Risk-Adjusted Portfolio Performance Excel Add-in Documentation

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Purpose of the Add-in

The add-in was written to provide an easy way to calculate risk-adjusted portfolio performance and related measures in Microsoft Excel. Once installed (see below), the functions can be used just like any built-in Excel function. I chose to use the C# programming language along with the Excel-DNA library (https://github.com/Excel-DNA/ExcelDna) because it is easier to provide function and argument documentation in the Insert Function dialog box than it is with VBA. Additionally, I wanted to learn the C# language.

Add-in Installation

Installing the add-in is easy. After you have downloaded the add-in, click the File tab and then choose Options. Next, click Add-ins and then the Go button. In the Add-ins dialog box, click the Browse button and then navigate to the directory where you saved the add-in. Select the PortfolioPerformance.xll file and then click Ok. You should see that the add-in has been added to the list and that the check box next to it has a check mark. From this point on, the add-in will be loaded and the functions available to be used every time you start Excel.

Removing the Add-in

If you decide that you don't want the add-in to load when you start Excel, or that you want to unload it, simply return to the Add-ins dialog box and uncheck the box. To remove it completely, delete the PortfolioPerformance.xll file. If you delete the file, then the next time Excel starts it will give a message saying that it can't find the add-in. If you return to the Add-ins dialog box after deleting the add-in, Excel will ask if you want to remove the add-in from the list.

Usage Instructions

Using the functions contained in the add-in is very similar to using Excel's built-in functions. You can access the functions directly by typing =Function (e.g., =SharpeRatio). Alternatively, you can access the functions through the Insert Function (Shift + F3) dialog box in the Portfolio Performance category.

Updates

This free and open-source project is hosted on GitHub, and you can always download the latest version at https://github.com/mayest/. To update it, make sure that Excel is not open and then download the appropriate version of the add-in to the same folder where you had originally saved it. The next time that you start Excel, the updated version of the add-in will be used.

Available Functions and Discussion

For most of the calculations, I follow Bacon (Bacon, 2013) backed up by original papers (see references) and my own experience teaching some of this material for many years at MSU Denver. However, while I believe that the functions are accurate, please understand that I do not make any guarantees. Use them at your own risk (pun fully intended).

Most of these functions can be fairly easily calculated in Excel as can be seen in the Test Data.xlsx file, which is available on the GitHub site along with the add-in and the source code. In many cases, I make use of array formulas and the new calculation engine that was made available to Office Insiders in 2019. I also use the new Filter function for a number of calculations, which makes some calculations much easier. You will need a fairly new version of Office 365 to effectively use the spreadsheet calculations. However, the add-in should work fine in any recent version of Excel. Unfortunately, the add-in is not compatible with Office for Mac.

Note: Some of the functions in the add-in accept optional arguments. Most often this is the range of risk-free returns. In this case, you have three choices:

1. Omit the argument. If you do this, then the risk-free rate is set to 0% for each period.
2. Provide a single number. If you do this, then that number is used for the risk-free rate in each period. If you provide 0%, then this is the same as omitting the argument.
3. Provide a range of risk-free returns. This range should contain the risk-free returns for each period, and it should be of the same length as the asset returns. If the number of risk-free returns does not equal the number of asset returns, then the *first* risk-free return will be used for all periods (this is effectively the same as option 2).

I believe that the above behavior makes sense for the risk-free returns because it is often assumed that the risk-free rate is a constant. On the other hand, you may wish to treat the risk-free rate as a variable, and that is allowed in the functions.

In addition, another frequent optional argument is the data frequency. By default, this is set to 1 and the result of the function will be in per period terms (not annualized unless your returns are already annual). However, if you set it to some other value (most commonly 12 for monthly returns) then the result of the function will be annualized according to the frequency that you have set. Most often, this means annualizing a variance or standard deviation. Note that for variance and standard deviation I use an approximation – specifically that the variance scales with time (e.g., $\sigma_A^2 = 12\sigma_M^2$) and that the standard deviation, therefore, scales with the square root of time (e.g., $\sigma_A = \sqrt{12}\sigma_M$) where "time" is the number of periods per year. This is *the* commonly used assumption. However, it should be noted that several authors have pointed out that this is based on the idea that annual returns are the sum of per period returns, which is obviously incorrect. Kaplan (Kaplan, What's Wrong with Multiplying by the Square Root of Twelve, 2012/2013) gives the correct annualization for variance and standard deviation, while Weber (Weber, 2017) extends this to covariance and several other measures. Where annualized returns are called for, I use the product of one plus the per period returns, which correctly accounts for compounding.

