Protecting Shorelines with Triply Periodic Minimal Surface (TPMS) Inspired Breakwaters

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Introduction

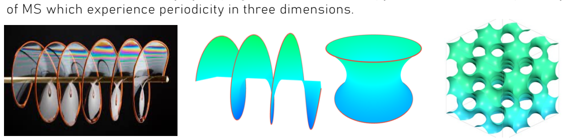
Breakwaters

Breakwaters are offshore structures that help to protect coastlines from coastal erosion. They have become increasingly important among coastal communities to mitigate effects of rising sea levels. Various breakwater designs such as vertical walls, lattice structures, and rubble-mound breakwaters help protect shorelines globally.

Breakwater at port of Granadilla, Spain

Triply Periodic Minimal Surface (TPMS)

Minimal surfaces (MS) are a family of mathematical surfaces which locally minimize surface area. They are characterized by having zero mean curvature and can be formed naturally by dipping wireframes into soapy water. TPMSs are special types of MS which experience periodicity in three dimensions.



Soap Film → Geometric Minimal Surface TPMS

Goal

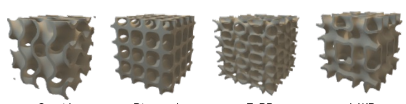
This study measures the effectiveness of TPMS-based breakwater designs by comparing them against traditional structures. The performance of different types of TPMSs with various design parameters, specifically those with graded density, are also investigated.

Background

Types of TPMSs

TPMSs are a large family of surfaces with varied symmetries. Some have been the focus of multiple studies concerning their capacity for heat dissipation, sound absorption and stress distribution with applications in heat sinks, sound isolation and artificial bones. TPMS architectures selected for this study include Gyroid, Diamond, F-RD, and I-WP.

TPMS Architectures

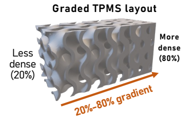


Gyroid Diamond F-RD I-WP

Graded TPMSs

A graded TPMS is a TPMS-based structure whose shape changes gradually in thickness according to the specified starting and ending cell densities.

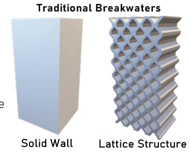
Graded TPMS layout



Traditional Breakwaters

Two types of traditional breakwaters, solid wall and lattice structure, are analyzed and compared with TPMS designs. The Lattice structure represents a traditional (non-TPMS) breakwater design, while the solid wall serves as a control group as well as a datum.

Traditional Breakwaters



Solid Wall Lattice Structure

Breakwater Design Metrics

Using the principles of linear wave theory[1], it can be shown that the mean energy flux per unit surface length of the wave (P) is proportional to the product of square of wave height (H) and wave phase velocity (c_p).

$$P = \frac{1}{8}\gamma H^2 c_g \quad \Rightarrow \quad P \propto H^2 c_g$$

$$c_g = n c_p$$

$$c_g \propto c_p \quad \Rightarrow \quad \boxed{P \propto H^2 c_p}$$

P = mean energy flux per unit surface length
γ = specific weight of water
H = wave height (double the amplitude)
c_g = wave group velocity
c_p = wave phase velocity (velocity of wave crest)
n = constant that depends on wavelength and mean water depth

Three metrics of the breakwater designs indicate wave dissipation efficiency: the relative wave height reduction ($\Delta H_{\%}$), the relative wave velocity decrease ($\Delta c_{\%}$), and the drag force against the structure. Drag can also be used as a measure of structural stress and therefore structural durability. Based on the above discussion, relative energy flux decrease ($\Delta P_{\%}$) can also be used as a unifying metric for $\Delta H_{\%}$ and $\Delta c_{\%}$ to measure dissipation efficiency.

Due to the proportionality relationship, $\Delta P_{\%}$ can be calculated using the equation

$$\Delta P_{\%} = 1 - (1 - \Delta c_{\%})(1 - \Delta H_{\%})^2$$

Related literature

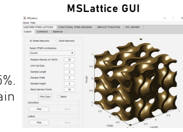
Recent research[2] featured the Gyroid TPMS structure being tested as a breakwater.

Methodology

TPMS Synthesis

Uniform and graded TPMS designs were created using MSLattice[3] software. All non-graded TPMS models have the same relative densities and volume. Graded models were created with density gradients ranging from 5%-25% to 15%-15%. Blender was utilized to reduce the amount of polygons on certain designs to improve computation performance.

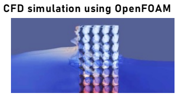
MSLattice GUI



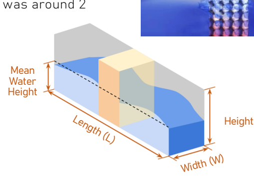
Computational Fluid Dynamics (CFD) Simulation

Openfoam and Olaflow[4] were used for CFD simulation. A numeric wave tank was created for testing various breakwater structures. The simulation time was around 2 days for each case on a 6-core computer.

CFD simulation using OpenFOAM



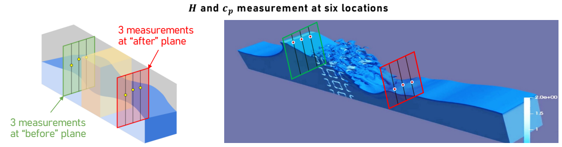
Simulation Parameters (m)	Values
Wave tank dimensions (LxWxH)	30x2x9
TPMS Breakwater and wall dimensions (LxWxH)	2x2x4
Graded TPMS (LxWxH)	4x2x4
Lattice (LxWxH)	1.6x2x4
Mean water height	2



Data Collection

The output data was imported into Paraview and extracted over three equidistant lines on two parallel planes each before and after the breakwater to measure the wave heights and wave velocity at each line. The data was outputted into CSV files.

