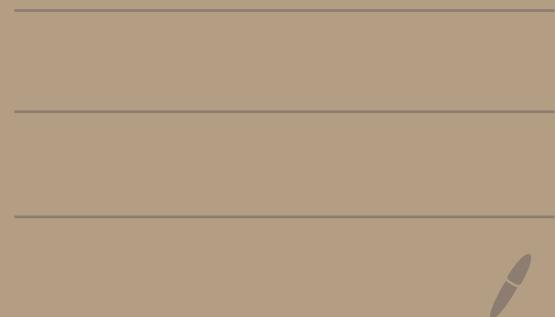


Lecture 21 VAE



$$z \sim P(z), \underbrace{N(0, 1)}$$

$$x \sim P(x; f(z; \theta))$$

$$x \sim N(\mu, \sigma^2)$$

$$\mu = f_c(z; \theta)$$

$$\sigma^2 = f_v(z; \theta)$$

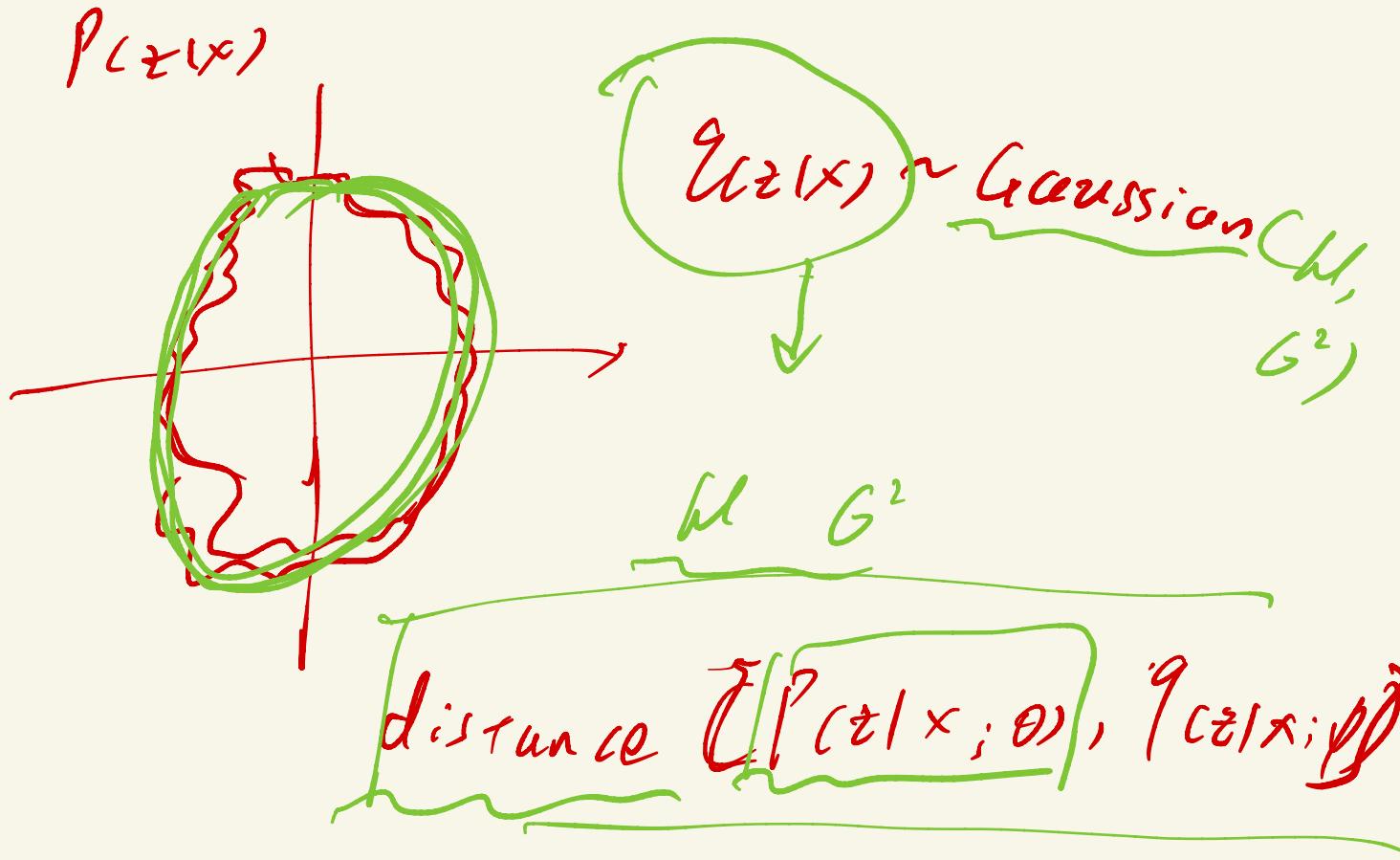
$$P(x) = \int_z P(z) P(x|z) \quad x \sim N(\mu, \sigma^2)$$

$$Q_{xz} = \boxed{P(z|x)} \times \boxed{\begin{matrix} P(z) & P(x|z) \end{matrix}}$$