Here is a list of the functions contained in the add-in:

Purpose	Function Name and Arguments
Sharpe Ratio	SharpeRatio(Asset Returns, Risk-Free Returns, Data Frequency)
Revised Sharpe Ratio	RevisedSharpeRatio(Asset Returns, Risk-Free Returns, Data Frequency)
Adjusted Sharpe Ratio	AdjustedSharpeRatio(Asset Returns, Risk-Free Returns, Data Frequency)
M-Squared (i.e., the Modigliani & Modigliani measure)	MSquared(Asset Returns, Market Returns, Risk-Free Returns, Data Frequency)
Roy Ratio (Safety First)	RoyRatio(Asset Returns, Risk-Free Returns, Data Frequency)

Information Ratio	InformationRatio(Asset Returns, Benchmark Returns, Data Frequency)
Treynor Index	TreynorIndex(Asset Returns, Risk-Free Returns, Asset Beta, Data Frequency)
Arithmetic Tracking Error	TrackingErrorArithmetic(Asset Returns, Benchmark Returns, Data Frequency)
Geometric Tracking Error	TrackingErrorGeometric(Asset Returns, Benchmark Returns, Data Frequency)
Beta	Beta(Asset Returns, Market Returns)
Adjusted Beta	AdjustedBeta(Asset Returns, Market Returns)
Bull Beta (beta in up markets)	BullBeta(Asset Returns, Market Returns)
Bear Beta (beta in down markets)	BearBeta(Asset Returns, Market Returns)
Beta Timing Ratio (ratio of bull beta to bear beta)	BetaTimingRatio(Asset Returns, Market Returns)
Jensen's Alpha	JensensAlpha(Asset Returns, Market Returns, Risk-Free Returns, Data Frequency)
Appraisal Ratio	AppraisalRatio(Asset Returns, Market Returns, Risk-Free Returns, Data Frequency)
Fama's Decomposition	FamaDecomposition(Asset Returns, Market Returns, Risk-Free Returns, Target Beta, Data Frequency)
Up Capture Ratio	UpCaptureRatio(Asset Returns, Benchmark Returns)
Down Capture Ratio	DownCaptureRatio(Asset Returns, Benchmark Returns)
Up Percentage Ratio	UpPercentageRatio(Asset Returns, Benchmark Returns)
Down Percentage Ratio	DownPercentageRatio(Asset Returns, Benchmark Returns)
Percentage Gain Ratio	PercentageGainRatio(Asset Returns, Benchmark Returns)
Percentage Loss Ratio	PercentageLossRatio(Asset Returns, Benchmark Returns)
Hurst Exponent	HurstExponent(Asset Returns)
Bias Ratio	BiasRatio(Asset Returns, Standard Deviations)
Market Risk	MarketRisk(Asset Returns, Benchmark Returns, Data Frequency)
Unique (Diversifiable) Risk	UniqueRisk(Asset Returns, Benchmark Returns, Data Frequency)
Lower Partial Moment	LowerPartialMoment(Asset Returns, Target Return, Degree, Data Frequency)
Upper Partial Moment	UpperPartialMoment(Asset Returns, Target Return, Degree, Data Frequency)
Semi-Variance	SemiVariance(Asset Returns, Target Return, Data Frequency)
Semi-Deviation	SemiDeviation(Asset Returns, Target Return, Data Frequency)
Omega Ratio	OmegaRatio(Asset Returns, Target Return, Degree, Data Frequency)
Kappa Index	KappaIndex(Asset Returns, Target Return, Degree, Data Frequency)
Jarque-Bera Test	JarqueBeraTest(Asset Returns)
K Ratio	KRatio(Asset Returns)
Total Return Index	TotalReturnIndex(Asset Returns, Start Value)
Maximum Drawdown	MaxDrawDown(Asset Returns)
Maximum Drawdown by Year	MaxDrawDownByYear(Asset Returns, Dates)
Average Annual Max Drawdown	AverageMaxDrawDown(Asset Returns, Dates)

Average Drawdown	AverageDrawDown(Asset Returns, Count)
Maximum Drawdown Duration	MaxDrawDownDuration(Asset Returns)
Calmar Ratio	CalmarRatio(Asset Returns, Risk-Free Returns, Data Frequency)
Sterling Ratio	SterlingRatio(Asset Returns, Risk-Free Returns, Count, Data Frequency)
Ulcer Index	UlcerIndex(Asset Returns)
Ulcer Performance Index	UlcerPerformanceIndex(Asset Returns, Risk-Free Returns, Data Frequency)
Parametric Value at Risk	ParametricVaR(Asset Returns, Confidence Level)
Modified Parametric VaR	ModifiedParametricVaR(Asset Returns, Confidence Level)
Historical Simulation VaR	HistoricalSimulationVaR(Asset Returns, Confidence Level)
Holding Period Return	HoldingPeriodReturn(Prices, Cash Flows)
HPR with Reinvestment	HPRWithReinvestment(Prices, Cash Flows)
Sub-Period Returns	SubPeriodReturns(Prices, Cash Flows)
Log Sub-Period Returns	LogSubPeriodReturns(Prices, Cash Flows)

Function Descriptions

Sharpe Ratio – SharpeRatio(Asset Returns, Risk-Free Returns, Data Frequency)

The Sharpe Ratio (Sharpe, 1966), also known as Reward to Variability, is the difference between the average portfolio return and the risk-free rate divided by the standard deviation of the portfolio returns:

$$S_p = \frac{\bar{R}_p - \bar{R}_f}{\sigma_p}$$

The function takes in a series of asset/portfolio returns and risk-free returns. As noted above, you can omit the risk-free returns, supply a single number, or supply a range. Also, it is important to note that the Sharpe Ratio (and its variants below) is often calculated using annualized returns. So, the function allows you to specify the frequency of the data. If you omit this, then the average per period returns are used (frequency is 1 by default). If you specify a frequency, then annualized returns are used and the standard deviation is also annualized.

