The list of statements to prove
Algebra,
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CS NRU HSE, 2023/2024, DSBA

1. There is at most one neutral element for a binary operation. Claim 11.
2. There is at most one inverse for an associative binary operation. Claim 13.
3. Properties of powers in a group. Claim 21.
4. Classification of cyclic groups. Claim 26.
5. Subgroups of the group $\mathbb{Z}$. Claim 27.
6. Subgroups of the group $\mathbb{Z}_n$. Claim 28.
7. Equivalent definitions of a normal subgroup. Claim 33.
8. Relation between cosets of a group. Claim 34.
9. Number of elements in a coset. Claim 36.
10. Formulas for the number of cosets. Claim 37.
11. The relation between the order of an element the order of a group. Corollary 2 of Claim 39.
12. A group of a prime order. Corollary 4 of Claim 39.
13. The Fermat Little Theorem. Corollary 5 of Claim 39.
14. A homomorphism of groups preserves the identity and the inverses. Claim 43.
15. Properties of the kernel of a group homomorphism. Claim 47 items 2 and 4.
16. Properties of the image of a group homomorphism. Claim 47 items 1 and 3.
17. The Additive Chinese Remainder Theorem for integers. Claim 52.
18. The Multiplicative Chinese Remainder Theorem for integers. Claim 56.
19. Ideals of the ring $\mathbb{Z}$. Claim 68.
20. Ideals of the ring $\mathbb{Z}_n$. Claim 69.
21. Properties of the kernel of a ring homomorphism. Claim 75 items 2 and 4.
22. Properties of the image of a ring homomorphism. Claim 75 items 1 and 3.
23. Ideals of the polynomial ring in one variable. Claim 82.
24. Expression of gcd as a linear combination of given polynomials. Claim 83 item 1.
25. Ideals of a ring of polynomial remainders. Claim 91.
26. The Chinese Remainder Theorem for the ring of polynomial remainders. Claim 92.
27. Options for the characteristic of a field. Claim 95.
28. When a ring of integer remainders is a field. Claim 98.
29. Number of elements of a finite field. Claim 103.
30. Structure of the multiplicative group of a finite field. Claim 104.
31. Property of a descending chain of monomials. Claim 111.
32. Prove that a reduction process in a polynomial ring in several variables terminates. Claim 122.
33. S-polynomial in case of coprime leading monomials. Claim 127.
34. Describe the Buchberger algorithm and prove its correctness (without halting). Section 8.5.
35. The Diamond Lemma. Claim 132.
36. The Buchberger Criterion. Claim 134.