Formative Assessment 5

Zyann Lynn Mayo

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GitHub Link: https://github.com/mayonnayz/Probability-and-Probability-Distribution.git

Exercise 7.1 Item 6:

Insert the given data into variables:

$$\begin{aligned} P(S_1) &= 0.40, \quad P(E|S_1) = 0.01 \\ P(S_2) &= 0.25, \quad P(E|S_2) = 0.02 \\ P(S_3) &= 0.35, \quad P(E|S_3) = 0.015 \end{aligned}$$

```
mess_s1 <- 0.40
mess_s2 <- 0.25
mess_s3 <- 0.35
error_s1 <- 0.01
error_s2 <- 0.02
error_s3 <- 0.015
```

(a) What is the probability of receiving an email containing an error?

Using the law of total probability,

$$P(E) = P(E|S_1)P(S_1) + P(E|S_2)P(S_2) + P(E|S_3)P(S_3)$$

```
prob_error <- (mess_s1*error_s1)+(mess_s2*error_s2)+(mess_s3*error_s3)
```

The answer will be **P(E) = 1.43%**.

(b) What is the probability that a message will arrive without error?

Using this solution to find the probability of messages without error,

$$P(E^c) = 1 - P(E)$$

```
prob_no_error <- 1 - prob_error
```

Thus the probability without an error email is $P(E)$ **= 98.58%**.

(c) If a message arrives without error, what is the probability that it was sent through server 1?

Using Bayes' Theorem:

$$P(S_1|E^c) = \frac{P(E^c|S_1)P(S_1)}{P(E^c)}$$

We must get the percentage of no error occurring in each servers by following the formula:

$$P(E^c|S_1) = 1 - P(E|S_1)$$

```
no_error_s1 <- 1-error_s1
no_error_s2 <- 1-error_s2
no_error_s3 <- 1-error_s3
```

Using the law of total probability to get $P(E^c)$

```
totalprob_no_error <- (no_error_s1*mess_s1)+(no_error_s2*mess_s2)+(no_error_s3*mess_s3)
```

Now we can input the data into Bayes'Theorem:

```
sent_thru_s1 <- (no_error_s1*mess_s1)/(totalprob_no_error)
```

Thus, we get that the probability that a successful message was sent through server 1 is **40.17%**

Exercise 7.1 Item 9:

Store the data into variables

$$\begin{aligned} P(A) &= 0.2, \quad P(G|A) = 0.1 \\ P(B) &= 0.7, \quad P(G|B) = 0.4 \\ P(C) &= 0.1, \quad P(G|C) = 0.2 \end{aligned}$$

```
#Probabilities of intentions
not_buy <- 0.2
yes_buy <- 0.7
undecided <- 0.1

#Probabilities of upgrading given intentions
g_not_buy <- 0.1
g_yes_buy <- 0.4
g_und_buy <- 0.2
```

(a) Calculate the probability that a manager chosen at random will not upgrade the computer hardware

Using the law of total probability to get the probability of upgrading:

```
will_upg <- (g_not_buy*not_buy)+(g_yes_buy*yes_buy)+(g_und_buy*undecided)
```

