DSS
Decision Sciences & Systems

Technische Universität München

Tutorial Business Analytics

Tutorial 6: Decision Trees

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Classifiers

Classifiers from previous lectures:

- Zero-Rule: class with the most instances (rule)
- One-Rule: rules for one attribute
- Naïve Bayes: conditional probability attribute - class

What is the difference between classification and regression?

Classification	Regression
Prediction of a class label by means of the attributes	Prediction of a numerical value by means of the attributes

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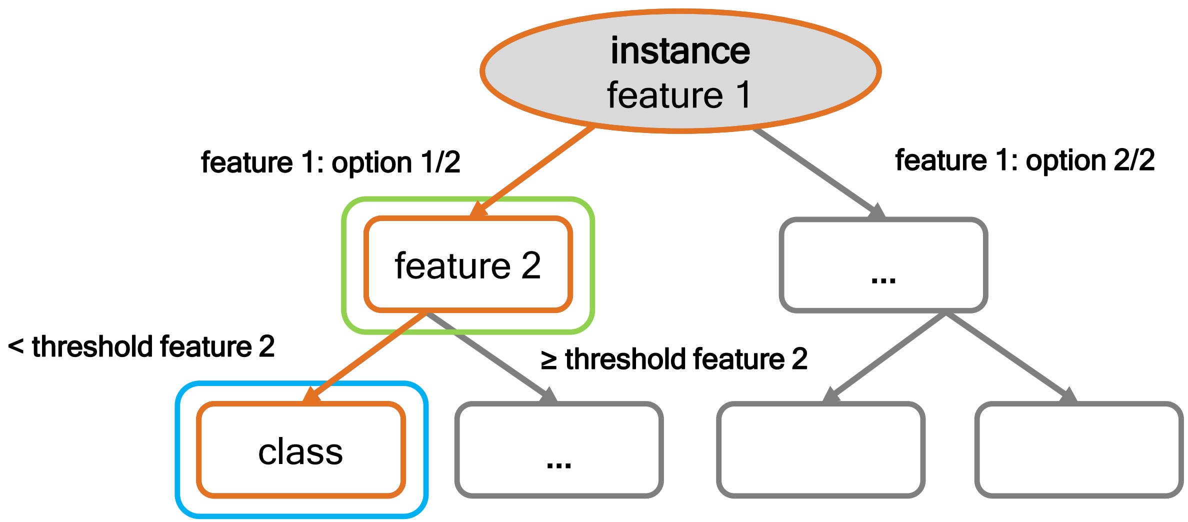
Classification Decision Trees

- A decision tree for n different classes is created based on some training data
- An internal node is a test on an attribute
- A branch represents an outcome of the test
- A leaf node represents a class
- A new instance is classified by following a matching path to a leaf node



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Optimal Tree

For m attributes and $n = 2$ classes, there are 2^{2^m} possible trees already

- That is equal to the number of Boolean functions

Finding the optimal tree is NP-complete

- Not feasible for data mining applications

Solution: **Greedy algorithm** for tree construction

- Top down approach: The tree is created recursively from the root node
- Every possible split is assessed with a measure
- The best split is chosen
- Repeat until all leaf nodes are pure or all attributes have been used

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Evaluating splits

Which split is better?

- Instances should be classified as easy as possible
- Good separation of classes (ideally leaf nodes contain instances of a single class only)
- In the worst case the separation does not affect the class distribution
- Possible measure: information

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Information and entropy

- Let us denote c_i to be the absolute number of training examples being in class i at the current stage
- The probability (relative frequency) of class i then is $p_i = \frac{c_i}{C}$ with $C = \sum_{i=1}^{n} c_i$

Entropy measures information content in bits (uncertainty of a node):

$$\text{entropy}(p_1, \dots, p_n) = -\sum_{i=1}^{n} p_i \cdot \log_2 p_i \,.$$

Information necessary to classify:

$$\text{info}([c_1, \dots, c_n]) = \text{entropy}\left(\frac{c_1}{C}, \dots, \frac{c_n}{C}\right).$$

Represents the expected amount of information that would be needed to specify the class of this node.

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Information gain

The quality of a split is equal to the gained information

gain(attribute) = info(before split by attribute) - info(after split by attribute)

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Formulas

Entropy:

$$\text{entropy}(p_1, \dots, p_n) = -\sum_{i=1}^{n} p_i \cdot \log_2 p_i$$

Information for $C = \sum c_i$:

$$\text{info}([c_1, \dots, c_n]) = \text{entropy}\left(\frac{c_1}{C}, \dots, \frac{c_n}{C}\right)$$

Average information for a numeric split into m branches, with $L_i = [c_{i,1}, \dots, c_{i,n}]$ being the set of class counts in this split, $C_i = \sum_k c_{i,k}$ the corresponding number of instances, and $L = \sum C_i$:

$$\text{info}(L_1, \dots, L_m) = \sum_{i=1}^{m} \frac{C_i}{L} \cdot \text{info}(L_i)$$

Information gain:

gain(attribute) = info(before split by attribute) - info(after split by attribute)

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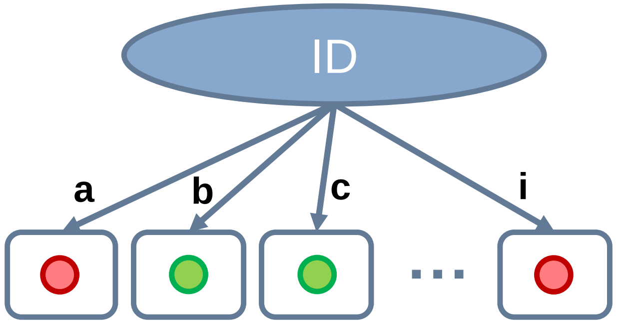
Information gain problems

Biased against attributes with a lot of edges

- For example: ID attribute
- Highest information gain because every leaf is pure
- Results in overfitting



Solution

- Take number and size of leafs into account: Intrinsic Information
- Intrinsic information: with s being the size of a leaf (number of affected instances)

$$\text{intrinsicInfo(attribute)} = \text{info}([s_1, \ldots, s_n])$$

New criterion: Gain ratio

$$\text{gainRatio(attribute)} = \frac{\text{gain(attribute)}}{\text{intrinsicInfo(attribute)}}$$

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Numerical attributes

Considering nominal attributes

- one edge per attribute value works well
- bad in case of numerical values

Solution: **Binary Splits**

- values are separated into two sections: below (<) and above (≥) some chosen threshold
- the split is evaluated with the information gain: set threshold to a value, s.t. information gain is maximized
- common practice to place numeric thresholds halfway between the values that delimit the boundaries
- Numeric attributes may be tested several times in a tree