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Fixture Capability Optimisation for Early-stage Design of Assembly System with Compliant Parts Using Nested Polynomial Chaos Expansion

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Abstract

This paper introduces the novel concept of *fixture capability* measure to determine fixture layout for the best assembly process yield by optimizing position of locators and reference clamps to compensate stochastic product variations and part deformation. This allows reducing the risk of product failures caused by product and process variation. The method is based on three main steps: (i) physics-based modelling of parts and fixtures, (ii) stochastic polynomial chaos expansion to calculate fixture capability, and (iii) fixture capability optimisation using surrogate modelling. The methodology is demonstrated and validated using the results of an aerospace wing sub-assembly joined by riveting technique.

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1. Introduction and Motivation

Fixtures are used to accurately locate and securely hold parts during machining, assembly and joining operations to ensure that quality requirements are achieved. Fixture design is one of the most important tasks during process design phase of a new product development as it involves the definition and layout of locators and restraining blocks which provide support to the parts being assembled, and as a result highly affect assembly process capability and the final product quality.

Over the years, researchers have addressed different issues related to the fixture design: *fixture planning*, *fixture configuration*, and *fixture construction*. In the fixture planning stage, issues related to number and type of locators and orientation of fixture, are addressed. The fixture configuration stage determines the layout of a set of locators and clamps. Finally, the fixture construction stage involves the 3D CAD modelling of fixture components with the definition of contact surfaces with the work-piece [1]. This paper deals with the problem of optimizing the fixture layout for assembly processes of compliant sheet-metal parts. The optimum fixture layout design (which falls under the domain of fixture configuration) aims to calculate the optimum

locator and clamps location, which satisfies given quality requirements on critical key performance indicators (KPIs). It has been reported that only 60-70% of *Right-First-Time* is reached during the design phase [2]. Failures not predicted during the design phase can appear during ramp-up, which in turn, require engineering changes thus leading not only to significant cost increase ('Rule of 10') but also cause delay in the launch of a new product. Moreover, most of the engineering changes [3], after the fixture design release, are triggered by geometrical and dimensional variation of parts being assembled. For example, for a single closure panel fixture around a hundred engineering changes are made after design release. For each change the fixture design has to be revised, verified and validated and finally, design needs to be updated, leading to further cost increases and production launch delays. However, research on fixture layout design for compliant sheet-metal parts is limited due to lack of *part variation stochastic model* ("non-ideal" part model) which has capability to expand the current ideal-part CAD/CAM models to emulate real parts variation at early design stage. Moreover, existing research has focused on the development of methods under the assumption of ideally rigid parts. Therefore, "3-2-1" locating scheme has been analysed both for single- and multi-station assembly systems [1,4].

Subsequently, more sophisticated models have been introduced aiming to model both part compliancy (“N-2-1” locating scheme) and part-to-part interactions. Recently, a more comprehensive methodology has been reported in [5], where a special-purpose finite element software, called EAVS (**Elastic Assembly Variation Simulation**), is implemented to simulate variation propagation in panel assembly. This involves using time-intensive Monte Carlo (MC) simulations which leads to several weeks of computational time when dealing with medium to large sub-assemblies as used in body-in-white automotive applications. Li et al. [6] improved the approach for robust fixture design optimisation by using **response surface methodology**. Production data were employed to model part variability and “non-ideal” CAD models were created by running FEM simulations. Then prediction and correction method was used to determine the fixture locating layout. For some assembly processes, such as (remote) laser welding or riveting, fixtures have to additionally maintain a proper part-to-part fit-up to satisfy quality constraints. This implies that fixture layout needs to be optimized not only for a single product variation, but for a production batch, as to accommodate stochastic part variation as coming from real manufacturing process. Li et al. [7] calculated the optimum fixture layout with normally-distributed product variation, as obtained based on production data.

Table 1. Literature review table on fixture layout optimisation.