Then subtracting it from 1 to get the probability of NOT UPGRADING

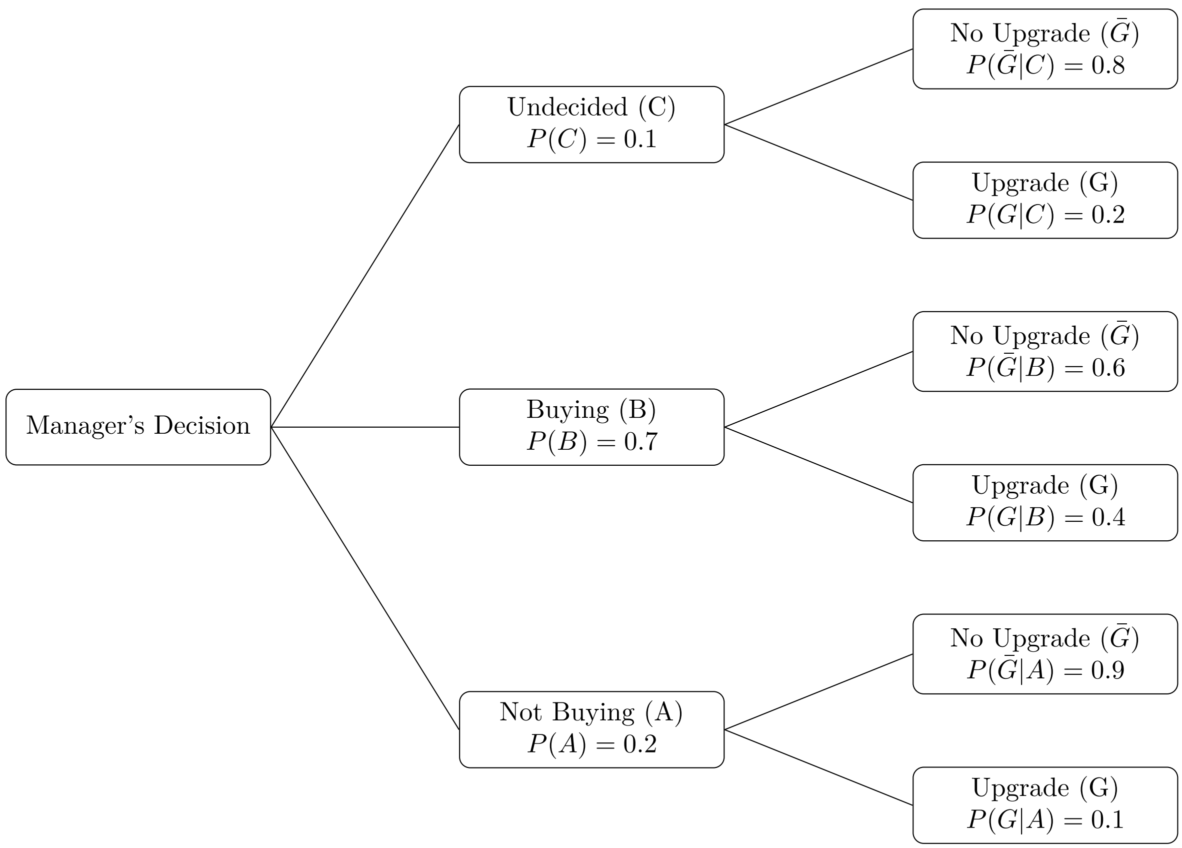
```
will_not_upg <- 1-will_upg
```

We get **68%**

(b) Explain what is meant by the posterior probability of B given G, $P(B|G)$

Firstly, the posterior probability refers to the probability of an event happening after incorporating new information that influences the event. This means that the posterior probability $P(B|G)$ shows the probability that a manager has the intentions to buy the graphics package (referring to **event B**), given that they show interest in upgrading their computer (referring to **event G**).

(c) Construct a tree diagram and use it to calculate the probabilities.



(i) Probability of Upgrading P(G): \ Using the Law of Total Probability

```
upgrade <- (g_not_buy*not_buy)+(g_yes_buy*yes_buy)+(g_und_buy*undecided)
```

Thus, $\mathbf{P(G) = 32\%}$

(ii) Probability of buying given that they upgraded P(B|G): Using Bayes'Theorem:

```
yes_buy_upg <- (g_yes_buy*yes_buy)/(will_upg)
```

Thus, $\mathbf{P(B|G) = 87.5\%}$

(iii) Probability of buying given no upgrade $P(B|\bar{G})$: Using Bayes'Theorem:

$$P(B|\bar{G}) = \frac{P(\bar{G}|B)P(B)}{P(\bar{G})}$$

Get $P(\bar{G})$ first, then compute:

```
no_upg <- ((1-g_not_buy)*not_buy)+((1-g_yes_buy)*yes_buy)+((1-g_und_buy)*undecided)

yes_buy_no_upg <- ((1-g_yes_buy)*yes_buy)/(no_upg)
```

Thus, $P(B|\bar{G})$ = **61.76%**

(iv) Probability of being undecided given upgrade P(C}G): Using Bayes' Theorem:

```
und_g_upg <- (g_und_buy*undecided)/(will_upg)
```

Thus, **P(C|G) = 6.25%**

(v) Probability of not being undecided given no upgrade $P(\bar{C}|\bar{G})$: This means $P(\bar{C}|\bar{G}) = 1 - P(C|\bar{G})$

Using Bayes' Theorem:

```
und_gnot_upg <- ((1-g_und_buy)*undecided)/(will_not_upg)
not_und_gnot_upg <- 1-und_gnot_upg
```

Thus, $P(\bar{C}|\bar{G})$ = **88.24%**

Exercise 7.1 Item 13:

Store the data into variables:

```
via_internet <- 0.7
via_email <- 0.3
det_internet <- 0.6
det_email <- 0.8
not_det_internet <- 1-det_internet
not_det_email <- 1-det_email
```

(a) What is the probability that this spyware infects the system?

Since a spyware only infects the system if it's not detected, we must get the probability that is it not detected $P(N)$.

Using the law of total probability:

$$P(N) = P(N|I)P(I) + P(N|E)P(E)$$

```
infects <- (not_det_internet*via_internet)+(not_det_email*via_email)
```

We get that the probability that the spyware infects the system is **34%**.

(b) If the spyware is detected, what is the probability that it came through the Internet?

Using Bayes' Theorem to get the probability that it came through the internet given that it was detected $P(I|D)$:

$$P(I|D) = \frac{P(D|I)P(I)}{P(D)}$$

We must first calculate the probability that is was detected $P(D)$ using law of total probability:

```
detected <- (det_internet*via_internet)+(det_email*via_email)
```

Then we insert the values into Bayes' Theorem:

```
spy_internet <- (det_internet*via_internet)/(detected)
```

We get that $P(I|D) = \mathbf{63.64\%}$