C, A→C←B, B→A←C
    - Triangle 2: B→C←D, B→D←C, C→B←D
- **Open Triplets** = 8
    - A→B←D, A→C←D, A→C←E
    - B→C←E, D→C←E
    - C→E←G, C→E←F
    - G→E←F

Global Clustering Co-Efficient = 6/14 = 0.42

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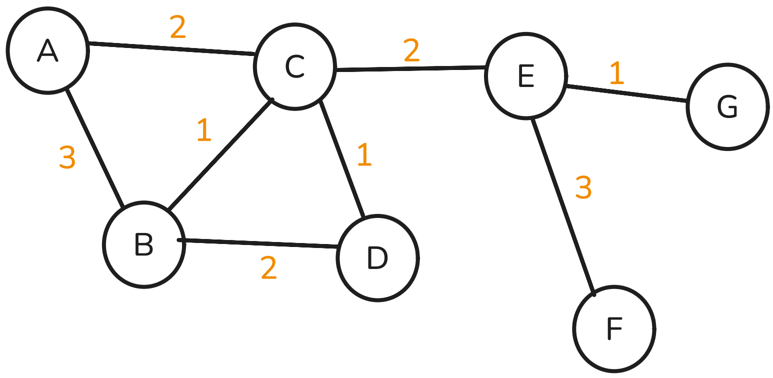
6. Weighted Global Clustering Co-efficient

Let's take the same network but this time the edges have weights.



There are 4 methods for this: Arithmetic Mean, Geometric Mean, Maximum and Minimum.

Triplet	Closed?	weights	A.M	G.M	Max	Min
A→B←C	Yes	3,1	2	1.7	3	1
A→C←B	Yes	2,1	1.5	1.4	2	1
B→A←C	Yes	3,2	2.5	2.4	3	2
B→C←D	Yes	1,1	1	1.0	1	1
B→D←C	Yes	2,1	1.5	1.4	2	1
C→B←D	Yes	1,2	1.5	1.4	2	1
A→B←D	No	3,2	2.5	2.4	3	2
A→C←D	No	2,1	1.5	1.4	2	1
A→C←E	No	2,2	2	2.0	2	2
B→C←E	No	1,2	1.5	1.4	2	1
D→C←E	No	1,2	1.5	1.4	2	1
C→E←G	No	2,1	1.5	1.4	2	1
C→E←F	No	2,3	2.5	2.4	3	2
G→E←F	No	1,3	2	1.7	3	1
Sum			**25**	**22.4**	**32**	**18**

Now sum all the closed triplets (for each method) and divide by the total.

$$\text{Arithmetic Mean} = \frac{2 + 1.5 + 2.5 + 1 + 1.5 + 1.5}{25} = \frac{10}{25} = 0.4$$

$$\text{Geometric Mean} = \frac{1.7 + 1.4 + 2.4 + 1.0 + 1.4 + 1.4}{22.4} = \frac{9.3}{22.4} = 0.415$$

$$\text{Maximum} = \frac{3 + 2 + 3 + 1 + 2 + 2}{32} = \frac{13}{32} = 0.406$$

$$\text{Minimum} = \frac{1 + 1 + 2 + 1 + 1 + 1}{18} = \frac{7}{18} = 0.389$$