Fixture model		“3-2-1” locating scheme	“N-2-1” locating scheme
Stochastic model			
Monte Carlo simulations based on:	Production data	Li et al. (2010) [14]	Cai et al. (2006) [5] Li et al. (2001) [12] Li & Shiu (2001) [13]
	Design data	Lu & Zhao (2014) [11]	Cai, (2008) [17] Camelio et al. (2004)[10] Cai et al. (1996) [19]
Surrogate models based on:	Production data	Phoomboplab & Ceglarek (2008) [1]	Li et al. (2009) [6] Li et al. (2003) [15] Li et al. (2002) [7]
	Design data	Kim & Ding (2004) [4]	Proposed in this paper

Table 1 summarizes the main methods developed over the years to optimize the fixture layout design. Some of the methods are mainly based on production data, making their applicability very limited during early-stage design when only CAD and tolerance specifications are available. This limitation has been partially overcome by stochastic models based on design data and relying on MC simulations. Although these methods provide accurate prediction of process capability, they are not suitable for handling complex fixture systems with high number of design parameters. To overcome the aforesaid limitations, this paper proposes a novel methodology for fixture layout design optimisation for a batch of compliant non-ideal parts by introducing the concept of *fixture capability*. Intuitively, *fixture capability* represents the capability of the fixture to deliver quality

requirements under given product and process variations. The proposed *fixture capability* is a measure of the probability that a given fixture satisfies product quality requirements during production. The developed method is based on: (i) modelling of product/process variation using CAD/CAM data and tolerance specifications at early-stage design; (ii) integration of physics-based modeller with nested polynomial chaos; and (iii) robust fixture layout optimisation based on surrogate modelling.

The rest of the paper is arranged as follows: Section 2 presents the problem formulation; Section 3 summarizes the proposed methodology; lastly, industrial case study and final remarks are depicted in Sections 4 and 5, respectively.

2. Problem Formulation

The dimensional quality of a manufactured product is evaluated by its KPIs, which are delivered by key control characteristics (KCCs). In case of fixture design, KPIs correspond to functional key features measured on the final assembly/sub-assembly (such as gap and flushness, part-to-part gap, spring-back deviations and/or residual stresses in automotive body assembly). KCCs refer to assembly process parameters, such as position of all locators and clamps.

Let us assume that the set of KPIs and KCCs are grouped as in Eq. (1), where N_{KPI} and N_{KCC} are the numbers of KPIs and KCCs, respectively.

$$\begin{cases} KPIs : \{KPI_i\} \in \mathfrak{R}, \forall i = 1, \dots, N_{KPI} \\ KCCs : \{KCC_i\} \in \mathfrak{R}, \forall i = 1, \dots, N_{KCC} \end{cases} \quad (1)$$

The objective of fixture layout design optimisation is to maximize the *fixture capability* in presence of stochastic manufacturing errors both at product and process levels. Let “ ξ ” and “ ψ ” be the stochastic and determinist parameters, respectively.

$$\begin{aligned} KPIs &\Rightarrow \xi_{KPIs} \\ \xi_{KPIs} &\subseteq KPIs : \{\xi_{KPI_i}\} \in \mathfrak{R}, \forall i = 1, \dots, N_{KPI} \end{aligned} \quad (2a)$$

$$\begin{aligned} \xi_{KCCs} &\subseteq KCCs : \{\xi_{KCC_i}\} \in \mathfrak{R}, \forall i = 1, \dots, N_{\xi_{KCC}} \\ \psi_{KCCs} &\subseteq KCCs : \{\psi_{KCC_i}\} \in \mathfrak{R}, \forall i = 1, \dots, N_{\psi_{KCC}} \\ KCCs &= \xi_{KCCs} \cup \psi_{KCCs} \end{aligned} \quad (2b)$$

For instance, **stochastic parameters** (Eq. (2a)) represent manufacturing errors and/or uncertainty (i.e., part variation and production batch variation or tooling variation); whereas, **deterministic parameters** (Eq. (2b)) represent nominal design intent. Moreover, design constraints (DCs) in terms of lower (DC_L) and upper (DC_U) allowance limits (as defined by quality and design specifications) are defined for each KPIs and KCCs (Eq. (3)).

