```
(*  2013-coq-final.v
 * Some axioms and theorems about real numbers
 *
 * Programmer: Mayer Goldberg, 2013
 *)
```

Section *Reals*.

Require Import *Setoid*.

Axiom *PNP*: $\forall\ p$: Prop, $p \lor \neg p$.

Variable R : Set.
Variable *zero* : R.
Variable *one* : R.
Variable $A : R \to R \to R$.
Variable $M : R \to R \to R$.

Axiom *x_add_comm*: $\forall\ x\ y : R,\ A\ x\ y = A\ y\ x$.

Axiom *x_add_assoc*: $\forall\ x\ y\ z : R,\ A\ (A\ x\ y)\ z = A\ x\ (A\ y\ z)$.

Axiom *x_left_dist*: $\forall\ x\ y\ z : R,\ M\ x\ (A\ y\ z) = A\ (M\ x\ y)\ (M\ x\ z)$.

Axiom *x_mult_comm*: $\forall\ x\ y : R,\ M\ x\ y = M\ y\ x$.

Axiom *x_mult_assoc*: $\forall\ x\ y\ z : R,\ M\ (M\ x\ y)\ z = M\ x\ (M\ y\ z)$.

Axiom *x_zero*: $\forall\ x : R,\ A\ x\ zero = x$.

Axiom *x_one*: $\forall\ x : R,\ M\ x\ one = x$.

Axiom *x_neg*: $\forall\ x : R,\ \exists\ xneg : R,\ A\ x\ xneg = zero$.

Axiom *x_inv*: $\forall\ x : R,\ \tilde{}(x = zero) \to \exists\ xinv : R,\ M\ x\ xinv = one$.

Definition *inverse_of* $(x\ y : R) := M\ y\ x = one$.

Theorem *T0*: $\forall\ x\ xneg\ xneg' : R,$
$(A\ x\ xneg = zero \land$
$A\ x\ xneg' = zero) \to$
$xneg = xneg'$.

Proof.
intros *x xneg xneg' H1*.
destruct *H1* as [*H1 H2*].
assert (*H3*: $A\ xneg\ (A\ x\ xneg) = A\ xneg\ (A\ x\ xneg')$).
rewrite *H1*, *H2*; reflexivity.
repeat rewrite ← *x_add_assoc*,
(*x_add_comm xneg*),
H1,
(*x_add_comm zero*),
x_zero in *H3*.
exact *H3*.

```
Qed.

Theorem T1: ∀ x y : R,
                (x ≠ zero) → (y ≠ zero) →
                ∃ xinv yinv : R,
                  inverse_of xinv x ∧
                  inverse_of yinv y ∧
                  M (M x y) (M xinv yinv) = one.
Proof.
  intros x y H1 H2.
  apply x_inv in H1.
  apply x_inv in H2.
  destruct H1 as [xinv H1].
  destruct H2 as [yinv H2].
  ∃ xinv.
  ∃ yinv.
  split.
  exact H1.
  split.
  exact H2.
  rewrite (x_mult_comm x y).
  rewrite ← x_mult_assoc.
  rewrite (x_mult_assoc y x xinv).
  rewrite H1.
  rewrite x_one.
  rewrite H2.
  reflexivity.
Qed.

Theorem T2: ∀ x y z : R, (x ≠ zero) →
                          ((M x y) = (M x z)) →
                          (y = z).
Proof.
  admit.
Qed.

Theorem T3: ∀ x y z : R, M (A x y) z = A (M x z) (M y z).
Proof.
  admit.
Qed.

Theorem T4: ∀ x : R, A zero x = x.
Proof.
  admit.
Qed.
```

`Theorem` $T5$: $\forall\ x : R,\ M\ one\ x = x$.
`Proof.`
  admit.
`Qed.`

`Theorem` $T6$: $\forall\ x\ y\ z : R,\ (A\ x\ y = A\ x\ z) \rightarrow (y = z)$.
`Proof.`
  admit.
`Qed.`

`Theorem` $T7$: $\forall\ m\ n\ p\ q : R,$
$M\ (A\ m\ n)\ (A\ p\ q) =$
$A\ (A\ (M\ m\ p)\ (M\ n\ p))\ (A\ (M\ m\ q)\ (M\ n\ q))$.
`Proof.`
  admit.
`Qed.`

`Theorem` $T8a$: $\forall\ m\ n\ p\ q : R,\ A\ m\ (A\ n\ (A\ p\ q)) = A\ (A\ m\ n)\ (A\ p\ q)$.
`Proof.`
  admit.
`Qed.`

`Theorem` $T8b$: $\forall\ m\ n\ p\ q : R,\ A\ m\ (A\ n\ (A\ p\ q)) = A\ (A\ (A\ m\ n)\ p)\ q$.
`Proof.`
  admit.
`Qed.`

`Theorem` $T9$: $\forall\ m\ n\ p : R,\ A\ m\ (A\ n\ p) = A\ (A\ p\ m)\ n$.
`Proof.`
  admit.
`Qed.`

`Theorem` $T10$: $\forall\ m\ n\ p : R,\ M\ m\ (M\ n\ p) = M\ p\ (M\ m\ n)$.
`Proof.`
  admit.
`Qed.`

`Theorem` $T11$: $\forall\ m\ n\ p\ q : R,\ M\ m\ (A\ n\ (A\ p\ q)) = A\ (A\ (M\ m\ n)\ (M\ m\ p))\ (M\ m\ q)$.
`Proof.`
  admit.
`Qed.`

`Theorem` $T12$: $\forall\ m\ n\ p\ q : R,$
$M\ (M\ m\ (A\ n\ p))\ q = A\ (M\ (M\ m\ n)\ q)\ (M\ m\ (M\ p\ q))$.
`Proof.`
  admit.
`Qed.`

`Theorem` $T13$: $\forall\ x : R,\ (\forall\ m : R,\ A\ m\ x = m) \rightarrow x = zero$.

Proof.
admit.
Qed.

Theorem $T14$: $\forall\ x\ :\ R,\ (\exists\ m\ :\ R,\ A\ m\ x = m) \to x = zero.$
Proof.
admit.
Qed.

Theorem $T15$: $\forall\ m\ :\ R,\ M\ m\ zero = zero.$
Proof.
admit.
Qed.

Theorem $T16$: $\forall\ x\ :\ R,\ (\forall\ m\ :\ R,\ M\ m\ x = m) \to x = one.$
Proof.
admit.
Qed.

Theorem $T17$: $\forall\ x\ :\ R,\ (\exists\ m\ :\ R,\ m \neq zero \land M\ m\ x = m) \to x = one.$
Proof.
admit.
Qed.

Theorem $T18$: $\forall\ m\ n\ :\ R,$
$\exists\ mneg\ nneg\ :\ R,$
$A\ m\ mneg = zero\ \land$
$A\ n\ nneg = zero\ \land$
$M\ mneg\ nneg = M\ m\ n.$
Proof.
admit.
Qed.

End Section.