Basketball: A Physics Study

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***Abstract*—This is a study of the physics of basketball. The overall goal of this model and simulation are to determine the various physical behaviors of a basketball and their effects on the success of the shot. Specifically, I hope to vary initial shot angle from horizontal and ball spin to determine which initial conditions are most related to shot success. I vary these parameters over a range of distances from the hoop to determine how optimal shot conditions vary with player distance to the hoop. The model measures shot success by introducing average human variation into each set of initial conditions and then determining which shots are most successful. The model incorporates aspects that include gravity, friction, air resistance, and spin. The model detects collisions with all parts of the backboard and the rim.**

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I. INTRODUCTION

In order to score points in basketball, players propel a ball through a ring, called the hoop, the basket, or the rim. The hoop is connected to a vertical rectangular board called the backboard. Players shoot the ball from various distances due to a few influences. Shots from behind a specific line are worth three points rather than two, and players may not be able to approach the basket because of the opposing team's defense. The ball, when shot by a player, may bounce multiple times on the rim and backboard before falling in the hoop. On certain shots, players may choose to aim so that the ball bounces directly off the backboard and into the hoop. The ball is generally shot as the player jumps and is shot from over his head. The ball also usually is released so that it spins backward.

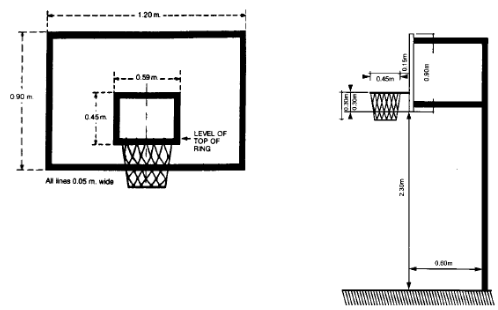

Fig. 1: Backboard Dimensions

II. GENERAL MODELING ASSUMPTIONS

- The ball is rigid. The ball does not deform or compress in the air or during collisions.
- The ball can be modeled as a spherical shell, which means that its rotational inertia is equal to $\frac{2}{3}mR^2$. [3]
- The rim and backboard are rigid and do not bend or deform.
- The rim is assumed to have no thickness. It is just a ring.

III. BASIC PRINCIPLES OF THE BALL'S FLIGHT

The ball's flight is the simplest part of the ball's motion. The only forces considered by the model are gravity and air resistance. Obviously, gravity is the major determinant of the objects flight path. Gravity, under ideal situations, causes the ball to follow a parabolic path. Under these conditions, the player can shoot the maximum range for a specified velocity by releasing the ball at a 45 degree angle with the horizontal. Air resistance causes the ball to slow as it is in flight. Its decrease in speed causes the parabolic shape to be deformed slightly, as the ball falls with a steeper angle near the end of its flight. In reality, there are other forces on the ball, such as lift caused by the ball's spin and its interaction with the air.

A. Modeling Decisions

Air resistance doesn't affect the shape of the path or optimal angle much, but its effects are included for multiple reasons. Air resistance doesn't dramatically affect the range of a ball, but its most important effects are prevalent at the end of the basketball's flight. Air resistance causes the ball to drop at a greater angle toward the end of its flight. The air resistance also causes the ball to land with slightly less velocity. Although slight in magnitude, both of these affects could dramatically affect the ball's success in landing inside the hoop. The other effects on the ball's flight, such as aerodynamic forces due to the spin of the ball, are disregarded here because they are very negligible. The slowing of the ball's spin in the air is also ignored because the change in rotational speed is negligible.

B. Equations of Motion for the Ball's Flight

$$F = m\mathbf{g} - \frac{1}{2}C_D A\rho\|\vec{\mathbf{v}}\|^2\hat{\mathbf{v}} \tag{1}$$

This produces the differential equation:

$$m\ddot{\vec{\mathbf{x}}} + \frac{1}{2}C_D A\rho\|\dot{\vec{\mathbf{x}}}\|^2\dot{\hat{\mathbf{x}}} = m\mathbf{g} \tag{2}$$

which is solved using a differential equations solver in Matlab. m stands for the mass of the basketball, C_D is the coefficient of drag, ρ is the air density, and A is the cross-sectional area of the ball.

C. Discussion of Air Resistance

Although its discussion is largely outside of the scope of this model, it is important to understand why this form of the drag equation was chosen. It is believed that at low velocities, the drag is proportional to the velocity, and at high velocites, the drag force is proportional to the square of the velocity. It turns out that the threshold for this change is dependent upon the size of the object, the kinematic air viscosity, and its speed. For the basketball at the assumed speeds, and at standard atmospheric pressure and temperature, this Reynolds number exceeds the threshold at which this drag equation becomes valid. Thus, the drag equation assuming proportionality to the square of velocity is valid. [1]

IV. Collisions

Collisions are a key part of the basketball shot. Only a few of shots are good enough to go in the hoop without hitting the rim or the backboard. Thus, in order for this model to be realistic, it must accurately model collisions.