$$\begin{aligned} DC_s^{(\xi_{KPIs})} &: \begin{cases} DC_L^{(\xi_{KPIs})} : \{DC_{i,L}^{(\xi_{KPIs})}\} \in \mathfrak{R} \\ DC_U^{(\xi_{KPIs})} : \{DC_{i,U}^{(\xi_{KPIs})}\} \in \mathfrak{R} \end{cases} \quad \forall i = 1, \dots, N_{KPI} \\ DC_s^{(\psi_{KCCs})} &: \begin{cases} DC_L^{(\psi_{KCCs})} : \{DC_{i,L}^{(\psi_{KCCs})}\} \in \mathfrak{R} \\ DC_U^{(\psi_{KCCs})} : \{DC_{i,U}^{(\psi_{KCCs})}\} \in \mathfrak{R} \end{cases} \quad \forall i = 1, \dots, N_{\psi_{KCC}} \end{aligned} \quad (3)$$

2.1. Variation Response Function

In automotive and aerospace fixture assembly systems one leading challenge is the identification of the relationship mapping between input KCCs and output KPIs. Let F_i be the **variation response function**, defined as:

$$\xi_{KPI_i} = F_i(\xi_{KCCs}, \psi_{KCCs}), \forall i = 1, \dots, N_{KPI} \quad (4)$$

Eq. (4) indicates that having defined the F_i function, any stochastic variation of the output KPIs can be analytically calculated knowing the stochastic variation of the input KCCs along with the deterministic parameters. For example, when optimizing the fixture design for remote laser welding joining process, the input stochastic variation can be imputed to part variability (ξ_{KCCs}), whereas the deterministic parameters (ψ_{KCCs}) are clamps and locators, whose location and number need to be optimized to achieve a satisfactory part fit-up (ξ_{KPIs}). **This paper proposes a systematic methodology to efficiently estimate the variation response function by integrating stochastic polynomial chaos (PC) expansion [8] with analytical surrogate modelling.** The concept of polynomial chaos expansion was originally proposed into the CAE/FEM simulation community for uncertainty quantification. However, to the best of our knowledge, to date, no research paper has been published to address the problem of fixture layout design optimisation. Eq. (4) can be approximated through a series of orthogonal polynomial functions, called chaos expansion of degree κ . It can be written as Eq. (5a), where $\alpha_{t,i}$ and P_t are the t -th PC coefficient and polynomial of degree “ t ”, respectively.

$$\xi_{KPI_i}(\xi_{KCCs}, \psi_{KCCs}) \approx \sum_{t=0}^{N_{PC}} \alpha_{t,i} P_t(\xi_{KCCs}), \forall i = 1, \dots, N_{KPI} \quad (5a)$$

N_{PC} is the number of PC coefficients equal to:

$$N_{PC} = \frac{(\kappa + N_{\xi_{KCC}})!}{\kappa! N_{\xi_{KCC}}!} \quad (5b)$$

It can be proved that the polynomial series has a unique solution for given probability density functions (PDFs). For example, in case of Gaussian processes it can be proved that the orthogonal polynomials correspond to the normalized Hermite series [8].

2.2. Fixture Capability

The *fixture capability*, FC , represents the capability of the assembly fixture to deliver KPIs under given product and process variations.

FC is maximized by enhancing the probability of satisfying all design constraints. Let P_r be the probability of satisfying the design constraints, for a given stochastic variation as in Eq. (6), where “*PDF*” stands for the probability density function.

The fixture capability is then expressed as a cumulative probability index, as stated in Eq. (7). The reader can notice that Eq. (7) simplifies to the product of the individual probability values when the KPIs are stochastically independent.