Revised Sharpe Ratio – RevisedSharpeRatio(Asset Returns, Risk-Free Returns, Data Frequency)

The revised Sharpe Ratio (Sharpe, 1994) is identical to the original Sharpe Ratio, except that the denominator is the standard deviation of the differences between the asset/portfolio returns and the risk-free returns:

$$Revised\ S_p = \frac{\bar{R}_p - \bar{R}_f}{\sigma_{p-Rf}}$$

It is the same as the Information Ratio if the benchmark was the risk-free asset.

Adjusted Sharpe Ratio – AdjustedSharpeRatio(Asset Returns, Risk-Free Returns, Data Frequency)

This version of the Sharpe Ratio was created (Pezier, 2004) to adjust for non-normal return distributions (skewness and kurtosis). It is calculated as:

$$Adj.S_p = S_p\left[1 + \frac{\mu_3}{6} \times S_p - \frac{\mu_4 - 3}{24} \times S_p^2\right]$$

where S_p is the Sharpe Ratio, μ_3 is skewness, and μ_4 is kurtosis.

M-Squared – MSquared(Asset Returns, Market Returns, Risk-Free Returns, Data Frequency)

M-squared (Modigliani & Modigliani, 1997) is a variation of the Sharpe Ratio that presents the result as a risk-adjusted return that can be directly compared to the market/benchmark return:

$$M^2 = \left(\frac{\sigma_m}{\sigma_p}\right)\left(R_p - R_f\right) + R_f = S_p\sigma_m + R_f$$

Essentially, the portfolio is levered up or down until its risk is the same as the market return. The return of the resulting portfolio is the M-squared.

Roy Ratio – RoyRatio(Asset Returns, Risk-Free Returns, Data Frequency)

Roy's "Safety First" Ratio is the grandfather of performance measures, coming long before (Roy, 1952) the Treynor and Sharpe ratios. Essentially, this ratio is the same as the Sharpe Ratio, except that a minimum target return ("disaster level") is substituted for the risk-free rate:

$$RR_p = \frac{\bar{R}_p - \bar{R}_T}{\sigma_p}$$

Note that Roy assumed that the "disaster level" was a constant, but I'm letting it vary and treating it the same as the risk-free rate. That is, you can omit it ($\bar{R}_T = 0$), specify a single number, or specify a range.

Information Ratio – InformationRatio(Asset Returns, Benchmark Returns, Data Frequency)

The information ratio (Treynor & Black, 1973) is very similar to the revised Sharpe Ratio, except that a benchmark portfolio is substituted for the risk-free asset:

$$IR_p = \frac{\bar{R}_p - \bar{R}_b}{\sigma_{p-b}}$$

Treynor Index – TreynorIndex(Asset Returns, Risk-Free Returns, Asset Beta, Data Frequency)

The Treynor Index (Treynor, 1965) is identical to the original Sharpe Ratio, except that it uses the security's beta in the denominator instead of the standard deviation:

$$T_p = \frac{\bar{R}_p - \bar{R}_f}{\beta_p}$$

If you do not supply an asset beta, then it will be treated as if it was 0 and the function will return a #DIV/0! error.

Arithmetic Tracking Error – TrackingErrorArithmetric(Asset Returns, Benchmark Returns, Data Frequency)

The tracking error is a measure of how closely an asset/portfolio matches the returns of its benchmark. Specifically, it is the standard deviation of the differences in the asset/portfolio returns and the benchmark returns:

$$Arithmetic\ Tracking\ Error = \sigma_{p-b}$$

If the difference in these returns is constant over time, then the asset/portfolio is said to perfectly track its benchmark (the tracking error would be 0).

Geometric Tracking Error – TrackingErrorGeometric(Asset Returns, Benchmark Returns, Data Frequency)

Same as the arithmetic tracking error, except that it is the standard deviation of the geometric differences (i.e., dividing instead of subtracting).

$$Geometric\ Tracking\ Error = \sigma_{(1+p)/(1+b)-1}$$

Beta – Beta(Asset Returns, Market Returns)

Beta (β) is an index of systematic risk; that is, it measures the sensitivity of the asset/portfolio returns to market risk factors. It is best known as the risk measure used in the Capital Asset Pricing Model (CAPM). It is also the denominator of the Treynor Index and is used in Fama's Decomposition of the excess return.

