Propriety A score S is **'proper'** (see for example [5]) if, for any pair of forecast pdfs p and q, the following inequality holds.

$$\int q(z)S(p(z), z)dz \geq \int q(z)S(q(z), z)dz \quad (1)$$

A score is **'strictly proper'** if the inequality is strict, this concept is made clear in Bernardo [1] but he uses the term Proper. Brown [2] uses the term 'admissable' instead of proper. The implication of this property is that if forecasters are incentivised using Proper scores they will do best to give a forecast p that is equal to their true beliefs q. McCarthy [3] calls this 'keeping the forecaster honest'. Selten [4] uses the phrase 'Incentive Compatible' in place of strictly proper. By this he notes that Improper scores encourage forecasters to deliberately issue a forecast that does not agree with their true beliefs (q). In recent times many forecasts have been fully automated largely removing such behavioural features. As will be illustrated in this chapter, however, it is also advisable to use Proper scores because, in the limit of many observations, they favour the true distribution above other forecasts. Here scores that do not have the Propriety property will be referred to as either Improper or non-Proper.

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