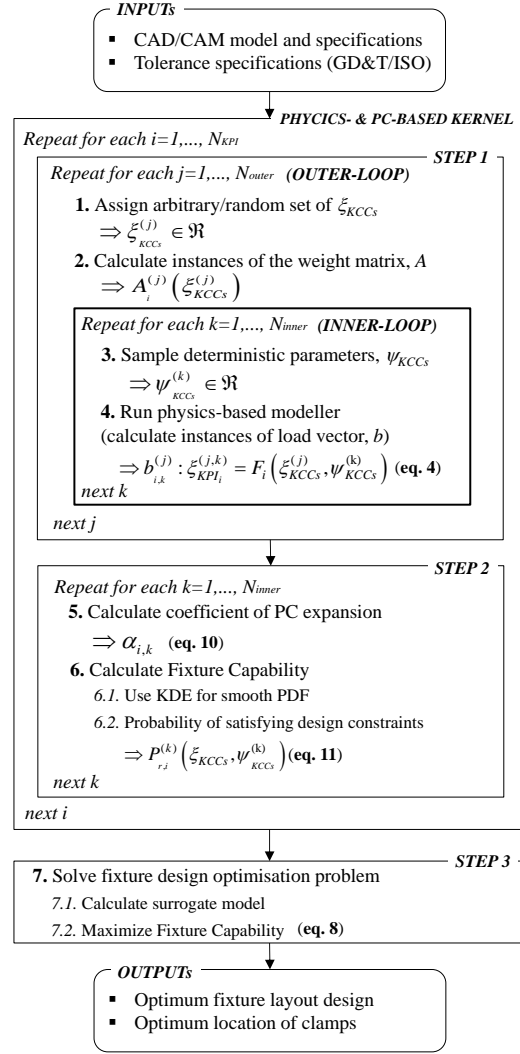


Fig. 1. Proposed methodology for fixture capability layout optimisation.

$$P_r : \{P_{r,i}(\xi_{KCCs}, \psi_{KCCs})\} \in [0,1], \forall i = 1, \dots, N_{KPI} \quad (6)$$

$$P_{r,i} : \begin{cases} \text{upper bound} \rightarrow \int_{-\infty}^{DC_{i,U}^{(\xi_{KPI})}} PDF_{\xi_{KPI_i}} d\xi_{KPI_i} \\ \text{lower bound} \rightarrow \int_{DC_{i,L}^{(\xi_{KPI})}}^{+\infty} PDF_{\xi_{KPI_i}} d\xi_{KPI_i} \\ \text{double bound} \rightarrow \int_{DC_{i,L}^{(\xi_{KPI})}}^{DC_{i,U}^{(\xi_{KPI})}} PDF_{\xi_{KPI_i}} d\xi_{KPI_i} \end{cases}$$

$$FC(\xi_{KCCs}, \psi_{KCCs}) = \prod_{i=1}^{N_{KPI}} P_{r,i}(\xi_{KCCs}, \psi_{KCCs}) \quad (7)$$

2.3. Fixture Layout Design Optimisation

The fixture layout design optimisation problem is formulated as in Eq. (8), which states as follows: in order to obtain the optimum configuration of deterministic parameters (ψ_{KCCs}), the fixture capability (FC) needs to be

maximized, for given stochastic variation (ξ_{KCCs}) at product and process levels.

$$\begin{aligned} \psi_{KCCs} : \max_{\psi_{KCCs} \in \mathcal{R}} (FC(\xi_{KCCs}, \psi_{KCCs})) \\ s.t. : \psi_{KCCs} \subseteq DCs^{(\psi_{KCCs})} \end{aligned} \quad (8)$$

3. Proposed Methodology

Fig. 1 shows the overall framework of the proposed methodology for fixture layout design optimisation. The main steps of the methodology are:

3.1. STEP 1 & STEP 2 - Physics- & PC-based Kernel

The key role is played by the nested inner- and outer-loops, which allows calculating the PC coefficients based on least squares technique. The inner-loop handles the deterministic parameters for a given set of stochastic parameters, which are generated by the outer-loop. Eq. (5a) can then be re-written as Eq. (9a) for any couple (j, k), “j” being an arbitrary sample of the j-th stochastic parameters (whose arbitrary samples are N_{outer}), and “k” the k-th deterministic parameters (whose arbitrary samples are N_{inner}).

$$\xi_{KPI_i}^{(j,k)} \approx \alpha_{0,i,k} P_0(\xi_{KCCs}^{(j)}) + \dots + \alpha_{N_{PC},i,k} P_{N_{PC}}(\xi_{KCCs}^{(j)}) \quad (9a)$$

Eq. (9a) turns to:

$$\xi_{KPI_i}^{(j,k)} \approx \underbrace{\left\{ P_0(\xi_{KCCs}^{(j)}) \dots P_{N_{PC}}(\xi_{KCCs}^{(j)}) \right\}}_{A_i^{(j)}} \underbrace{\begin{bmatrix} \alpha_{0,i,k} \\ \vdots \\ \alpha_{N_{PC},i,k} \end{bmatrix}}_{b_{i,k}^{(j)}} \quad (9b)$$

$$\forall j = 1, \dots, N_{outer} \quad \forall k = 1, \dots, N_{inner} \quad \forall i = 1, \dots, N_{KPI}$$

which leads to Eq. (10), where $A_{i,k}$ ($N_{outer} \times N_{PC}$) is called weight matrix, whereas $b_{i,k}$ ($N_{outer} \times 1$) is denoted as load vector. The number of samples (i.e., N_{outer}) needs to be bigger or equal to the number of unknown PC coefficients (i.e., N_{PC}) as in Eq. (5b)).

