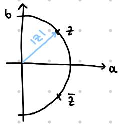
NUMPROG 2SMF. TEIL 3

Komplexe Zahlen wdh

Fourier-Transformation : DFT , FFT

WDH zu **KOMPLEXE ZAHLEN**

kann man nicht oft genug wiederholen lol



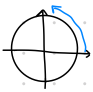
$z = a + ib$

$\bar{z} = a - ib$ (konjugiert (bzw. an a-Achse gespiegelt))

$|z| = \sqrt{a^2+b^2}$ Betrag (Länge des Vektors)

$e^{it} = \underset{\substack{\uparrow \\ a}}{\cos(t)} + i \cdot \underset{\substack{\uparrow \\ b}}{\sin(t)}$

Mathematische Rotation ist gegen den Uhrzeigersinn



$e^{i \cdot 0} = \underbrace{\cos(0)}_{=1} + i \cdot \underbrace{\sin(0)}_{=0} = 1$

$e^{i\frac{\pi}{2}} = \underbrace{\cos(\tfrac{\pi}{2})}_{0} + i \cdot \underbrace{\sin(\tfrac{\pi}{2})}_{1} = i$

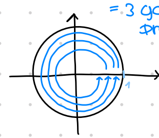
BSP: Wie viele Lösungen hat $\omega^3 = 1$ in...

a) $\omega \in \mathbb{R}$?

b) $\omega \in \mathbb{C}$?

„3 Drehungen um wieder auf die 1 zu kommen"

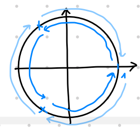
a) 1 Lösung: $1 \times 1 \times 1 = 1$



b) 3 Lösungen:

1. $1 \times 1 \times 1 = 1$ (wie in $\mathbb{R}$)
2. $e^{\frac{2}{3}\pi \cdot i}$ => $\frac{1}{3}$ Drehung gegen Uhrzeigersinn
3. $e^{-\frac{2}{3}\pi \cdot i}$ => $\frac{1}{3}$ Drehung im Uhrzeigersinn

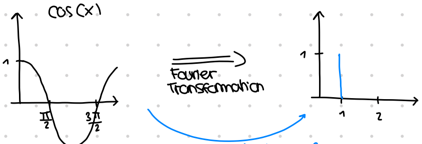
Darstellung als NST: $(x-1)(x-e^{\frac{2}{3}\pi \cdot i})(x-e^{-\frac{2}{3}\pi \cdot i})$



FourierTransformation

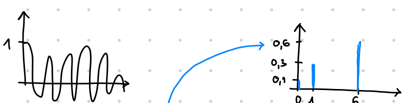
some pictures to get the idea :)

VISUALISIERUNG



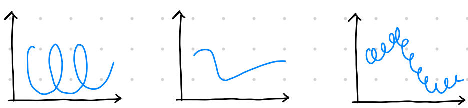
Signale in Frequenzen zerlegen

$0{,}5\cos(x) + 0{,}5\cos(2x)$

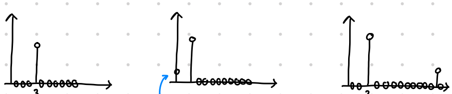


$0{,}1 + 0{,}3\cos(x) + 0{,}6\cos(6x)$

Weitere Beispiele:



⇩ FOURIER-TRANSFORMATION ⇩



große Frequenz an 3 => obere Kurve dreht sich 3x

Nullversatz

2 große Drehungen

Drehungen ccccccc

DISKRETE FOURIER-TRANSFORMATION DFT

$$c_k = (DFT(v))_k := \frac{1}{n} \sum_{j=0}^{n-1} v_j \cdot \bar{\omega}^{jk} \qquad k = 0,1,\dots,n-1$$

sehr ugly besser als Matrix

$$\begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{n-1} \end{pmatrix} = c = M_{DTF} \cdot v = \boxed{\frac{1}{n} \cdot \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \bar{\omega}^1 & \bar{\omega}^2 & \dots & \bar{\omega}^{(n-1)} \\ \vdots & \vdots & \vdots & & \\ 1 & \bar{\omega}^{n-1} & \bar{\omega}^{2(n-1)} & \dots & \bar{\omega}^{(n-1)(n-1)} \end{pmatrix} \cdot \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ v_{n-1} \end{pmatrix}}$$

„That's the important stuff!"

