CO-1 – Probability
Session-01 & 02:
Session Topic: **Introduction to probability: Sample space and Events**

Probability, Statistics and Queueing Theory
(Course code: 21MT2103RA)

(DEEMED TO BE UNIVERSITY)
COMPUTER SCIENCE & ENGINEERING

CO1

(Probability)

- **Introduction to Probability: Sample Space and Events**
- **Probabilities defined on Events, Conditional Probabilities**
- **Independent Events, Bayes Formula.**
- **Random Variables, Probability Distribution Function**
- **Cumulative Distribution Function**
- **Discrete Random Variables: Bernoulli, Binomial, and Poisson.**

Course Handout

- **Complete Discussion About Course Handout**
- **LTP Structures**
- **Moocs Details**
- **Evaluation Components and their details**

source: https://ncert.nic.in/textbook.php?kemh1=16-16

Probability: Review of concepts

Probability is a measure of uncertainty of various phenomenon. The role of probability theory is to provide a framework for analyzing phenomena with uncertain outcomes.

The three approaches:

1. The **classical theory** of probability: The probability of an event is computed as the ratio of the number of outcomes favorable to the event, to the total number of equally likely outcomes. This could be a **thought** experiment; example: tossing a coin; outcomes: head or tail
2. The **statistical approach** of probability: the probability on the basis of **observations** and collected data.

   The above two approaches assume that all outcomes are equally likely.

3. The **axiomatic approach** of probability: Here, the outcomes need not have equal chances of occurrence. We may have reason to believe that one outcome is more likely to occur than the other. In this approach, some axioms are stated to interpret probability of events. To understand this approach, let us learn a few basic terms viz. random experiment, sample space, events

source: https://ncert.nic.in/textbook.php?kemh1=16-16

Random experiments

Random experiment: In life, we perform many experimental activities, where the result may not be same, when the experiments are repeated under identical conditions. We are not sure which one of many possible results will actually be obtained. Such experiments are called random experiments.

A possible result of a random experiment is called its **outcome**.

An experiment is called **random experiment** if it satisfies the following two conditions:

(i) It has more than one possible outcome.

(ii) It is not possible to predict the outcome in advance.

Two steps in description of a random experiment:

1. Describe possible **outcome**s of a random experiment
2. Describe beliefs about **likelihood** (chance) of outcomes

Random experiments: examples

Example 1: **Tossing a fair coin**

Two outcomes: Head (H) and Tail (T)

Both are equally likely.

Example 2: **Tossing a fair dice**

Set of outcomes = {1,2,3,4,5,6} = sample space

All 6 outcomes are equally likely.

Example 3: **Mathematics examination grade**

The likelihood of a student getting first class is smaller than

the likelihood of getting pass class.

Example 4: **Winner of world cup**

The likelihood of India winning the next world cup in cricket is …

Sample space

The sample space S is a list (set) of possible outcomes.

The list must be

- Mutually exclusive, and
- Collectively exhaustive

Types of outcomes:

1. Discrete
2. Continuous

Examples of experiments with discrete outcomes:

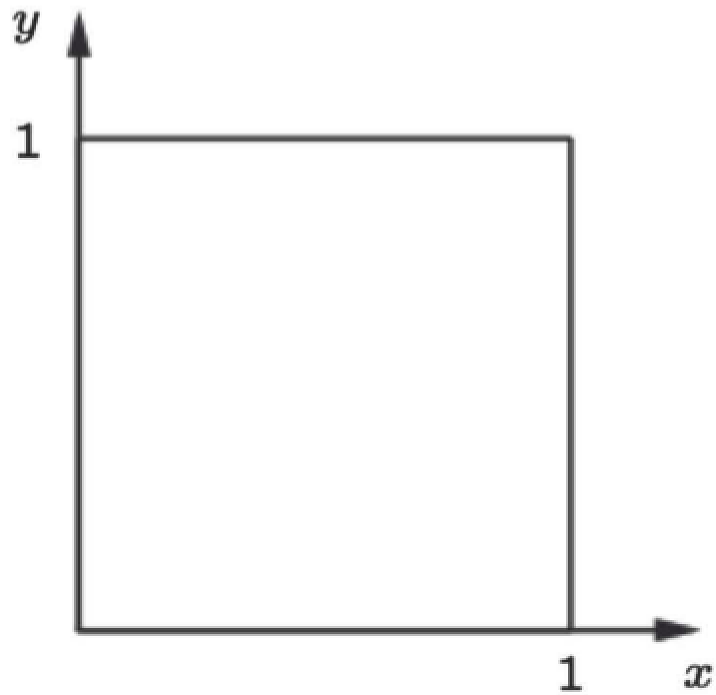
Tossing a coin: sample space, S = {H, T}

Tossing a dice: sample space, S = {1,2,3,4,5,6}

Each element of the sample space is called a **sample point**. In other words, each outcome of the random experiment is also called sample point.

Sample space: continuous example

- (x, y) such that $0 \le x, y \le 1$



source: https://ocw.mit.edu/courses/res-6-012-introduction-to-probability-spring-2018/91864c7642a58e216e8baa8fcb4a5cb5_MITRES_6_012S18_L01.pdf

Event

Any subset E of a sample space S is called an **event**.

