

Continuous Ranked Probability Score [Proper, Not Local, Not Feasible]

The Continuous Ranked Probability Score (CRPS) is defined by:

$$S(p, v) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^z p(t) dt - H(z - v) \right)^2 dz \quad (1)$$

Where the Heaviside (step) function H is defined as follows:

$$H(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases} \quad (2)$$

Hence, the CRPS is defined as the square of the L^2 distance between the distribution function of the forecast and a step distribution function centred on the outcome. See Ferro et al [1] for a similar definition to the one above. The CRPS score, whilst proper, is not Local. Also, CRPS is not Feasible as shown by Maynard [3]. Ficker et al [2] show that CRPS is not Fair for ensembles, but they also derive an extension (not shown) to CRPS that does have this property.

Bibliography

- [1] C. Ferro, D. Richardson, and A. Weigel. On the effect of ensemble size on the discrete and continuous ranked probability scores. RMetS, 15:19–24, 1998.
- [2] T. Fricker, C. Ferro, and D. Stephenson. Three recommendations for evaluating climate predictions. Meteorological Applications RMetS, 20:246–255, 2013.
- [3] T. Maynard. Extreme Insurance and the Dynamics of Risk. PhD thesis, London School of Economics and Political Science, 2016.