Gaussian Gaussian

$P(z|x)$ is not Gaussian

$$\boxed{E_{z \sim Q_{xz}} \log P(x, z; \theta)}$$



$$\log P_\theta(x) \geq \text{ELBO}$$

unrelated
to ϕ

$$Q(z) = P(z|x)$$

$$\log P_\theta(x) = \text{ELBO}$$

$$\text{ELBO} = \underbrace{\log P_\theta(x)}_{\text{arg max } \phi} - \text{KL}[Q_\phi(z|x) || P(z|x)]$$

$$\text{arg max}_\phi \text{ELBO} = \text{arg min}_\phi \text{KL}[Q_\phi(z|x) || P(z|x)]$$

We cannot compute $P_{CZ}(x)$



$$\underbrace{q_{CZ}(x)}_{\text{Exponential}} \rightarrow P_{CZ}(x)$$

$$\arg \max_{q_{CZ}(x)} \text{ELBO}$$

Exponential

$\overbrace{\hspace{10em}}$

Variational EM

$\overbrace{\hspace{10em}}$
 $P_{CZ}(x)$

$$\arg \max_{\theta} E_{z \sim q_{CZ}(x; \phi)} \log \frac{p_{Cx, z; \theta}}{q_{CZ}(z; \phi)}$$

$z \sim q_{CZ}(x; \phi)$

$$\arg \max_{\phi} E_{z \sim q_{CZ}(x; \phi)} \log \frac{p_{Cx, z}}{q_{CZ}(z)} \text{ function}$$

$$z = g_z - \dots$$

$z \sim q_{CZ}(x; \phi)$

$$\log \frac{p_{Cx, z^{(1)}}}{q_{CZ^{(1)}}(z^{(1)})} \text{ of } \phi$$

$$\varepsilon \sim N(0, 1)$$

$$z = \underbrace{\mu}_{\text{---}} + \underbrace{G \odot \varepsilon}_{\downarrow}$$

Element-wise multipl:

what is the distribution of z



$$z \sim N(\mu, G)$$

$$q(z|x)$$

$$z = \mu + G \odot \varepsilon$$

$$z \sim q_{\pi}(z|x; \phi)$$

$$\boxed{z = g(x, \phi, \varepsilon)}$$

z discrete $\underbrace{q_{\pi}(z|x)}_{\text{not Gaussian}}$

$\boxed{\text{Gumbel / softmax}}$

$$\text{ELBO} = \log P(x) - \text{KL}(q_{cz|x}, P_{cz|x})$$

$$\mathbb{E}_{z \sim q_{cz|x}} [\log P(x|z)] + \text{KL}(q_{cz|x} || P_{cz|x})$$

Diagram illustrating the components of the ELBO:

- $q_{cz|x}$: A red oval representing the latent variable distribution.
- $\mathbb{E}_{z \sim q_{cz|x}} [\log P(x|z)]$: A red oval representing the expected log likelihood term.
- $P_{cz|x}$: A red oval representing the true conditional distribution.
- $\text{KL}(q_{cz|x} || P_{cz|x})$: A red oval representing the KL divergence term.
- $N(0, 1)$: A red box representing a standard normal distribution.