BSP: Berechne $DTF((\overset{v_0}{1}\ \overset{v_1}{0}\ \overset{v_2}{-1})^T)$

$$\begin{pmatrix} c_0 \\ c_1 \\ c_2 \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & \bar{\omega}_3^1 & \bar{\omega}_3^2 \\ 1 & \bar{\omega}_3^2 & \bar{\omega}_3^{\cancel{4}\,1} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 0 \\ 1-\bar{\omega}_3^2 \\ 1-\bar{\omega}_3^{\cancel{4}\,1} \end{pmatrix}$$

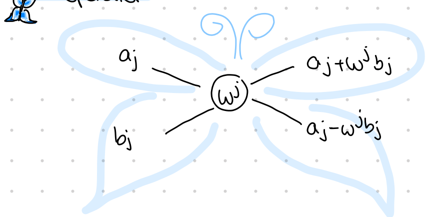
$\bar{\omega}_3^4 = \bar{\omega}_3^1$

mit: $1 - \bar{\omega}_3^2 = 1 - \cos(\frac{2\pi}{3}) - i \cdot \sin(\frac{2\pi}{3}) = 1 - \frac{1}{2} \cdot i \frac{\sqrt{3}}{2}$

FFT: Schnelle Fourier Transformation

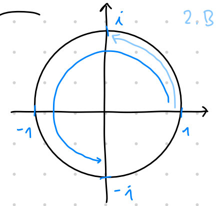
FFT : fast Fourier transformation

Butterfly-operator



$\omega_4^0 = 1$
$\omega_4^1 = i$
$\omega_4^2 = -1$
$\omega_4^3 = -i$

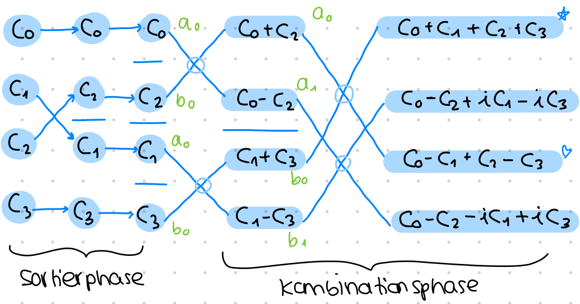
explaination for this :)



z.B. $\omega_4^1 = \frac{1}{4}$ Drehung $= i$

$\omega_4^3 = \frac{3}{4}$ Drehung $= -i$

$\omega_4^4 = \frac{4}{4}$ Drehung $= 1$



Umrechnen:

$a_0 = C_0$, $b_0 = C_2$, $\omega^{j=0}$: $C_0 + C_2$, $C_0 - C_2$

$a_0 = C_1$, $b_0 = C_3$, $\omega^{j=0}$: $C_1 + C_3$, $C_1 - C_3$

$a_0 = C_0 + C_2$, $b_0 = C_1 + C_3$, $\omega^{j=0}$:

$C_0 + C_2 + \underbrace{\omega^0}_{=1}(C_1 + C_3) = C_0 + C_1 + C_2 + C_3$

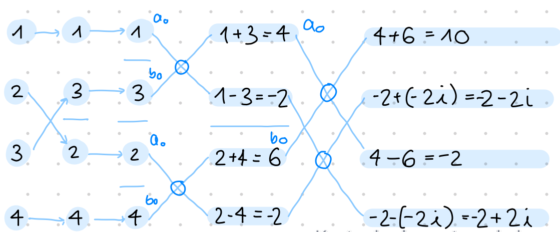
$C_0 + C_2 - \omega^0 (C_1 + C_2) = C_0 - C_1 + C_2 - C_3$

$a_1 = C_0 - C_2$, $b_1 = C_1 - C_3$, $\omega^{j=1}$:

$C_0 + C_2 + \underbrace{\omega^1}_{=i}(C_1 + C_3) = C_0 - C_2 + iC_1 - iC_3$

$C_0 + C_2 - \underbrace{\omega^1}_{=i}(C_1 + C_2) = C_0 - C_2 - iC_1 + iC_3$

konkretes BSP:



Konzentration[1000], damit man nicht verrutscht lol

me does not like this topic...

good luck