H and c_p measurement at six locations

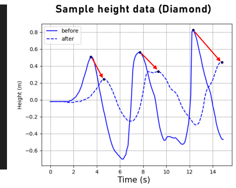


Data Processing

Using Python and external modules including Matplotlib, SciPy, and NumPy, the output data (height, velocity, drag) were directly extracted from the CSVs. $\Delta H_{\%}$ and $\Delta c_{\%}$ for each structure was calculated by taking the average percent decrease of height/velocity peaks as the waves passed through the structure.

```
import csv
import matplotlib.pyplot as plt
from scipy.signal import find_peaks
import numpy as np



class TPMS:
    def __init__(self, Types:list) -> None:

        frames = 150
        Type1, Type2 = Types

        # rawdata processing

        ...
        rawdata is in form
        np.array([num 1 data],
                 [num 2 data],
                 [num 3 data])
        ...

        self.h_before = np.zeros((3,frames))
        self.h_after = np.zeros((3,frames))
        self.v_before = np.zeros((3,frames))
        self.v_after = np.zeros((3,frames))
```

Sample height data (Diamond)



Sample height decrease calculation (third peak)

Peak before passing structure: 0.8233 m
Peak after passing structure: 0.4457 m

$$\Delta H_3 = \frac{0.8233 - 0.4457}{0.8233} = 45.86\%$$

$$\Delta H_{\%} = \frac{1}{3}\sum_{i=1}^{3} \Delta H_i$$

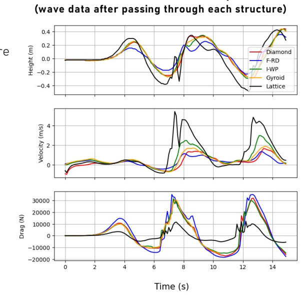
Results

Uniform TPMSs vs. Lattice

The wave heights and velocities for all four types of TPMSs and Lattice structure were plotted, and the percentages of reduction are shown. The Lattice structure is a clear outlier in terms of velocity reduction and drag; Lattice seems to not have decreased the velocity to the same degree as all other TPMS structures have, and the lattice experienced less drag.

Structure	$\Delta H_{\%}$	$\Delta c_{\%}$	$\Delta P_{\%}$
Diamond	46.07%	66.69%	90.31%
F-RD	62.61%	75.21%	96.53%
I-WP	44.99%	53.84%	86.03%
Gyroid	45.04%	61.72%	88.44%
Lattice	38.56%	28.41%	72.98%

TPMS vs. Lattice comparisons in H, c_p, and drag (wave data after passing through each structure)



Graded Diamond TPMSs

Diamond with various graded density have been tested. All the graded structures has larger $\Delta H_{\%}$ and $\Delta c_{\%}$ than the Uniform Diamond structure.

$\Delta H_{\%}$ and $\Delta c_{\%}$ for Graded TPMS

Density Gradient	$\Delta H_{\%}$	$\Delta c_{\%}$
5% - 25%	48.36%	76.77%
10% - 20%	46.78%	75.85%
15% - 15%	46.47%	75.79%
20% - 10%	47.08%	76.17%
25% - 5%	47.41%	75.14%

Drag Force

Lattice experienced the lowest drag, with a relative drag of 0.2360. The TPMS designs of Gyroid, Diamond, and I-WP structures show impressive drag decrease relative to a wall with the same dimensions

Drag data for various structures

Structure	Maximum Drag (N)	Relative Drag (Wall = 1)
Diamond	30,587	0.6220
F-RD	35,077	0.7133
I-WP	31,660	0.6438
Gyroid	30,509	0.6204
Lattice	11,604	0.2360
Wall	49,175	1

Conclusion

All tested TPMS designs have greater $\Delta H_{\%}$ and $\Delta c_{\%}$ than the Lattice structure, implying that TPMS designs are more effective at reducing the P of waves. Some TPMS structures also take in higher drag forces compared to Lattice as a byproduct of greater wave reduction; However, they demonstrate impressive drag reduction with respect to a wall with identical dimensions. Similarly, the graded Diamond designs performed better in wave reduction than the non-graded Diamond design. F-RD seems to be the most effective TPMS design tested for wave reduction due to the maximal $\Delta P_{\%}$ metric. Ultimately, however, Diamond seems to be the most effective TPMS structure for breakwater use considering its $\Delta P_{\%}$ is comparable to that of F-RD yet experiences one of the lower drag forces in the TPMS group. Graded versions of Diamond also, in general, are more efficient at wave reduction than uniform versions.

Future Work

- Explore other TPMS designs as possible breakwaters (e.g. Lidinoid, Neovius, Primitive)
- Inclusion of other non-TPMS porous lattice structures as potential breakwaters
- Investigate the graded forms of other TPMS designs

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Citations

[1] Holmes, P. (2011). Linear Wave Theory. OAS. https://www.oas.org/cdcm_train/courses/course21/chap_05.pdf

[2] Dang, B. Nguyen-Van, V. Tran, P. Wahab, M. Lee, J. Hackl, K. Nguyen-Xuan, H.(2022, April). Mechanical and hydrodynamic characteristics of emerged porous Gyroid breakwaters based on triply periodic minimal surfaces

[3] Alketan, Oraib & Abu Al-Rub, Rashid. (2020). MSLattice: A free software for generating uniform and graded lattices based on triply periodic minimal surfaces. Material Design & Processing Communications. 3. 10.1002/mdp2.205.

[4] Higuera, P. (2018, June). CFD for waves