$$\begin{aligned} A_i = \begin{bmatrix} A_i^{(1)} \\ \vdots \\ A_i^{(N_{outer})} \end{bmatrix} \quad b_{i,k} = \begin{bmatrix} b_{i,k}^{(1)} \\ \vdots \\ b_{i,k}^{(N_{outer})} \end{bmatrix} \\ \alpha_{i,k} \approx (A_i^T \cdot A_i)^{-1} \cdot A_i^T \cdot b_{i,k} \end{aligned} \quad (10)$$

The over-sampling is necessary to make the system of linear equations in (10) over-determined (i.e., more equations than unknowns) and then solvable using the least squares method.

Although input stochastic parameters can be arbitrarily assigned, it seems an attractive strategy to randomly sample them, based on the input PDF (1 as in Fig. 1). For example, for a given part being assembled, product variation can be assigned in the form of shape errors as defined by GD&T or ISO tolerance specifications. In this paper, we have implemented the morphing method, originally developed in [9].

This technique allows to parametrize any 3D complex geometry and to embed geometrical tolerance deviations for given probability distributions. Instances of the weight matrix are simply calculated (2) by evaluating the orthogonal chaos polynomials at the sampled stochastic parameters. Deterministic parameters are sampled as well (3). Uniform, random and/or space filling techniques (i.e., full factorial, fractional factorial, etc.) can be used.

The physics-based modeller (4) plays a critical role to determine instances of the load vector, as formally defined in Eq. (4). For this purpose computer simulation CAT (Computer Aided Tolerancing) tools, such as 3D-DCS Compliant Modeller, EAVS, Robust Design and Tolerance (RT&D), and Tolerance Analysis of deformable Assembly (TAA) can be used to emulate part and fixture stack-ups. This paper uses Variation Response Method (VRM) [3] because of its advanced capability to automatically parametrize KCCs for given product and process variations, including: fixture constraints; compliancy of parts being assembled; and part-to-part interaction.

Having calculated the weight and the load vector, PC coefficients are then solved (5) by Eq. (10). In this paper, fixture capability (6) involves calculating the probability of satisfying design constraints, which relies on the PDFs of the given KPIs. Generally, even though the input stochastic parameters (ξ_{KCCs}) have Gaussian distribution, the output PDFs related to ξ_{KPIs} cannot be approximated through a Gaussian function because of the non-linearity of the variation response function (i.e., F_i). Therefore, we have used kernel density estimation (KDE) to get a smooth PDF with few samples. It has:

$$PDF_{\xi_{KPI_i}^{(k)}} = \frac{1}{N_{KDE}} \sum_{z=1}^{N_{KDE}} \Theta \left(\frac{\xi_{KPI_i}^{(k)} - \xi_{KPI_i}(\xi_{KCCs}^{(z,k)}, \psi_{KCCs}^{(k)})}{h} \right) \quad (11)$$

$$\forall k = 1, \dots, N_{inner} \quad \forall i = 1, \dots, N_{KPI}$$

where h and Θ are the bandwidth and the probability kernel, respectively. N_{KDE} is the number of samples randomly generated by Eq. (5a).

3.2. STEP 3 - Surrogate Model & Optimisation

Fixture capability's instances are generated for given input stochastic parameters and sampled deterministic parameters. Sampled parameters are used to train a surrogate model (7.1), analytically linking input stochastic parameters and deterministic parameters; adaptive polynomial fitting, spline or Kriging methods can be utilized for this purpose.

The last step of the methodology involves the calculation of optimum deterministic parameters (7.2) through Eq. (8).

4. Industrial Case study

The proposed methodology has been applied with respect to an aerospace wing sub-assembly joined using riveting technique (see Fig. 2). The sub-assembly mainly consists of one large bended skin with three stringers joined by NAS1097AD4-4 rivets. All parts are made by Aluminium alloy (material thickness: 1.02 mm).

