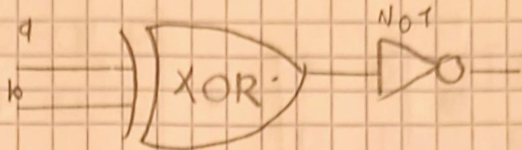
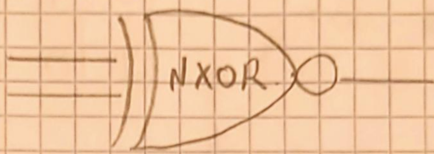
Taller de bits, Qubits y compuertas clásicas

1)

- a NXOR b

a	b	XOR	NXOR
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1





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$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{array} = NXOR$$

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$$NOT = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0 & 1 \\ 1 & 1 & 0 \end{array} \qquad XOR = \begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{array}$$

NOT · XOR = NXOR.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

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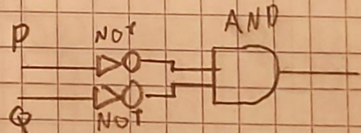
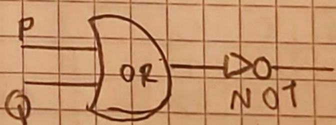
a	b	XOR
0	0	0
0	1	1
1	0	1
1	1	0

2) (NOT P) AND (NOT Q) = NOT (P OR Q)

P	Q	¬P	¬Q	(¬P) ∧ (¬Q)	¬(P ∨ Q)
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0





$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{array} =$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

NOT ⊗ NOT

· AND = $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

AND · (NOT ⊗ NOT)

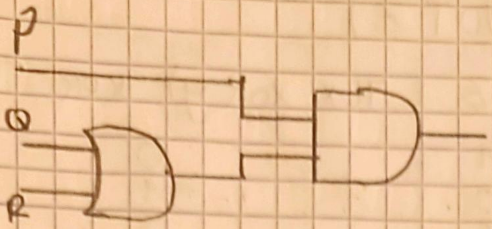
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

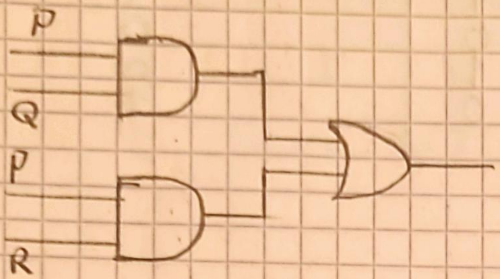
(NOT) (OR)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

3) P AND (Q OR R) = (P (AND Q) OR (P AND R)

P	Q	R	Q∨R	P∧(Q∨R	P∧Q	P∧R	(P∧Q) ∨ (P∧R)
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1





$$\begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cccccccc} 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} = M$$

se cumple por igualdad.

AND (OR $\otimes$ I)

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

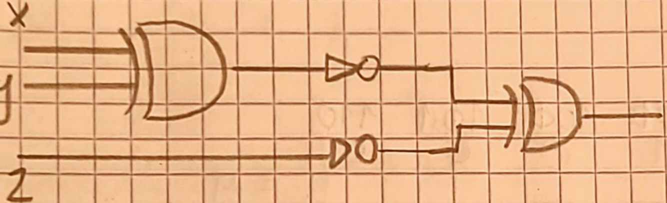
$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Reorganizando la matriz resultante se obtiene.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

4) (NOT(X XOR y)) XOR (NOT Z)

x	y	z	x XOR y	¬(x XOR y)	¬z	¬(x XOR y) XOR ¬z
0	0	0	0	1	1	0
0	0	1	0	1	0	1
0	1	0	1	0	1	1
0	1	1	1	0	0	0
1	0	0	1	0	1	1
1	0	1	1	0	0	0
1	1	0	0	1	1	0
1	1	1	0	1	0	1



$$M = \begin{array}{c|cccccccc} & 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \\ \hline 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{array}$$

(X XOR y) XOR (NOT Z)

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$