# Exercises of chapter 3

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## 1 Turing machines

Turing machine (TM) was invented by Alan Turing in 1993, and consists of a tape with an infinite length, on which read and write operations can be done, using a certain alphabet, and with the instructions of a transition state table, it solves a certain task.

On this document we demonstrate some examples of the use of a TM on different tasks. First we define the special symbols that show up in every example:

- $\Sigma$ is our alphabet.
- $\triangleright$ is the start of the tape.
- $-$ means no movement.
- $\sqcup$ is a blank space.
- $\leftarrow$ means move to the left.
- $\rightarrow$ means move to the right.

### 1.1 Example 1: Multiply a binary number by two

Define a Turing machine that produces a binary number (provided as input) multiplied by two.

In this TM our alphabet is $\Sigma = \{0, 1, \triangleright, \sqcup, \leftarrow, \rightarrow, -\}$ and our transition states are $s$, $q$ and *stop*.

The transition table is shown in Table 1.

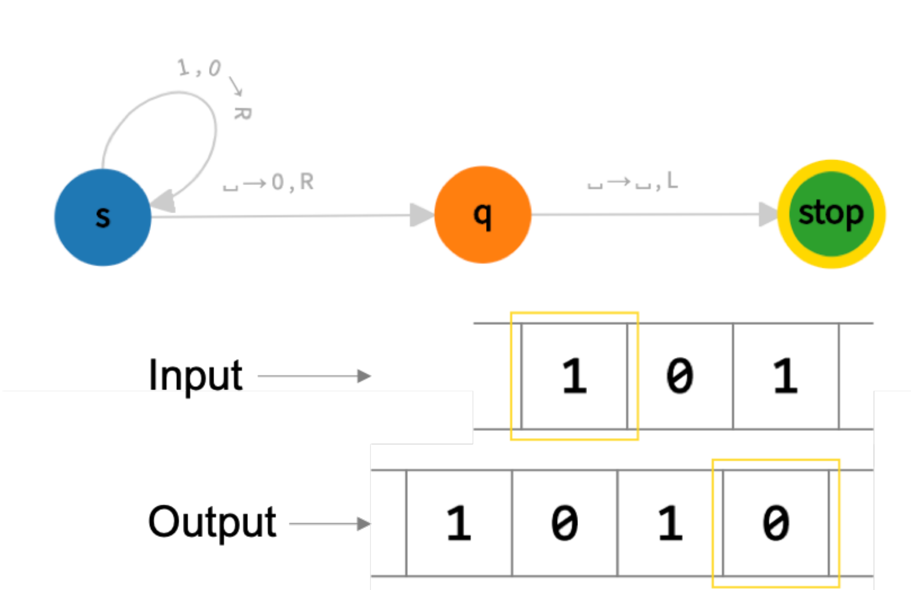
Table 1: Transition table for example 1

| State | Transition |
|---|---|
| $s$, $\triangleright$ | (s, $\triangleright$, $\rightarrow$) |
| $s$, 0 | ($s$, 0, $\rightarrow$) |
| $s$, 1 | ($s$, 1, $\rightarrow$) |
| $s$, $\sqcup$ | ($q$, 0, $\rightarrow$) |
| $q$, $\sqcup$ | ($stop$, $\sqcup$, $-$) |

```
input: '101'
blank: ' '
start state: s
table:
  s:
    [1,0]: R
    ' '  : {write: 0, R: q}
  q:
    ' ' : {write: ' ', L: stop}
  stop:


```

Listing 1: TM Visualization code

Figure 1: Diagram of all the transitions and the result of an example.



## 1.2 Example 2: Contains at least one letter

Given the alphabet $\Sigma = \{a, b, c, \triangleright, \sqcup, \leftarrow, \rightarrow, -\}$, the TM must check that at least one of each letter $a$, $b$, $c$ appears on a string. This time we have seven states ($start$, and $case2$ to 7), and two acceptance states $accept$ and $reject$.

The transition table is shown in Table 2.

