

### **Proper Linear Score [Proper, Not Local, Feasible]**

$$S(p, v) = \int_{-\infty}^{\infty} p^2(z) dz - 2p(v) \quad (1)$$

The improperness of the Naive Linear score can be ‘fixed’ by adding the integral term to define the Proper Linear Score in equation 1; as described in in Friedman 1983 [1]. By fixing the propriety of the score it is no-longer Local due to the integral term. For a given  $p$  the integral term is constant, hence if the observation,  $v$ , occurs at a point of zero probability the score is maximised, hence the score is Feasible. Selten [2] shows that the Proper Linear score is the unique scoring rule up to affine transformation that is symmetric, elongation invariant, Proper and neutral.

# Bibliography

- [1] D. Friedman. Effective scoring rules for probabilistic forecasts. Management Science, 29(4), April 1983.
- [2] R. Selten. Axiomatic characterization of the quadratic scoring rule. Experimental Economics, 1:43–62, 1998.