E-step

$$q_{Ct|X} \approx P_{Ct|X}$$

θ

optimize $q_{Ct|X; \theta}$ till convergence

$$P_{Ct}$$

$$x_C$$

$$P_{Ct, X}$$

$$z \sim P_{Ct}$$

$$x \sim P_{X|Ct}$$

VAE model $P(x|z_1, z_2)$

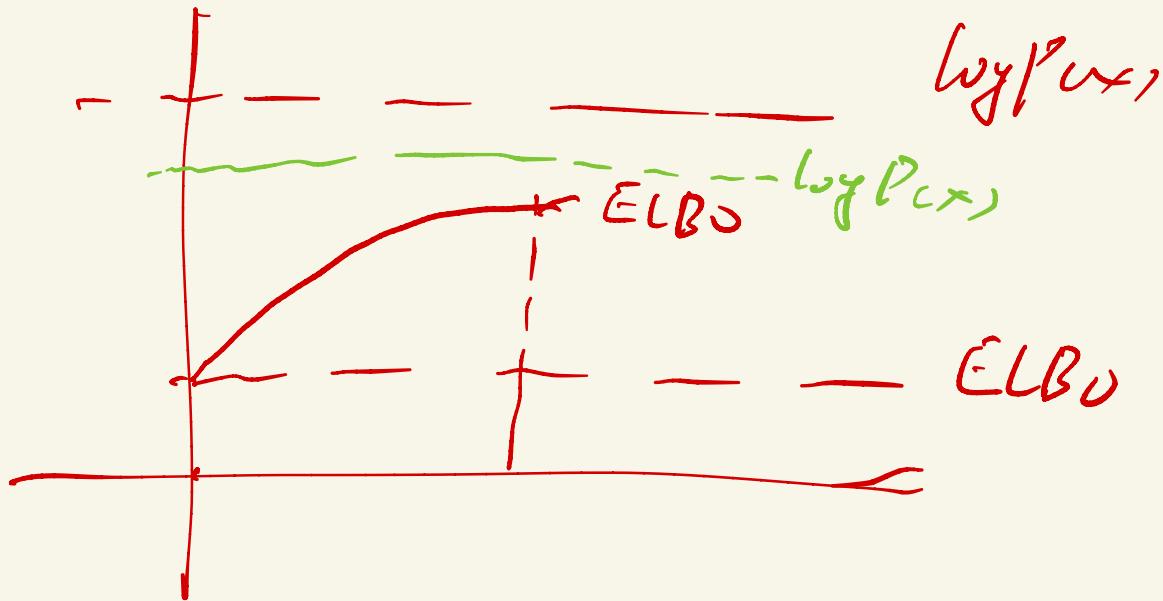
VAE inference

$$\mathcal{L} = \text{ELBO}$$

$\log P(x)$

E-stop

$$\underbrace{q(z|x) \approx p(z|x)}$$



$P(x|z)$

does not depend on z
practically

$$E_2 \sim q_{\phi}(z|x), [\log P_{\phi}(x|z)]$$

~~~~~

auto encoder

$$P(z) \sim \mathcal{N}(0, 1)$$

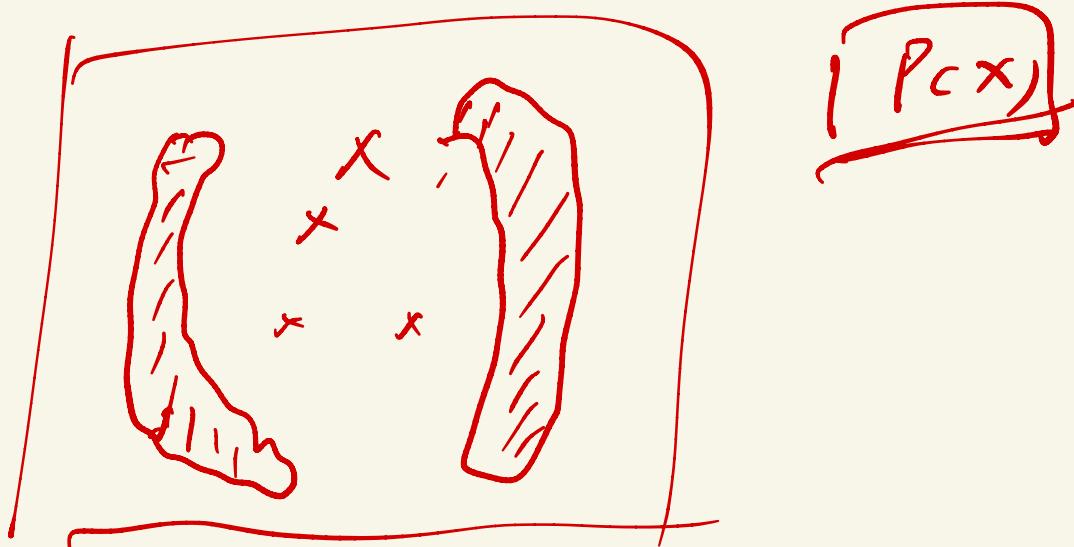
( $z$ )

$z \sim$  how to sample  $z$

$$z \sim \mathcal{N}(0, 1)$$

$\tilde{z} \sim N(0, 1)$

$\boxed{P(z)}$



VAE:

$$\begin{cases} x \sim N(\mu, G^t) \\ \mu, G^t = f(z; \phi) \end{cases}$$

$$\underline{N(0, 1)}$$

GAN:

$$P(x) = \begin{cases} P(x|z), & z \\ P(-z), & -z \end{cases}$$

$x = G(z)$

$$\begin{cases} x \sim N(\mu, G^t) \end{cases}$$

$E_{z \sim q_{CZ}(x)}$ ,  $\log P_{CZ}(z)$

$t = \infty$

$\tilde{z}^{(t)}$   $\sim q_{\pi}(z|x)$

$\tilde{x}$