Table 2: Transition table for example 2

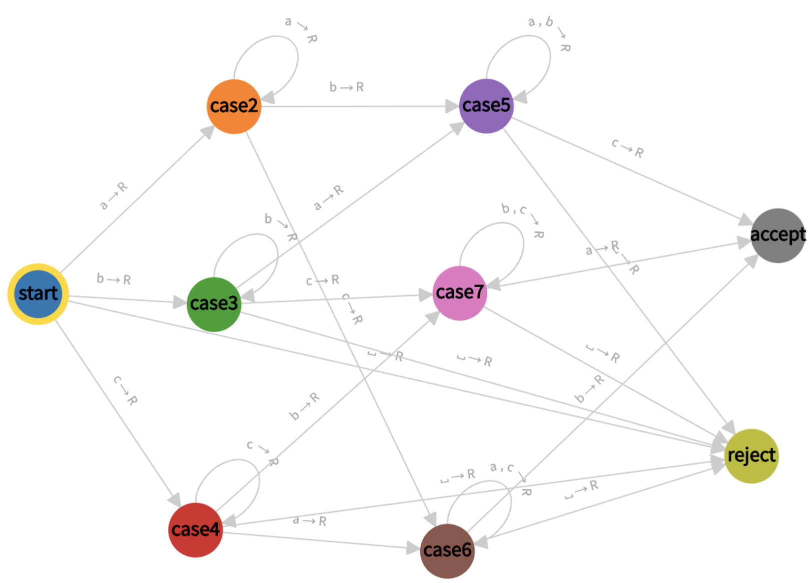
| State | Transition | State | Transition | State | Transition |
|---|---|---|---|---|---|
| $start$, $\triangleright$ | $(start, \triangleright, \rightarrow)$ | $case2$, $a$ | $(case2, a, \rightarrow)$ | $case3$, $a$ | $(case5, a, \rightarrow)$ |
| $start$, $a$ | $(case2, 0, \rightarrow)$ | $case2$, $b$ | $(case5, b, \rightarrow)$ | $case3$, $b$ | $(case3, b, \rightarrow)$ |
| $start$, $b$ | $(case3, 1, \rightarrow)$ | $case2$, $c$ | $(case6, c, \rightarrow)$ | $case3$, $c$ | $(case7, c, \rightarrow)$ |
| $start$, $c$ | $(case4, 0, \rightarrow)$ | $case2$, $\sqcup$ | $(reject, \sqcup, -)$ | $case3$, $\sqcup$ | $(reject, \sqcup, -)$ |
| $start$, $\sqcup$ | $(reject, \sqcup, -)$ | | | | |
| **State** | **Transition** | **State** | **Transition** | **State** | **Transition** |
| $case4$, $a$ | $(case6, a, \rightarrow)$ | $case5$, $a$ | $(case5, a, \rightarrow)$ | $case6$, $a$ | $(case6, a, \rightarrow)$ |
| $case4$, $b$ | $(case7, b, \rightarrow)$ | $case5$, $b$ | $(case5, b, \rightarrow)$ | $case6$, $b$ | $(accept, b, -)$ |
| $case4$, $c$ | $(case4, c, \rightarrow)$ | $case5$, $c$ | $(accept, \sqcup, -)$ | $case6$, $c$ | $(case6, c, \rightarrow)$ |
| $case4$, $\sqcup$ | $(reject, \sqcup, -)$ | $case5$, $\sqcup$ | $(reject, \sqcup, -)$ | $case6$, $\sqcup$ | $(reject, \sqcup, -)$ |
| **State** | **Transition** | | | | |
| $case7$, $a$ | $(accept, a, -)$ | | | | |
| $case7$, $b$ | $(case7, b, \rightarrow)$ | | | | |
| $case7$, $c$ | $(case7, c, \rightarrow)$ | | | | |
| $case7$, $\sqcup$ | $(reject, \sqcup, -)$ | | | | |

```
input: 'ccaacb'
blank: ' '
start state: start
synonyms:
  accept: {R: accept}
  reject: {R: reject}

table:
  start:
    a: {R: case2}
    b: {R: case3}
    c: {R: case4}
    ' ': reject
  case2:
    a: R
    b: {R: case5}
    c: {R: case6}

  case3:
    b: R
    a: {R: case5}
    c: {R: case7}
    ' ': reject
```

```
case4:
  c: R
  a: {R: case6}
  b: {R: case7}
  ' ': reject

case5:
  [a,b]: R
  c: accept
  ' ': reject
case6:
  [a,c]: R
  b: accept
  ' ': reject

case7:
  [b,c]: R
  a: accept
  ' ': reject

accept:
reject:


```

Listing 2: TM Visualization code

Figure 2: Diagram of all the transitions and the result of an example.




The input for this TM is the string *ccaacb*, and the output is *accept*. If we change that string to *caacc* the output is *reject*.

### 1.3 Example 3: Know if a string is a palindrome

Given the alphabet $\Sigma = \{a, b, c, \triangleright, \sqcup, \leftarrow, \rightarrow, -\}$, the TM must take the string and check if is a palindrome or not. This time we have eight states (*start*, and *case*2 to 8), and two acceptance states *accept* and *reject*.

The transition table is shown in Table 3.