Note that beta is usually calculated as the slope of the single index model regression:

$$\beta_p = \frac{\sigma_{p,m}}{\sigma_m^2} = \frac{Cov(p,m)}{Var(m)}$$

However, it is sometimes calculated by using excess returns (i.e., returns in excess of the risk-free rate). If you wish to do this, then you can subtract the range of risk-free rates from the asset and market returns in the function. This will make it an array formula. This also applies to the other beta functions.

Adjusted Beta – AdjustedBeta(Asset Returns, Market Returns)

This is the beta, adjusted for the tendency to revert to the mean of 1.00. That is, it has been noted by Blume and others that beta tends move towards 1 over time. The adjustment that I use here is to multiply beta by 2/3 and then to add 1/3:

$$\beta_{Adj} = \frac{2}{3}\beta_p + \frac{1}{3}$$

This is the same beta adjustment that was historically used by Merrill Lynch (Levy & Sarnat, 1984). This adjustment, though, is slightly different than that used in Blume (Blume, 1975), who had different adjustments for different time periods. I'm not sure where this exact adjustment comes from originally, though it is a close approximation to Blume's.

Bull Beta – BullBeta(Asset Returns, Market Returns)

This is the same as the beta, except it uses only returns from those periods when the market portfolio had a positive return. Again, note that the risk-free returns are optional.

Bear Beta – BearBeta(Asset Returns, Market Returns)

This is the same as the beta, except it uses only returns from those periods when the market portfolio had a negative return. Again, note that the risk-free returns are optional.

Beta Timing Ratio – BetaTimingRatio(Asset Returns, Market Returns)

This is the ratio of the bull beta to the bear beta. Ideally, we would like to see a portfolio manager with a beta much greater than one when the market is rising, and much less than one when it is falling. This ratio attempts to measure that ability.

$$Beta\ Timing\ Ratio = \frac{Bull\ \beta_p}{Bear\ \beta_p}$$

Note, however, that if the portfolio manager has a negative bear beta (which would indicate exceptional bear market performance) then this ratio would be negative. So, it does have the ability to be misleading.

Jensen's Alpha – JensensAlpha(Asset Returns, Market Returns, Risk-Free Returns, Data Frequency)

Jensen's alpha (α) is a per period measure of the excess return earned by the asset/portfolio, where excess means "in excess of that expected based on the CAPM."

$$Jensen's\ \alpha = \bar{R}_p - \bar{R}_f - \beta_p(\bar{R}_m - \bar{R}_f)$$

This is somewhat different from what portfolio managers usually mean when they talk about alpha. They generally mean the excess return over their benchmark portfolio, which is not a risk-adjusted return (i.e., they could have earned that excess return by taking on a lot of extra risk).

Appraisal Ratio – AppraisalRatio(Asset Returns, Market Returns, Risk-Free Returns, Data Frequency)

The appraisal ratio was defined by Treynor and Black (Treynor & Black, 1973) as a modification of Jensen's alpha. Specifically, it is Jensen's alpha per unit of unique risk (see below):

Fama's Decomposition – FamaDecomposition(Asset Returns, Market Returns, Risk-Free Returns, Target Beta, Data Frequency)

Fama's decomposition of the excess return (Fama, 1972) is one of the earliest attempts at attribution analysis. It calculates the excess return as the asset/portfolio return minus the risk-free rate, and then decomposes this into the excess return due to selectivity (same as Jensen's alpha) and the excess return due to risk (based on the beta). These can be further decomposed into manager's risk and investor's risk (if you have a target beta) and diversification and net selectivity.

This function returns an array of values that it can calculate, depending on if a target beta is given. Therefore, in older versions of Excel you will need to pre-select the output area and use the Ctrl + Shift + Enter keystroke combination to enter it. The newer Excel versions, with the new calculation engine, do not require that "magic" keystroke combination. If you wrap the function with Transpose(), then it will output in two rows, instead of two columns.

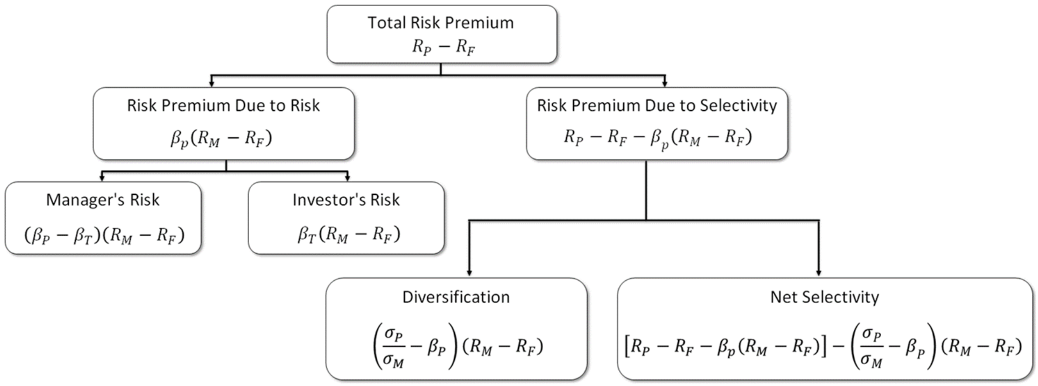

Figure 1: Fama's Decomposition of the Excess Return

Most textbooks show a graph of the security market line (SML) with the components of Fama's decomposition, but I've never felt that was all that helpful. I prefer to think of it in terms of the hierarchy (org chart) shown in Figure 1.

