Extra: Discussion of phase transitions.

Mayra Cristina Berrones Reyes

May 29, 2020

1 Discussion

For this discussion, we read some publications and articles that explained the phase transition problem. The most basic form that we understood, was the notion that a phase transition is the one that happens once a certain feature of the problem changes and it reaches what we know as a critical point. A widely used example is the one of the water temperature. In this case, the temperature is going to change little by little, and the critical point in this experiment will happen when the water boils and turns into gas, or solidifies because is too cold.

Since most of the examples and common phase transition examples we found were about physics, magnetic examples and such, we had a hard time placing optimization problems in this type of solutions. In a conference transcript of a phase transitions workshop [Zhang, 2002] we found a graphic that helped us understand a little bit better this concept. First they start with a complexity analysis on different types of problems, such as a tree search. In Figure 1 we see the easy-hard transition in optimal tree search.

In this figure we have the order parameter as:

- b is the mean branching factor.
- p is the probability that an edge has a cost 0.
- bp is the expected number of children having the same cost as the parent.

So in this case the complexity is determined by bp.

In this same conference transcript we found another example of a TSP problem, similar to the one described in the documentation for this problem [Schaeffer, 2020]. In this case the control parameter changes. In Figure 2 we can see the comparison they made. They used as the changing parameter the number of the distance, and the different iterations are from different size of cities they have to travel to.

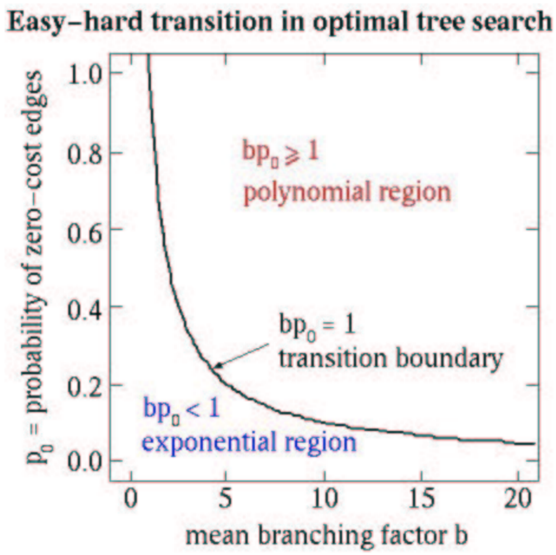

Figure 1: Easy-hard transition in optimal tree search.

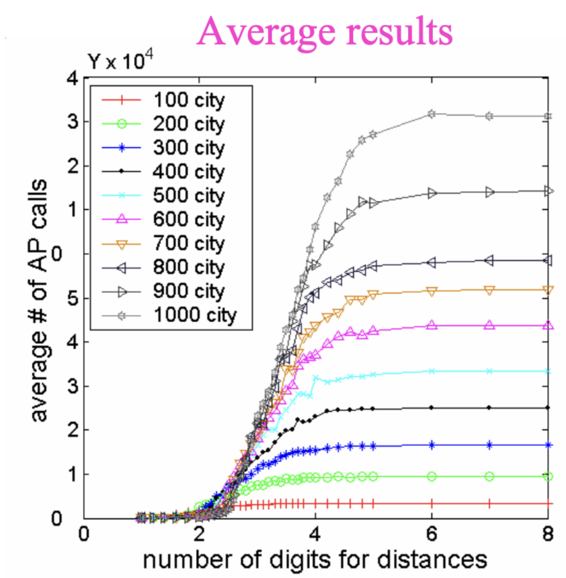

Figure 2: Average results for the TSP with the changing parameter of distance between cities.

After this images, the practice 15 came to mind, because we found a similar problem than this. In that practice we made some experimentation with the number of cities we had to visit using a method of branch and bound to find the solution. In Figure 3 we can see how after 10 cities the complexity starts to rise and by the 25 iteration the complexity becomes exponential.

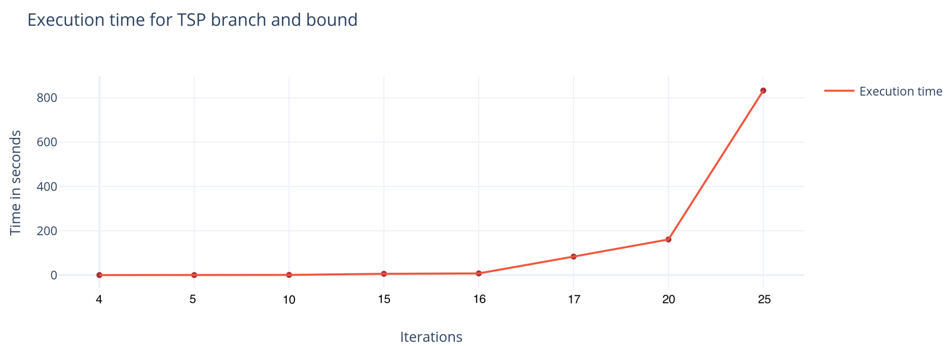

Figure 3: Execution time for branch and bound for TSP.

To solve this kind of problems we found various explanations with the Ising Model. The simplified way they explained is that, we suppose we have what they called spins. These spins have a direction of up and down. Now what if one of the spins gets flipped at a random position because of a change in the changing parameter, which can cause the neighboring spins to flip in the same direction. And this process will go on and on, and completely ordered state will not remain stable [Singh, 2020].

This was easy to understand with magnetic forces or the temperature example, but we failed to connect it to the problems we first described as to how or when this type of critical points are going to be reached, other than trial and error, as we tried to do in Figure 3.

References

[Schaeffer, 2020] Schaeffer, E. (2020). Complejidad computacional de problemas y el analisis y diseño de algoritmos. Available "`https://elisa.dyndns-web.com/teaching/aa/pdf/aa.pdf`".

[Singh, 2020] Singh, S. P. (2020). The ising model: Brief introduction and its application. DOI: 10.5772/intechopen.90875.

[Zhang, 2002] Zhang, W. (2002). Phase transitions, backbones, measurement precision, and phase-inspired approximation. Available "http://www.ipam.ucla.edu/abstract/?tid=1654&pcode=PTAC2002".