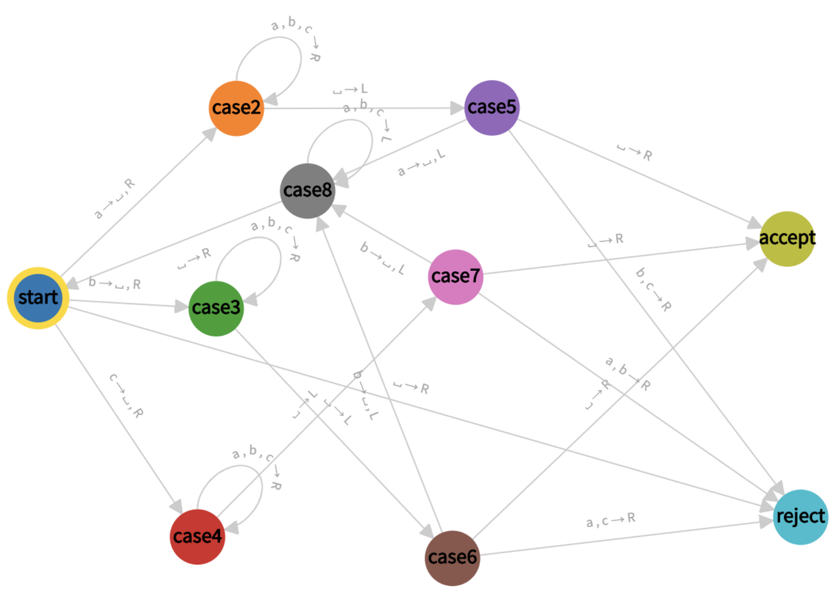
Table 3: Transition table for example 3

| State | Transition | State | Transition | State | Transition |
|---|---|---|---|---|---|
| $start$, $\triangleright$ | ($start$, $\triangleright$, $\rightarrow$) | $case2$, $a$ | ($case2$, $a$, $\rightarrow$) | $case3$, $a$ | ($case3$, $a$, $\rightarrow$) |
| $start$, $a$ | ($case2$, $\sqcup$, $\rightarrow$) | $case2$, $b$ | ($case2$, $b$, $\rightarrow$) | $case3$, $b$ | ($case3$, $b$, $\rightarrow$) |
| $start$, $b$ | ($case3$, $\sqcup$, $\rightarrow$) | $case2$, $c$ | ($case2$, $c$, $\rightarrow$) | $case3$, $c$ | ($case3$, $c$, $\rightarrow$) |
| $start$, $c$ | ($case4$, $\sqcup$, $\rightarrow$) | $case2$, $\sqcup$ | ($\sqcup$, $\sqcup$, $\leftarrow$) | $case3$, $\sqcup$ | ($\sqcup$, $\sqcup$, $\leftarrow$) |
| $start$, $\sqcup$ | ($reject$, $\sqcup$, $-$) | | | | |
| **State** | **Transition** | **State** | **Transition** | **State** | **Transition** |
| $case4$, $a$ | ($case4$, $a$, $\rightarrow$) | $case5$, $a$ | ($case8$, $\sqcup$, $\leftarrow$) | $case6$, $a$ | ($reject$, $a$, $\rightarrow$) |
| $case4$, $b$ | ($case4$, $b$, $\rightarrow$) | $case5$, $b$ | ($reject$, $b$, $\rightarrow$) | $case6$, $b$ | ($case8$, $\sqcup$, $\leftarrow$) |
| $case4$, $c$ | ($case4$, $c$, $\rightarrow$) | $case5$, $c$ | ($reject$, $c$, $\rightarrow$) | $case6$, $c$ | ($reject$, $c$, $\rightarrow$) |
| $case4$, $\sqcup$ | ($\sqcup$, $\sqcup$, $\leftarrow$) | $case5$, $\sqcup$ | ($accept$, $\sqcup$, $-$) | $case6$, $\sqcup$ | ($accept$, $\sqcup$, $-$) |
| **State** | **Transition** | **State** | **Transition** | | |
| $case7$, $a$ | ($reject$, $a$, $\rightarrow$) | $case8$, $a$ | ($case8$, $a$, $\leftarrow$) | | |
| $case7$, $b$ | ($reject$, $b$, $\rightarrow$) | $case8$, $b$ | ($case8$, $b$, $\leftarrow$) | | |
| $case7$, $c$ | ($case8$, $\sqcup$, $\leftarrow$) | $case8$, $c$ | ($case8$, $c$, $\leftarrow$) | | |
| $case7$, $\sqcup$ | ($accept$, $\sqcup$, $-$) | $case8$, $\sqcup$ | ($start$, $\sqcup$, $\rightarrow$) | | |

```
input: 'aacbbcaa'
blank: ' '
start state: start
synonyms:
  accept: {R: accept}
  reject: {R: reject}

table:
  start:
    a: {write: ' ', R: case2}
    b: {write: ' ', R: case3}
    c: {write: ' ', R: case4}
    ' ': reject
  case2:
    [a,b,c]: R
    ' ' : {L: case5}
  case3:
    [a,b,c]: R
    ' ' : {L: case6}
```

```
case4:
  [a,b,c]: R
  ' ' : {L: case7}
case5:
  [b,c]: {R: reject}
  a: {write: ' ', L: case8}
  ' ': accept
case6:
  [a,c]: {R: reject}
  b: {write: ' ', L: case8}
  ' ': accept

case7:
  [a,b]: {R: reject}
  b: {write: ' ', L: case8}
  ' ': accept
case8:
  [a,b,c]: L
  ' ' : {R: start}

accept:
reject:

```
Listing 3: TM Visualization code

Figure 3: Diagram of all the transitions and the result of an example.




The input for this TM is *aacbbcaa* and the output is *accept*. If we enter as input *abbca* the output is *reject*.

## 2 Conclusions

As a personal opinion, I think is easier to understand how a TM works if we drafted as a graph first. It also helps to visualize the transition tables and see where it can lead to an error. One of the tricks that I learned at the end of this exercise is to start with a small example, and then ask yourself more difficult questions every time you successfully finish one TM, to see if it still works properly.