In addition to the outputs shown in the figure, the function also reports the hypothetical beta, which is what the beta would be if the portfolio was perfectly diversified (i.e., perfectly correlated with the market portfolio). Note that:

$$Total\ Risk = \sigma_p^2 = \beta_p^2 \sigma_m^2 + \sigma_e^2$$

If the portfolio is perfectly diversified, then the company-specific risk will be 0, so we have:

$$\sigma_p^2 = \beta_p^2 \sigma_m^2$$

and solving for β_p we get:

$$\beta_p = \sqrt{\frac{\sigma_p^2}{\sigma_m^2}} = \frac{\sigma_p}{\sigma_m} = Hypothetical\ Beta$$

We can insert the hypothetical beta into the CAPM to get the return that the portfolio would have earned if it was perfectly diversified. If we then subtract the risk-free rate, we get the hypothetical risk premium, which can be directly compared to the actual risk premium.

Up Capture Ratio – UpCaptureRatio(Asset Returns, Benchmark Returns)

This function calculates the average returns of the asset/portfolio and benchmark when the benchmark has a positive return, and then computes the ratio:

$$Up\ Capture\ Ratio = \frac{\bar{R}_p^+}{\bar{R}_b^+}$$

Obviously, we would like to see the up capture ratio be greater than 1. A value less than 1 would indicate that the manager is underperforming in up markets.

Down Capture Ratio – DownCaptureRatio(Asset Returns, Benchmark Returns)

This function calculates the average returns of the asset/portfolio and benchmark when the benchmark has a negative return, and then computes the ratio:

$$Down\ Capture\ Ratio = \frac{\bar{R}_p^-}{\bar{R}_b^-}$$

Obviously, we would like to see the down capture ratio be less than 1. A value greater than 1 would indicate that the manager is underperforming in down markets.

Up Percentage Ratio – UpPercentageRatio(Asset Returns, Benchmark Returns)

This function returns the percentage of the time that the manager outperforms the benchmark in up markets. Ideally, this would be 100%, though that is unlikely. For example, say that the benchmark was up in 25 of the last 36 months, and that in 20 of those 25 months the portfolio outperformed the benchmark. This would give an up percentage ratio of 80% (= 20/25).

Down Percentage Ratio – DownPercentageRatio(Asset Returns, Benchmark Returns)

This function returns the percentage of the time that the manager outperforms the benchmark in down markets. Ideally, this would be 100%, though that is unlikely. For example, say that the benchmark was down in 12 of the last 36 months, and that in 8 of those 12 months the portfolio outperformed the benchmark. This would give a down percentage ratio of 66.67% (= 8/12).

Percentage Gain Ratio – PercentageGainRatio(Asset Returns, Benchmark Returns)

This compares the number of positive asset returns to the number of positive benchmark returns, without regard for when they occur. For example, assume that during the evaluation period the portfolio had a positive return in 18 months and the benchmark was positive for 15 months. This ratio would be 1.20 (= 18/15). Again, the positive returns of the asset and benchmark do not have to coincide. Higher numbers are better.

Percentage Loss Ratio – PercentageLossRatio(Asset Returns, Benchmark Returns)

This compares the number of negative asset returns to the number of negative benchmark returns, without regard for when they occur. For example, assume that during the evaluation period the portfolio had a negative return in 6 months and the benchmark was down for 7 months. This ratio would be 0.86 (= 6/7). Again, the negative returns of the asset and benchmark do not have to coincide. Lower numbers are better.

Hurst Exponent – HurstExponent(Asset Returns)

Calculates the Hurst Exponent, which indicates how predictable a time series is. According to Qian and Rasheed (Qian & Rasheed, 2004) values near 0.5 indicate randomness, values less than 0.5 indicate mean-reversion, and values greater than 0.5 indicate a strongly trending series. The range can be from 0 to 1. We would expect most asset return series to have $H \cong 0.5$ if markets are reasonably efficient. Most importantly for our purposes, a portfolio manager exhibiting skill would have H well above 0.5. The relevant formula is:

$$H = \frac{\ln\left(\frac{Range\ of\ Cumulative\ Deviations\ from\ Mean}{Standard\ Deviation\ of\ Returns}\right)}{\ln(Number\ of\ Observations)}$$

Bias Ratio – Bias Ratio(Asset Returns, Standard Deviations)

