Power Rule Scores [Proper, Not Local, Feasible]

$$S(p, v) = (\alpha - 1) \int_{-\infty}^{\infty} p^{\alpha}(z)dz - \alpha p(v)^{\alpha - 1} \tag{1}$$

The power rule score family is a generalisation of the Proper Linear score defined for $\alpha > 1$; all are proper. The rule is discussed in Selten [1] although his presentation is for categorical forecasts; the definitions have been converted to a continuous formulation above. When $\alpha = 2$ the Power Rule score is equal to the Proper Linear score. In this chapter: a power rule score with parameter α is denoted **'powerrule** α**'**. Since each value of α defines a Proper scoring rule, Selten notes '... the power family shows the there are infinitely many incentive compatible scoring rules'. Again the integral terms implies that the score is not Local. As with the Proper Linear score the integral term is constant for any particular forecast hence this score is Feasible.

Bibliography

[1] R. Selten. Axiomatic characterization of the quadratic scoring rule. Experimental Economics, 1:43–62, 1998.