Consider the experiment of throwing a dice.

Description of events	**Corresponding subset of S**
the number is exactly 2	A = {2}
the number is an even integer	B = {2,4,6}
the number is greater than 6	φ = {}

Occurrence of an event: The event E of a sample space S is said to have occurred if the outcome ω of the experiment is such that $\omega \in E$.

If the outcome ω is such that $\omega \notin E$, we say that the event E has not occurred.

In the above example, if the outcome is 6, event B has occurred, and event A has not occurred. On the other hand, if the outcome is 2, both events A and B have occurred.

Types of events

Impossible event : The null set $\varphi=\{\}$ is called an impossible event

Sure event : S, i.e., the whole sample space is called the sure event.

Simple Event : If an event E has only one sample point of a sample space, it is called a simple (or elementary) event.

Compound Event : If an event has more than one sample point, it is called a Compound event.

Algebra of events

We can combine two or more events to form new events.

Let A, B, C be events associated with an experiment whose sample space is S.

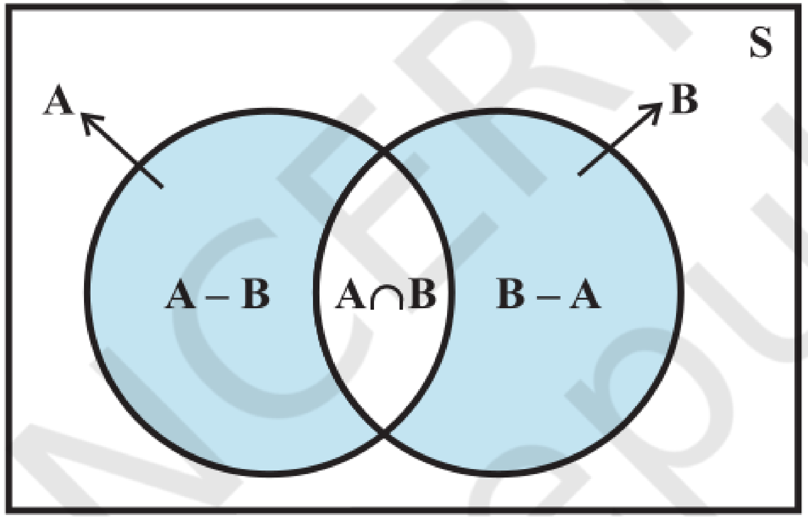
Complementary event : For every event A, there corresponds another event **A′** called the complementary event to **A**. It is also called the event 'not A'.

$$\mathbf{A' = \{\omega : \omega \in S \text{ and } \omega \notin A\} = S - A}$$

A or B $\mathbf{A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}}$

A and B $\mathbf{A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}}$

A but not B $\mathbf{A - B = A \cap B'}$



source: https://ncert.nic.in/textbook.php?kemh1=16-16

Events (contd.)

Mutually exclusive events : two events A and B are called mutually exclusive events if the occurrence of any one of them excludes the occurrence of the other event, i.e., if they can not occur simultaneously. In this case the sets A and B are disjoint.

For example, If A = {2,4,6} and B = {1,3},

A and B are mutually exclusive events.

Exhaustive Events : if E_1 , E_2 , ..., E_n are n events of a sample space S and if

$$\mathbf{E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = \bigcup_{i=1 \text{ to } n} E_i = S}$$

then E_1 , E_2 , ..., E_n are called exhaustive events.

In other words, events E_1 , E_2 , ..., E_n are said to be exhaustive if at least one of them necessarily occurs whenever the experiment is performed.

Events (contd.)

Mutually exclusive and exhaustive events

if $\mathbf{E\ i \cap E\ j = \varphi}$ **for** $\mathbf{i \neq j}$ i.e., events E i and E j are pairwise disjoint

and

$$\mathbf{E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = \bigcup_{i = 1 \text{ to } n} E_i = S}$$

then events E_1 , E_2 , ..., E_n are called mutually exclusive and exhaustive events.

A **partition of a set S** is a set of nonempty subsets of S, such that every element x in S is in exactly one of these subsets.

References

* https://ncert.nic.in/textbook.php?kemh1=16-16

* Notes: sections 1 to 1.3 of

  http://www.statslab.cam.ac.uk/~rrw1/prob/prob-weber.pdf

- Chapter 1 of TP1: William Feller, An Introduction to Probability Theory and Its Applications: Volume 1, Third Edition, 1968 by John Wiley & Sons,Inc.

- ***Sample space***: The set of all possible outcomes
- ***Sample points***: Elements of sample space
- ***Event***: A subset of the sample space
- ***Impossible event*** : The empty set
- ***Sure event***: The whole sample space
- ***Complementary event or 'not event'*** : The set A′ or S – A
- ***Event* A *or* B**: The set $A \cup B$
- ***Event* A *and* B**: The set $A \cap B$
- ***Event* A *and* not B**: The set A – B
- ***Mutually exclusive event***: A and B are mutually exclusive if $A \cap B = \phi$
- ***Exhaustive and mutually exclusive events***: Events $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive if $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and $E_i \cap E_j = \phi \;\; \forall\, i \neq j$