The bias ratio (Abdulali, 2006) attempts to identify smoothing of returns. It is said to be particularly useful in identifying smoothing of returns by hedge funds that trade a lot of illiquid (i.e., hard to value due to infrequent trades) assets. Values near 1.0 to 1.5 indicate a lack of smoothing, while values well-above 1.0 (say, above 2.5) are indicative of smoothing and possible fraudulent valuation. The bias ratio compares returns between 0% and one standard deviation above 0% to those that are between one standard deviation below 0% up to, but not including 0%:

$$Bias\ Ratio = \frac{Count\left(R_p^+\right)}{Count\left(R_p^-\right)}$$

where R_p^+ are those returns in the range $[0\%, +1\sigma]$ and R_p^- are the returns in the range $[-1\sigma, 0\%)$.

Market Risk – MarketRisk(Asset Returns, Benchmark Returns, Data Frequency)

Sometimes, incorrectly, people refer to beta as systematic (or market) risk. Actually, beta is an index of systematic risk. That is, it is a relative measure of systematic risk. True systematic risk can be determined by decomposing the total risk (σ_p^2) into its component parts:

$$\sigma_p^2 = \beta_p^2\sigma_m^2 + \sigma_e^2 = Market\ Risk + Unique\ Risk$$

Note that market risk is given by $\beta_p^2\sigma_m^2$. This value is what is returned by this function.

Unique (Diversifiable) Risk – MarketRisk(Asset Returns, Benchmark Returns, Data Frequency)

The unique (or company-specific, or diversifiable) risk is the risk of a company that is not explained by variation in the market portfolio. This is the type of risk that can be diversified away when securities are combined into a portfolio. As shown above, unique risk is given by σ_e^2. Note that Bacon (Bacon, 2013) refers to this value as "specific risk," though he uses the square root of this number.

Lower Partial Moment – LowerPartialMoment(Asset Returns, Target Return, Degree, Data Frequency)

Calculates the lower partial moment of Fishburn (Fishburn, 1977). This is a family of downside risk measures, where the target return and degree can be specified. Note that the most common usage of this would be where the target return is the mean return and the degree is 2 (this will give the semi-variance). Degree can be any number greater than zero and represents the tolerance for risk below the target of the investor (larger is more risk averse). The function calculates:

$$Lower\ Partial\ Moment = \frac{1}{N}\sum_{i=1}^{N}\left(R_{Target} - R_i\right)^{Degree}\ \forall\ R_i < R_{Target}$$

Note that when $Degree = 0$, this is the probability of the return being less than the target return, and when $Degree = 1$, this is the expected shortfall below the target return. As noted above, if $Degree = 2$, this is the semi-variance.

Upper Partial Moment – UpperPartialMoment(Asset Returns, Target Return, Degree, Data Frequency)

This is the same as the lower partial moment function, except that it looks at the upper side of the distribution:

$$Upper\ Partial\ Moment = \frac{1}{N}\sum_{i=1}^{N}\left(R_i - R_{Target}\right)^{Degree} \ \forall\, R_i > R_{Target}$$

Semi-Variance – SemiVariance(Asset Returns, Target Return, Data Frequency)

This is a measure of downside risk. It is the variance of the returns that are below the target return, where, by default, the target return is the mean of all of the returns:

$$Semivariance = \frac{1}{N}\sum_{i=1}^{N} Max\left[\left(R_{Target} - R_i\right), 0\right]^2$$

There seems to be some controversy as to whether N should be the total number of returns, or the number of returns less than the mean. I follow Nawrocki's (Nawrocki, 1999) calculation of the 2nd degree lower partial moment, so I'm using the total number of returns in the denominator.

Semi-Deviation – SemiDeviation(Asset Returns, Target Return, Data Frequency)

This is the square root of the semi-variance.

Omega Ratio – OmegaRatio(Asset Returns, Target Return, Degree, Data Frequency)

Originally created by Keating and Shadwick (Keating & Shadwick, 2002), the Omega ratio compares the upside potential to the downside potential (using Bacon's terminology). This is the ratio of the first-degree upper partial moment above the Target Return, to the first-degree lower partial moment below the Target Return and thus accounts for both sides of the return distribution:

$$Omega\ Ratio = \Omega_p = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(R_i - R_{Target}\right)^1 \ \forall\, R_i > R_{Target}}{\frac{1}{N}\sum_{i=1}^{N}\left(R_{Target} - R_i\right)^1 \ \forall\, R_i < R_{Target}}$$

This is a convenience function that uses the UpperPartialMoment and LowerPartialMoment functions described above with $Degree = 1$. Because Omega looks at the entire distribution, it naturally accounts for skewness and kurtosis, so it is useful when returns are not normally distributed. Higher values are better.