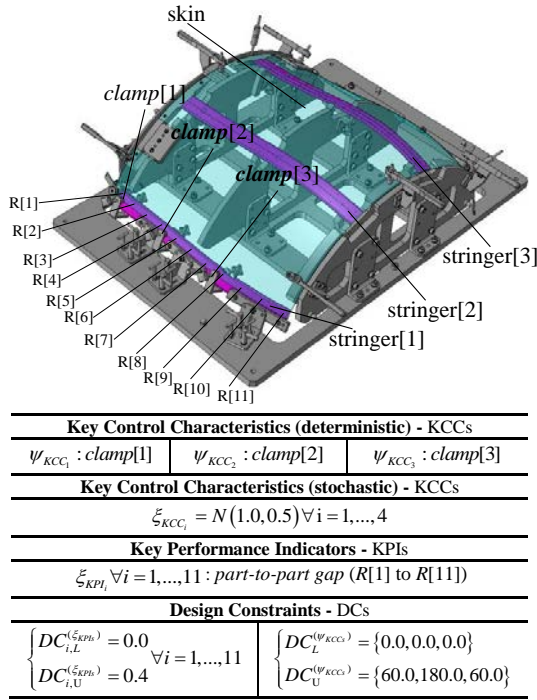


Fig. 2. Aerospace wing sub-assembly joined by riveting technique.

The skin and the stringers are assembled on the fixture schematically, as shown in Fig. 2 which highlights all blocks and support elements.

The attention of this study is oriented to the part-to-part gap between skin and stringer[1]. Indeed, excessive gap values may cause over-stress on the rivets because of unwanted elastic spring-back. Therefore, it is of interest to optimize the location of clamp[1] to clamp[3] so that the part-to-part gap for all rivets (R[1] to R[11] – $N_{KPI}=11$) is under the acceptable limit of 0.4 mm. Clamp[1] to clamp[3] are assumed as deterministic parameters and their position is limited to the translation along the longitudinal axis of stringer[1].

Stochastic part variation (Gaussian distributed with mean 1.0 mm and standard deviation 0.5 mm) has been assumed for both skin and stringer[1]. Morphing mesh [9] has been implemented to model part variation. For this purpose 4 control points (i.e., $N_{KCC}=4$) have been used. The steps of the methodology are illustrated as follows.

STEP 1 & STEP 2 - Physics- & PC-based Kernel

Deterministic control parameters have been sampled using full factorial approach (3 levels for each deterministic parameter, which equals $N_{inner}=27$ (3^3)).

The physics-based model has been developed using VRM simulation toolkit. Chaos expansion has been approximated using 2nd degree ($\kappa=2$) polynomial, thus leading to $N_{outer}=30$ ($2 \cdot N_{PC}$). The total number of physics-based simulations solved for equals 810 (N_{outer} times N_{inner}).