A. Collision Modeling Assumptions

- The ball is rigid. The ball does not deform or compress in the air or during collisions.
- The rim and backboard are rigid and do not bend or deform.
- The component of the ball's velocity that is parallel to the line between the ball's center of mass and the collision point is reversed. This means that, barring any effects from spin, the ball would bounce such that the angle of incedence equals the angle of reflection.
- The ball's spin causes friction that affects both the perpendicular component of the velocity and the angular velocity. The spin does not affect the reversal of the parallel component of velocity.
- The change is velocity is governed by a coefficient of restitution. This means that the ball's speed after a collision is a set percentage of the ball's speed before the collision. In the simulation, for convenience's sake, this effect is applied after the effect of the spin.
- The ball's coefficient of friction is sufficiently high that, before the ball has rebounded from the collision totally, the ball has achieved a rolling situation with the hoop. This means that the equations governing the relationship between angular velocity and velocity will hold. I believe that this assumption is valid because the ball is made of a rubbery material that is very sticky. This means that the coefficient of friction is very high.

B. Spin and Its Effects on Collisions

Kinetic friction is caused when two surfaces are touching but not moving at the same velocity. This friction, in the case of the basketball, not only changes the velocity of the ball, but also changes the ball's angular velocity. These are related because the same frictional force causes the change in both linear momentum and rotational momentum.

C. Equations Governing Post-Collision Values

As shown in the following figure, initial velocity is broken into parallel and perpendicular components.

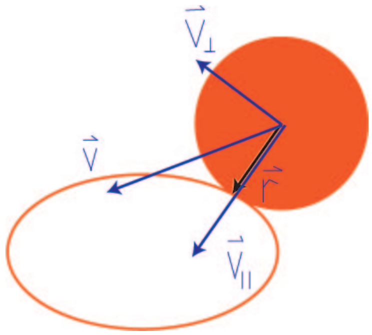

Fig. 2: The initial velocity components

These are the equations that govern post-collision values:

$$\int f \times dt = m\Delta v_\perp = \frac{I}{R}\Delta\omega \tag{3}$$

$$\omega = \frac{v_\perp}{R} \tag{4}$$

$$v_{\|,f} = -v_{\|,0} \tag{5}$$

$$\vec{v_f} = k(\vec{v_{\perp,f}} + \vec{v_{\|,f}}) \tag{6}$$

There are enough equations that we can solve for final parallel, perpendicular, and angular velocities.

V. Shot Variation due to Human Factors

In order to measure the success of certain initial conditions, we must introduce human error into the initial shot conditions. This essentially simulates a person trying to shoot the ball with the given conditions. This is the most realistic way of measuring shot success because it measures success over the range of normal human variation for a given shot.

VI. Results

The simulation's results were plausible, within expected ranges, and realistic. The collision detection and handling worked very well, and the simulation worked in three dimensions. Example result figures are as follows:

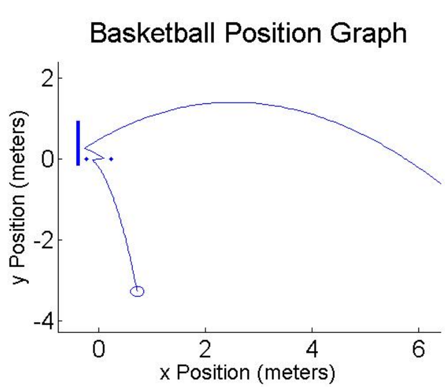

Fig. 3: A Basketball Trajectory plotted in 2D

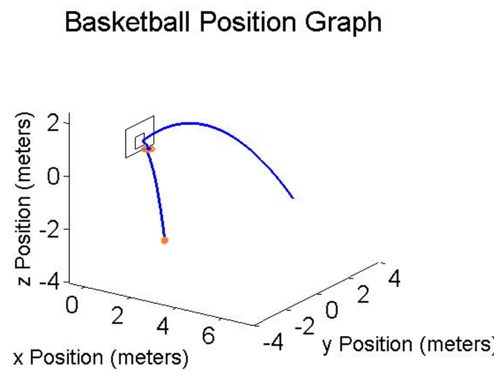

Fig. 4: A Basketball Trajectory plotted in 3D

VII. Conclusions

Initial simulation results suggest that the optimal shooting angle for a shot from the three-point line (21 feet from the basket) is around 48 degrees from horizontal. This agrees very well with results obtained by Tran and Silverberg. [5] This simulation predicts accuracies of around 50-55%, which is consistent with the average percentage of three point shots made in the NBA. Error bounds used in this model reflect the skill set of an average player, which explains why the model predicts similar numbers as those of NBA players, when the NBA players make that percentage of shots with defense on them.

Simulations varying two parameters have yet to be run. I believe that the findings of this model are valid and accurate because it agrees well with the results of other studies. More work needs to be done with varying shot spin and shooting angle at the same time, and for various distances from the basket. Ideally, results will include a "sweet spot" for each of the distances from the basket, a combination of angular velocities and shot angles that produce the highest odds of making the shot.

Appendix

Link to a repository containing simulation files: https://github.com/runnersaw/basketball

References

[1] National Aeronautics and Space Administration. *Drag of a Sphere*. Ed. by Tom Benson. Sept. 2010. URL: http://www.grc.nasa.gov/WWW/k-12/airplane/dragsphere.html.

[2] GR Hamilton and C. Reinschmidt. "Optimal trajectory for the basketball free throw". In: *Journal of Sports Sciences* (2008).

[3] Hans C. Ohanian and John T. Markert. *Physics for Engineers and Scientists*. Third. W. W. Norton, 2007.

[4] H. Okubo and M. Hubbard. “Dynamics of basketball-rim interactions”. In: *Sports Engineering* (Mar. 2004), pp. 15–29.

[5] Chau M. Tran and Larry M. Silverberg. “Optimal release conditions for the free throw in men’s basketball”. In: *Journal of Sports Sciences* (2008).