Kappa Index – KappaIndex(Asset Returns, Target Return, Degree, Data Frequency)

Kappa (Kaplan & Knowles, 2004) is a generalized performance measure utilizing any lower partial moment (specified by Degree) as the measure of risk. As a result, Kappa with Degree = 2 is equivalent to the Sortino Ratio, and Kappa with Degree = 1 is related to the Omega Ratio ($\Omega_p = \mathrm{K}_1 + 1$). It is calculated as:

$$\mathrm{K}_{Degree} = \frac{\bar{R}_p - \bar{R}_{Target}}{\sqrt[Degree]{\frac{1}{N}\sum_{i=1}^{N}\left(R_{Target} - R_i\right)^{Degree} \ \forall\, R_i < R_{Target}}} = \frac{\bar{R}_p - \bar{R}_{Target}}{\sqrt[Degree]{LPM_{Degree}}}$$

Jarque-Bera Test – JarqueBeraTest(Asset Returns)

Performs the Jarque-Bera test for normality of the returns and returns the test statistic. This is a test of the joint hypotheses that both skewness and excess kurtosis are equal to zero, which is a feature of the normal distribution. However, other distributions may also have skewness and excess kurtosis equal to zero, so this a small value of this test does not necessarily indicate that the data are normally distributed or not (e.g., see https://stats.stackexchange.com/questions/62291/can-one-measure-the-degree-of-empirical-data-being-gaussian/62320#62320). For asset returns, which typically have a reasonably symmetrical and unimodal distribution, a large value of this test would be indicative that the data do not come from a normal distribution.

K Ratio – KRatio(Asset Returns)

This ratio, created by Kestner (Kestner L. , 2013) measures the consistency of an asset/portfolio return series. Also known as the Zephyr K-Ratio, this is the ratio of the slope of the cumulative returns to the standard deviation of the error terms of that slope:

$$K\ Ratio = \frac{Slope\ of\ Cumulative\ Returns}{Standard\ Error\ of\ Slope\ of\ Cumulative\ Returns}$$

Total Return Index – TotalReturnIndex(Asset Returns, Start Value)

This function returns an array of values that show the growth of a portfolio of Start Value dollars over time. If the optional Start Value is omitted, then the function uses 1 as the Start Value.

Maximum Drawdown – MaxDrawDown(Asset Returns)

This is the maximum percentage loss experienced from a peak value. This is measured from the peak to the deepest trough prior to reaching the next peak.

Maximum Drawdown by Year – MaxDrawDownByYear(Asset Returns, Dates)

Given a range of dates and returns, this function returns an array containing the maximum drawdown for each year. The output will be a column array containing the year and the maximum drawdown for that year. This is probably best used for full years, but there is no such limitation. Note that the data returned may not contain the overall maximum drawdown because drawdowns can extend across calendar years.

Average Annual Max Drawdown – AverageMaxDrawDown(Asset Returns, Dates)

This is a convenience function that calculates the average of the annual maximum drawdowns. This could be replicated by wrapping MaxDrawDownByYear in the Average function.

Average Drawdown – AverageDrawDown(Asset Returns, Count)

This is the average of the drawdowns during the period covered by the asset returns. Note that a drawdown is a peak to trough measurement, so this function does not average all of the negative returns – it averages the percentage drawdowns.

The Count argument is optional. If it is omitted, then this function returns the average of all maximum losses during drawdown periods. If Count is specified, then the function returns the average loss of the Count largest drawdowns. For example, assume that there were 10 drawdown periods in a return series. If Count = 5, then this function would return the average of the largest five drawdowns. If it was omitted, then the function would return the average of all 10 drawdowns. If Count is greater than the number of drawdowns, then the function will return the average of all drawdowns.

Maximum Drawdown Duration – MaxDrawDownDuration(Asset Returns)

This function returns the longest number of periods from one peak to the next higher peak. This is not necessarily the length of time spent during the maximum drawdown phase as there can be several peaks after the maximum drawdown trough that do not surpass the previous peak.

Calmar Ratio – CalmarRatio(Asset Returns, Risk-Free Returns, Data Frequency)

Similar to the Sharpe ratio, except that the risk measure is the maximum drawdown:

$$C_p = \frac{\bar{R}_p - \bar{R}_f}{Max\ Drawdown}$$

Sterling Ratio – SterlingRatio(Asset Returns, Risk-Free Returns, Count, Data Frequency)

Same as the Calmar ratio (see above), except that it uses the absolute value of the average drawdown in the denominator. Note that the optional Count argument determines how many drawdowns are included in the average. If Count is omitted or exceeds the number of drawdowns, all are used.

$$SR_p = \frac{\bar{R}_p - \bar{R}_f}{|Avg\ Drawdown|}$$

Ulcer Index – UlcerIndex(Asset Returns)

This function calculates Martin's Ulcer index (Martin, 2011). The calculation is:

$$Ulcer\ Index = \sqrt{\frac{1}{N}\sum_{i=1}^{N} Drawdown_i^2}$$

where N is the number of drawdowns (length of the series + 1). Note that my result differs slightly from Bacon (but matches Martin exactly) because Bacon ignores the initial drawdown of 0.