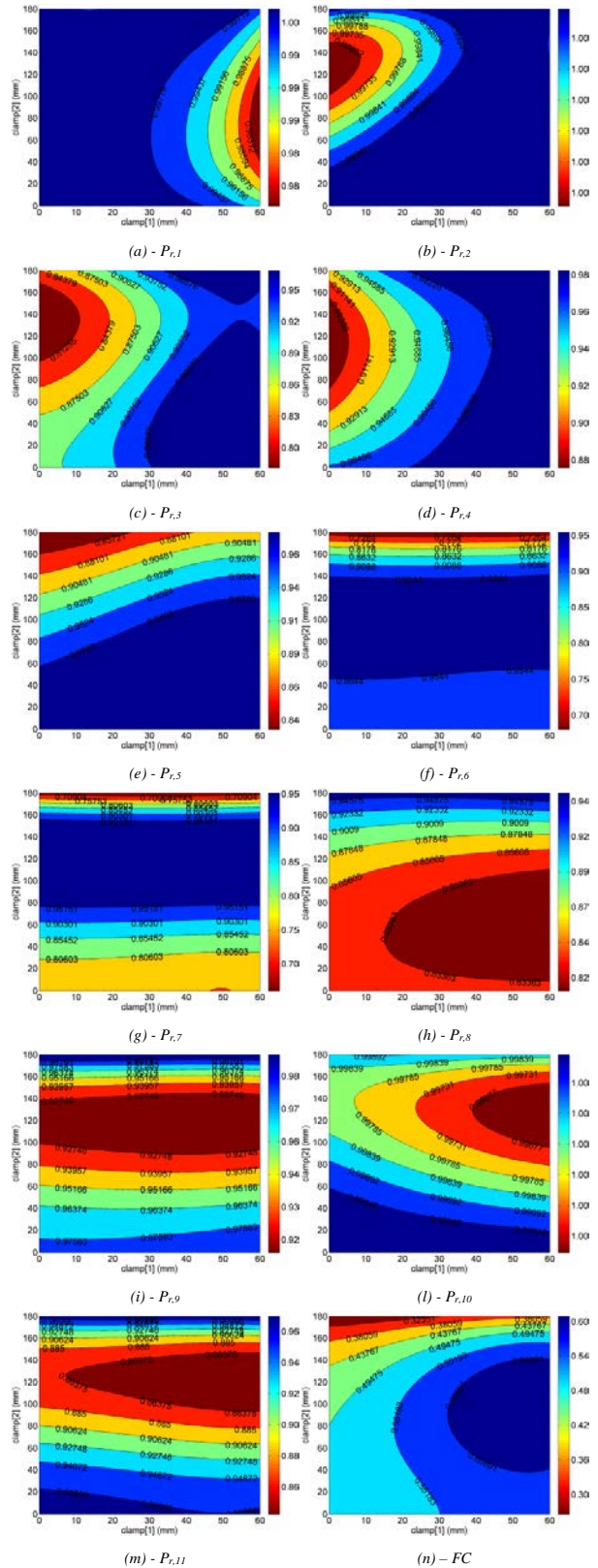


Fig. 3. Surrogate model development (clamp[3]=30 mm).

STEP 3 - Surrogate Model & Optimisation

The probability of satisfying the design constraints has been developed using Gaussian-based KDE estimation with adaptive bandwidth ($N_{KDE}=1000$); whereas, adaptive polynomial surrogate model with leave-one-out cross validation has been used for surrogate model calculation. Fig. 3 shows the calculated individual probability functions (Figs. 3a-m) and the cumulative fixture capability (Fig. 3n). The constrained optimisation problem stated in Eq. (8) has been solved using Nelder-Mead algorithm. Table 2 shows the probability values related to the optimum fixture clamp layout: $\psi_{KCC3}=\{56.1, 102.0, 45.2\}$.

Table 2. Optimum fixture layout results.

$P_{r,1}$	$P_{r,2}$	$P_{r,3}$	$P_{r,4}$	$P_{r,5}$	$P_{r,6}$
0.99	1.00	0.99	0.99	1.00	1.00
$P_{r,7}$	$P_{r,8}$	$P_{r,9}$	$P_{r,10}$	$P_{r,11}$	FC
1.04	0.84	0.76	0.99	1.00	0.67

5. Conclusions and Final Remarks

This paper proposes a new methodology for fixture layout design optimisation of batch of compliant non-ideal parts by introducing the concept of fixture capability. The paper makes two contributions: (1) *industrial* - optimisation of product/process capability at design stage to reduce costly engineering changes at installation and commissioning; (2) *research* - development of a computationally efficient optimisation methodology which analytically evaluates fixture capability considering product/process variations. The results have several benefits. First, the method can be applied in the early design stages, when no data are available (only CAD/CAM information and tolerance specifications are provided). Next, the approach is computationally efficient and can be used to optimize both single- and batch-of-part assembly. Finally, fixture capability is analytically estimated, thereby implying that critical design parameters are tracked and corrective actions can be taken to improve the assembly quality. The method has been applied in fixture layout optimisation for aerospace wing sub-assembly joined by rivet technique. However, it can also be beneficial for fixture layout optimisation for any assembly process involving sheet-metal parts. The proposed method goes beyond the state-of-art by developing an efficient methodology based on polynomial chaos expansion to optimize fixture design using design data; whereas, existing methods are mainly limited to expensive MC simulations using production data. This research will be further explored to model heterogeneous design tasks (such as, joining optimisation) to achieve both cost and cycle time improvement.

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