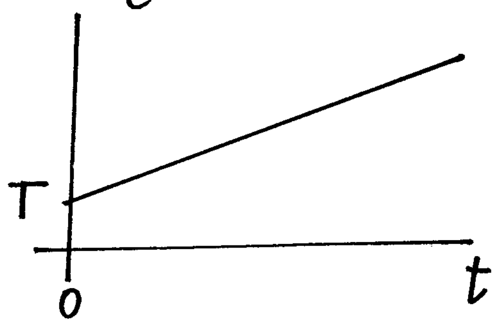
Consider the system

$$x(t) \longrightarrow \boxed{\begin{matrix}\text{Delay} \\ \tau(t)\end{matrix}} \longrightarrow y(t) = x(t - \tau(t))$$

where $\tau(t) = T + \alpha t$




Then
$$\begin{aligned} y(t) &= x(t - \tau(t)) \\ &= x(t - T - \alpha t) \\ &= x((1-\alpha)t - T) \end{aligned}$$

This is a temporal scaling of the signal $x(t)$. It changes the frequencies of the signal $x(t)$.

A sinusoidal signal $x(t)$ will produce a sinusoidal output signal $y(t)$ but $y(t)$ will have a different frequency.

If $x(t) = \cos(\omega_0 t)$

Then $y(t) = x(t - \tau(t))$ $\qquad \tau(t) = T + \alpha t$

$$= x((1-\alpha)t - T)$$

$$= \cos(\omega_0((1-\alpha)t - T))$$

$$= \cos(\omega_1 t - \theta)$$

where $\omega_1 = (1-\alpha)\omega_0$

$\theta = \omega_0 T.$

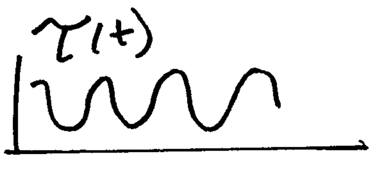
The input freq. $\omega_0$ is multiplied by $(1-\alpha)$.

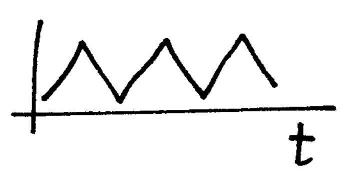
NOTE: $\tau(t) = T + \alpha t$ is not realizable.

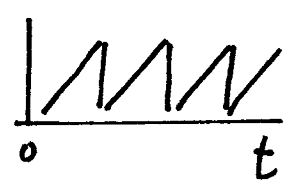
If $\alpha > 0$, the system needs past signal values arbitrarily far in the past. The system can not be implemented with finite memory.

If $\alpha < 0$, the system will eventually be non-causal.

To be realizable, $\tau(t)$ must be positive and bounded.

E.g. 

or 

or 

# INSTANTANEOUS FREQUENCY

If $x(t) = \sin(\phi(t))$

then the instantaneous frequency (i.f.) of $x(t)$ is given by

$$\text{i.f.}(t) = \frac{1}{2\pi} \frac{d}{dt} \phi(t)$$

① For example, if $x(t) = \sin(2\pi f_0 t)$

then $\phi(t) = 2\pi f_0 t$

$$\phi'(t) = 2\pi f_0$$

$$\frac{1}{2\pi} \phi'(t) = f_0$$ AS EXPECTED.

② $x(t) \rightarrow \boxed{\begin{array}{c}\text{Delay}\\ \tau(t)\end{array}} \rightarrow y(t) = x(t - \tau(t))$

If $x(t) = \sin(2\pi f_0 t)$

then $y(t) = \sin(2\pi f_0 (t - \tau(t)))$

and $\phi_y(t) = 2\pi f_0 (t - \tau(t))$

$$\phi_y'(t) = 2\pi f_0 (1 - \tau'(t))$$

$$\frac{1}{2\pi} \phi_y'(t) = f_0 - f_0 \tau'(t)$$

i.f. of $y(t)$ is

$$\frac{1}{2\pi}\phi_y'(t) = f_0 - f_0\tau'(t)$$

If $\tau(t) = T + W\sin(2\pi f_1 t)$

then $\tau'(t) = W\,2\pi f_1 \cos(2\pi f_1 t)$

and $\frac{1}{2\pi}\phi_y'(t) = f_0 - A\cos(2\pi f_1 t)$

The instantaneous frequency of $y(t)$ oscillates around $f_0$.

This explains what we perceive in the <u>vibrato</u> effect.