Formative Assessment 3

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GitHub Link: https://github.com/mayonnayz/FA1_Probability.git

2. Binary Communication Channel

> A binary communication channel carries data as one of two sets of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1, and a transmitted 1 is sometimes received as a 0. For a given channel, it can be assumed that a transmitted 0 is correctly received with probability 0.95, and a transmitted 1 is correctly received with probability 0.75. Also, 70

Let's insert the given into variables

```
# ZERO

# probability of correctly received 0
prob_c_rzero <- 0.95
# probability of incorrectly received (1 was received)
prob_w_rzero <- (1-0.95)
# probability of 0 transmitted
prob_tzero <- 0.70

# ONE

# probability of correctly received 1
prob_c_rone <- 0.75
# probability of incorrectly received (0 was received)
prob_w_rone <- (1-0.75)
# probability of 1 transmitted
prob_tone <- (1-0.70)
```

(a) a 1 was received

The formula of the probablity of the event that 1 was received is the union of receiving 1 from either 0 and 1 that was transmitted.

$$P(R_1) = (T_1 \cap R_1) \cup (T_0 \cap R_1)$$

which is also:

$$P(R_1) = P(T_1)P(R_1|T_1) + P(T_0)P(R_1|T_0)$$

Substituting the values…

```
prob_rone <- (prob_tone*prob_c_rone) + (prob_tzero*prob_w_rzero)
#          = (0.3 * 0.75) + (0.7 * 0.05)
```

We get that the probability of receiving 1 is: **26%**

(b) a 1 was transmitted given that a 1 was received

Using Bayes' rule for two events, the formula to be used will be:

$$P(T_1|R_1) = \frac{P(T_1)P(R_1|T_1)}{P(R_1)}$$

Using the initial given values and the value of that was acquired in the previous item…

```
prob_tone_rone <- (prob_tone*prob_c_rone) / prob_rone
#               = (0.3 * 0.75) / (0.26)
```

Thus, the probability of a 1 transmitted given that a 1 was received is: **86.54%**

7. Three Employees, One Error; Who is Most Likely to?

First, we must define the given conditional probabilities.

There are three employees working at an IT company: Jane, Amy, and Ava, doing 10%, 30%, and 60% of the programming, respectively. 8% of Jane's work, 5% of Amy's work, and just 1% of Ava's work is in error.

```
Prob_Jane <- 0.10
Prob_Amy <- 0.30
Prob_Ava <- 0.60
ProbError_Jane <- 0.08
ProbError_Amy <- 0.05
ProbError_Ava <- 0.01
```

What is the overall percentage of error?

We may now calculate the probability of error. We can use the Law of Total Probability.

```
P_TotalError <- (ProbError_Jane * Prob_Jane) + (ProbError_Amy * Prob_Amy) + (ProbError_Ava * Prob_Ava)
```

Therefore, the overall percentage of error is: **2.9%**

If a program is found with an error, who is the most likely person to have written it?

To figure out who is the most likely to have written a program with an error, we can calculate the posterior probability of each employee. We can use Bayes' Theorem that states the probability of A after B has occurred.

```r
PPError_Jane <- (ProbError_Jane * Prob_Jane) / P_TotalError
PPError_Amy <- (ProbError_Amy * Prob_Amy) / P_TotalError
PPError_Ava <- (ProbError_Ava * Prob_Ava) / P_TotalError
```

Jane's posterior probability that she wrote the program with an error: 27.59%
Amy's posterior probability that she wrote the program with an error: 51.72%
Ava's posterior probability that she wrote the program with an error: 20.69%

To summarize,

```r
summary <- data.frame(
  Employee_Names = c("Jane", "Amy", "Ava"),
  Probability_of_Error = c(ProbError_Jane, ProbError_Amy, ProbError_Ava),
  Contribution = c(Prob_Jane, Prob_Amy, Prob_Ava),
  Posterior_Probability = c(PPError_Jane, PPError_Amy, PPError_Ava)
)
print(summary)
```

```
##   Employee_Names Probability_of_Error Contribution Posterior_Probability
## 1           Jane                 0.08          0.1             0.2758621
## 2            Amy                 0.05          0.3             0.5172414
## 3            Ava                 0.01          0.6             0.2068966
```

Thus,

After careful consideration of the results provided above, we can conclude that **Amy** is the employee that is most likely to have written an error in the program with approximately **51.72%**.