Topics in Applied Operational Research
Mathematical Model for the SoM Course Timetabling Problem

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Here we highlight the mathematical model that we have designed as per some requirements of the School of Mathematics for timetabling courses. We have included constraints for which data can be/has been obtained. The objective of our model is to minimize clashes between classes of courses that belong to the same categorization. These categorizations include Statistics, Operational Research, Algebra, Mathematical Physics, Analysis, Applied & Computational, etc.

Parameters

$\mathcal{C}$: Set of courses to be timetabled.

$\mathcal{D} = \{1, 2, 3, 4, 5\}$: Set of days (from Monday to Friday).

$\mathcal{T} = \{1, 2, \ldots, 9\}$: Set of time periods in a day, given by '9:00-10:00', '10:00-11:00', ..., '17:00-18:00'.

$\mathcal{R}_{\text{lec}}$: Set of rooms used for lectures.

$\mathcal{R}_{\text{lab}}$: Set of rooms used for computer labs.

$\mathcal{R}_{\text{ws}}$: Set of rooms used for workshops.

$\mathcal{R} = \mathcal{R}_{\text{lec}} \cup \mathcal{R}_{\text{ws}} \cup \mathcal{R}_{\text{lab}}$: Set of all rooms.

s_c: course size for course $c \in C$.

s_c^{lec}: lecture size for course $c \in C$.

s_c^{ws}: workshop group size for course $c \in C$.

s_c^{lab}: computer lab group size for course $c \in C$.

m_r: capacity of room $r \in \mathcal{R}$.

n_c^{lec}: number of lectures for course c per week.

n_c^{ws}: number of workshops for course c per week.

n_c^{lab}: number of computer labs for course c per week.

w_c: number of workshop groups for course c.

l_c: number of computer lab groups for course c.

C_G: sets of compulsory course groupings. e.g., Year 1 compulsory courses for Bsc (Hons) Mathematics, Year 2 compulsory courses, MSc Computational Applied Mathematics compulsory courses, etc.

U: Set of categorized course groupings, e.g., set of Statistics courses, set of Algebra courses, set of Analysis courses, etc.

Decision Variables

We have three binary decision variables that to denote whether a lecture/workshop/computer workshop has been assigned to a room on a day at a time period or not.

$$x_{cdtr} = \begin{cases} 1, & \text{if a lecture for course } c \in \mathcal{C} \text{ is assigned to time } t \in \mathcal{T} \text{ in room } r \in \mathcal{R} \text{ on day } d \in \mathcal{D} \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{cdtr} = \begin{cases} 1, & \text{if a workshop for course } c \in \mathcal{C} \text{ is assigned to time } t \in \mathcal{T} \text{ in room } r \in \mathcal{R} \text{ on day } d \in \mathcal{D} \\ 0, & \text{otherwise.} \end{cases}$$

$$z_{cdtr} = \begin{cases} 1, & \text{if a computer workshop for course } c \in \mathcal{C} \text{ is assigned to time period } t \in \mathcal{T} \text{ in room } r \in \mathcal{R} \text{ on day } d \in \mathcal{D} \\ 0, & \text{otherwise.} \end{cases}$$

Constraints

- Two classes cannot take places in the same room at the same time.

$$\sum_{c \in \mathcal{C}} (x_{cdtr} + y_{cdtr} + z_{cdtr}) \leq 1 \quad \forall t \in \mathcal{T}, r \in \mathcal{R}, d \in \mathcal{D}$$

- Classes of each course must be assigned to distinct periods.

$$\sum_{r \in \mathcal{R}} (x_{cdtr} + y_{cdtr} + z_{cdtr}) \leq 1 \quad \forall t \in \mathcal{T}, c \in \mathcal{C}, d \in \mathcal{D}$$

- Classes must be assigned to appropriate rooms. In particular, do not assign a non-lecture room to a lecture and do not assign a non-computer lab to a computer workshop.

$$\sum_{r \in \mathcal{R} \setminus \mathcal{R}_{\text{lec}}} \sum_{c \in \mathcal{C}} \sum_{d \in \mathcal{D}} \sum_{t \in \mathcal{T}} x_{cdtr} = 0$$
$$\sum_{r \in \mathcal{R} \setminus \mathcal{R}_{\text{lab}}} \sum_{c \in \mathcal{C}} \sum_{d \in \mathcal{D}} \sum_{t \in \mathcal{T}} z_{cdtr} = 0$$

- Class sizes should not exceed room capacity for lectures/workshops/computer labs.

$$x_{cdtr} \cdot s_c^{\text{lec}} \leq m_r \quad \forall d \in \mathcal{D}, t \in \mathcal{T}, c \in \mathcal{C}, r \in \mathcal{R}$$
$$y_{cdtr} \cdot s_c^{\text{ws}} \leq m_r \quad \forall d \in \mathcal{D}, t \in \mathcal{T}, c \in \mathcal{C}, r \in \mathcal{R}$$
$$z_{cdtr} \cdot s_c^{\text{lec}} \leq m_r \quad \forall d \in \mathcal{D}, t \in \mathcal{T}, c \in \mathcal{C}, r \in \mathcal{R}$$

- Every lecture must have the required number of lectures/workshops/computer labs.

$$\sum_{d \in \mathcal{D}} \sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}} x_{cdtr} = n_c^{\text{lec}}$$
$$\sum_{d \in \mathcal{D}} \sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}} y_{cdtr} = n_c^{\text{ws}} \cdot w_c$$
$$\sum_{d \in \mathcal{D}} \sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}} z_{cdtr} = n_c^{\text{lab}} \cdot l_c$$

- Lectures of two different courses belonging to the same groups of compulsory courses should not clash.

$$\sum_{c \in C_{G_i}} \sum_{r \in \mathcal{R}} x_{cdtr} \leq 1 \quad \forall C_{G_i} \in C_G, d \in \mathcal{D}, t \in \mathcal{T}$$

Objective Function

The objective is to minimize clashes between classes (lectures, workshops and computer labs) of courses within the same categorization of courses. This is a soft constraint for each category of courses. We introduce a weight λ_i for each category $i \in U$ which denotes the importance of violations associated with class clashes between courses within category $i \in U$. The soft constraint is given by

$$\sum_{c \in U_i} \sum_{r \in R} (x_{cdtr} + y_{cdtr} + z_{cdtr}) \leq 1 \quad \forall U_i \in U, \forall t \in \mathcal{T}, d \in \mathcal{D}$$

Thus, the objective function for the problem is given by

$$\sum_{i=1}^{|U|} \lambda_i \left(1 - \sum_{c \in U_i} \sum_{r \in R} (x_{cdtr} + y_{cdtr} + z_{cdtr}) \right)$$