Ulcer Performance Index – UlcerPerformanceIndex(Asset Returns, Risk-Free Returns, Data Frequency)

The Ulcer Performance Index is similar to the Sharpe Ratio, except that the risk measure in the denominator is the Ulcer Index:

$$UPI_p = \frac{\bar{R}_p - \bar{R}_f}{\sqrt{\frac{1}{N}\sum_{i=1}^{N} Drawdown_i^2}}$$

Parametric Value at Risk – ParametricVaR(Asset Returns, Confidence Level)

Calculates *historical* Value at Risk (VaR) assuming that the returns are normally distributed:

$$Parametric\ VaR_p = \frac{1}{N}\sum_{t=1}^{N} R_t + \mathcal{Z}_\alpha \times \sigma_p$$

where $\mathcal{Z}_\alpha$ is the value of the inverse of the normal CDF at $\alpha = 1 - confidence\ level$. Most commonly, the confidence level will be 95% ($\mathcal{Z}_\alpha \cong -1.65$) or 99% ($\mathcal{Z}_\alpha \cong -2.33$). This approach is also called the variance/covariance method. This function should not be considered a forecast of potential future VaR, except under the very restrictive assumption that future returns will mirror those of the past.

Modified Parametric Value at Risk – ModifiedParametricVaR(Asset Returns, Confidence Level)

Introduced by Favre and Galeano (Favre & Galeano, 2002), this is the same as parametric VaR, except that it is adjusted to account for skewness and kurtosis. Note that Cavenaile and Lejeune (Cavenaile & Lejeune, 2012) have warned that this statistic is inconsistent with investor preferences when the confidence level is less than 95.84%. The calculation is:

$$MVaR_p = \frac{1}{N}\sum_{t=1}^{N} R_t + \mathcal{Z}_{CF,\alpha}\sigma_p$$

where $\mathcal{Z}_{CF,\alpha}$ is the Cornish Fisher approximation of the % quantile of the distribution:

$$\mathcal{Z}_{CF,\alpha} = \mathcal{Z}_\alpha + \frac{1}{6}(\mathcal{Z}_\alpha^2 - 1)S + \frac{1}{24}(\mathcal{Z}_\alpha^3 - 3\mathcal{Z}_\alpha)K - \frac{1}{36}(2\mathcal{Z}_\alpha^3 - 5)S^2$$

with S as the skewness and K as the *excess* kurtosis.

Historical Simulation VaR – HistoricalSimulationVaR(Asset Returns, Confidence Level)

Calculates *historical* Value at Risk using the historical simulation method. Essentially, this method sorts the returns and then returns the value at the 1 – Confidence Level position. For example, if you have 100 returns and ask for the 95% VaR, this function will return the value in the 5^{th} position after sorting the data in ascending order. The function replicates what you could do using Percentile.Exc(Asset Returns, 1 – Confidence Level) in Excel. If the exact position doesn't exist, then the function interpolates between the two surrounding positions. If the interpolation can't be carried out (usually because the return series isn't long enough) then the function will return a #NUM! error. This is the same behavior as the Percentile.Exc function in Excel.

Holding Period Return – HoldingPeriodReturn(Prices, Cash Flows)

Calculates the holding period return without reinvestment of the cash flows. This is typically abbreviated as HPR:

$$HPR = \frac{Price_N + \sum_{t=1}^{N} Cash\ flow_t}{Price_0} - 1$$

Cash Flows is optional. If omitted, it is assumed that all cash flows are 0. Note that this function does not annualize returns because it is calculating the return for the specific holding period. The user can easily annualize the result if desired.

HPR with Reinvestment – HPRWithReinvestment (Prices, Cash Flows)

Calculates the holding period return assuming that the cash flows are reinvested. This function calculates the HPR for each subperiod and then geometrically links them. Note that the Cash Flows are optional, and the length of the Cash Flows range can differ from the length of the Prices. If that is the case, then the function assumes that the first cash flow occurs at the same time as the second price, and so on (the first price is the price at the beginning):

$$HPR_{Reinvestment} = \prod_{t=1}^{N}(1 + HPR_t) - 1$$

Note that this function does not annualize returns because it is calculating the return for the specific holding period. The user can easily annualize the result if desired.

Sub-Period Returns – SubPeriodReturns(Prices, Cash Flows)

This function returns a column array that contains the holding period returns (see the HPRWithReinvestment function above) for each sub-period. Note that the number of cash flows must be less than or equal to the number of prices. If the number of cash flows is less than the number of prices, then the first cash flow is assumed to be matched to the second price, and so on. The first price is assumed to be the starting price (period 0). The function will return a #VALUE! error if the number of cash flows exceeds the number of prices.

Log Sub-Period Returns – LogSubPeriodReturns(Prices, Cash Flows)

This function is identical to the SubPeriodReturns function, except that it returns the log-price relatives (i.e., the natural log of 1 + return).

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