Formative Assessment 4

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2025-03-02

GitHub Link: https://github.com/mayonnayz/Probability-and-Probability-Distribution.git

Item 5: Law of Total Probability

> A geospatial analysis system has four sensors supplying images. The percentage of images supplied by each sensor and the percentage of images relevant to a query are shown in the following table.

```
sensor_data <- data.frame(
  Sensor = c(1, 2, 3, 4),
  Percentage_of_Images_Supplied = c(15, 20, 25, 40),
  Percentage_of_Relevant_Images = c(50, 60, 80, 85)
)

library(knitr)
kable(sensor_data, caption = "Sensor Data Table")
```

Table 1: Sensor Data Table

Sensor	Percentage_of_Images_Supplied	Percentage_of_Relevant_Images
1	15	50
2	20	60
3	25	80
4	40	85

> What is the overall percentage of relevant images?

First, let's put the data into variables

$$\begin{aligned} P(S_1) &= 0.15, \quad P(R|S_1) = 0.50 \\ P(S_2) &= 0.20, \quad P(R|S_2) = 0.60 \\ P(S_3) &= 0.25, \quad P(R|S_3) = 0.80 \\ P(S_4) &= 0.40, \quad P(R|S_4) = 0.85 \end{aligned}$$

```
#Sensor 1
s1_img_sup <- .15
s1_rel_img <- .50
#Sensor 2
```

```
s2_img_sup <- .20
s2_rel_img <- .60
#Sensor 3
s3_img_sup <- .25
s3_rel_img <- .80
#Sensor 4
s4_img_sup <- .40
s4_rel_img <- .85
```

Using the law of total probability, for any event A:

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i)P(B_i)$$

Inserting the given values,

```
overall_rel_img <- (s1_img_sup*s1_rel_img) +
  (s2_img_sup*s2_rel_img) +
  (s3_img_sup*s3_rel_img) +
  (s4_img_sup*s4_rel_img)
```

We get that the overall percentage of relevant images is **73.5%**

Item 6: Independence of Two Events

> A fair coin is tossed twice. Let E_1 be the event that both tosses have the same outcome, that is, $E_1 = \{HH, HT\}$.
> Let E_2 be the event that the first toss is a head, that is, $E_2 = \{HH, HT\}$.
> Let E_3 be the event that the second toss is a head, that is, $E_3 = \{TH, HH\}$.
> Show that E_1, E_2, and E_3 are pairwise independent but not mutually independent.

Possible outcomes = {HH, HT, TH, TT}

$$E_1 = \{HH, TT\} = \frac{2}{4} = \frac{1}{2}$$
$$E_1 = \{HH, HT\} = \frac{2}{4} = \frac{1}{2}$$
$$E_1 = \{TH, HH\} = \frac{2}{4} = \frac{1}{2}$$

Insert values into variables:

```
prob_e1 <- 1/2
prob_e2 <- 1/2
prob_e3 <- 1/2
```

Checking for Pairwise Independence

We have to check if $P(E_i \cap E_j) = P(E_i)P(E_j)$ for all pairs.

```r
library(MASS)
prob_e1_e2 <- fractions(prob_e1*prob_e2)
prob_e1_e3 <- fractions(prob_e1*prob_e3)
prob_e2_e3 <- fractions(prob_e2*prob_e3)
```

For E_1 and E_2:

$$
\begin{aligned}
P(E_1 \cap E_2) &= \{HH\} = \mathbf{1/4} \\
P(E_1)P(E_2) &= 1/2 \times 1/2 = \mathbf{1/4} \\
P(E_1 \cap E_2) &= P(E_1)P(E_2)
\end{aligned}
$$

For E_1 and E_3:

$$
\begin{aligned}
P(E_1 \cap E_3) &= \{HH\} = \mathbf{1/4} \\
P(E_1)P(E_3) &= 1/2 \times 1/2 = \mathbf{1/4} \\
P(E_1 \cap E_3) &= P(E_1)P(E_3)
\end{aligned}
$$

For E_2 and E_3:

$$
\begin{aligned}
P(E_2 \cap E_3) &= \{HH\} = \mathbf{1/4} \\
P(E_2)P(E_3) &= 1/2 \times 1/2 = \mathbf{1/4} \\
P(E_2 \cap E_3) &= P(E_2)P(E_3)
\end{aligned}
$$

Thus, all pairs are **PAIRWISE INDEPENDENT**.

Checking for Mutual Independence
In order to prove that $E_1, E_2, and E_3$ are not mutually independent, we must show that

$$P(E_1 \cap E_2 \cap E_3) \neq P(E_1)P(E_2)P(E_3)$$

```r
library(MASS)
prob_e1_e2_e3 <- fractions(prob_e1*prob_e2*prob_e3)
```

$$
\begin{aligned}
P(E_1 \cap E_2 \cap E_3) &= \{HH\} = \mathbf{1/4} \\
P(E_1)P(E_2)P(E_3) &= 1/2 \times 1/2 \times 1/2 = \mathbf{1/8}
\end{aligned}
$$

Since the probability of the intersection of the three events is 1/4, which is not equal to the product of the probability of all events which is 1/8, the events are **NOT MUTUALLY INDEPENDENT**