**Mean squared error Score [Not Proper, Not Local, Not Feasible]**

$$S(p, v) = \int_{-\infty}^{\infty} (v - z)^2 p(z) dz \tag{1}$$

The Mean Squared Error (MSE) (see for example Ferro et al [1]) is another example of an Improper score. The further the observation ($v$) is from the part of the forecast distribution that has the highest density, the greater weight will be given to the squared 'error' term. Hence a high (bad) score is given when high density is ascribed to values far from where the outcome actually occurs. The MSE score can be seen as a generalisation of the Root Mean Squared Error average score. The latter is not a score at all but an average over many forecasts - it suffers from non-properness and as it includes information about the whole forecast distribution is not Local. It is easy to show that $S_{MSE}(p, v) = \sigma^2 + \mu^2 + v(v - 2\mu)$, where $\sigma^2$ is the variance of the forecast and $\mu$ the mean. Therefore, the Mean Squared Error has both components 2 and 3 as described above. This score is not Feasible.

# Bibliography

[1] C. Ferro, D. Richardson, and A. Weigel. On the effect of ensemble size on the discrete and continuous ranked probability scores. RMetS, 15:19–24, 1998.