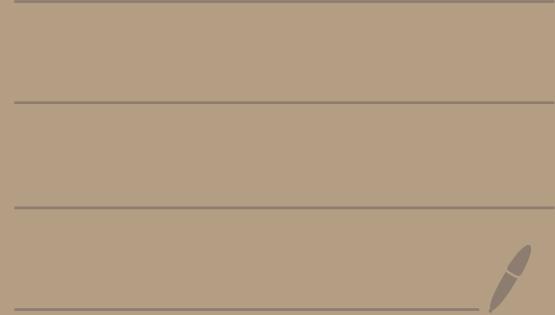


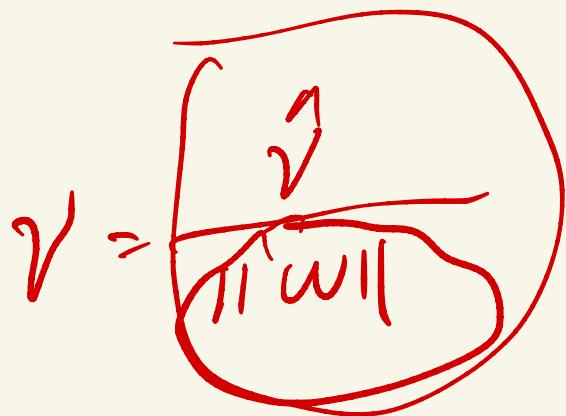
Lecture 7 SVM



$\gamma^{(i)}$ geometric margin

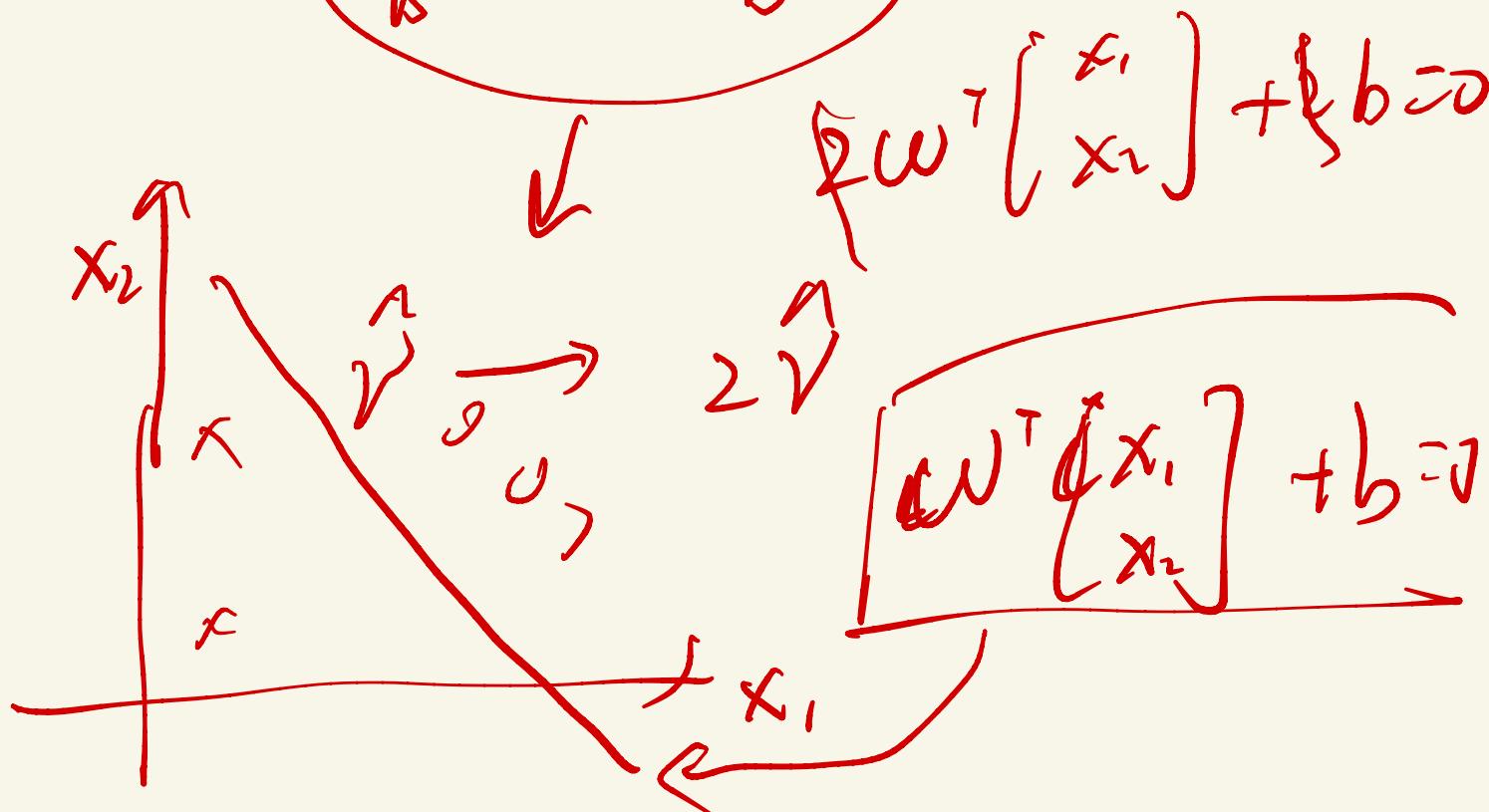
$$\|w\|$$

$\hat{\gamma}$ functional margin



$$\hat{y} = g(w^T x + b)$$

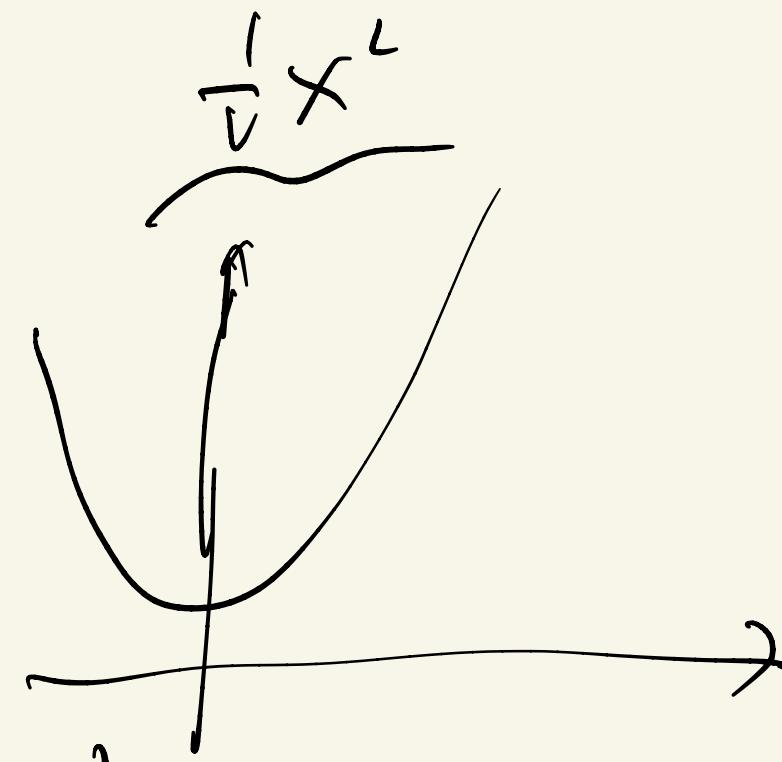
$$w \rightarrow 2w$$
$$b \rightarrow 2b$$



$$\|\omega\| = 1, 2$$

$$\hat{V} = 1$$

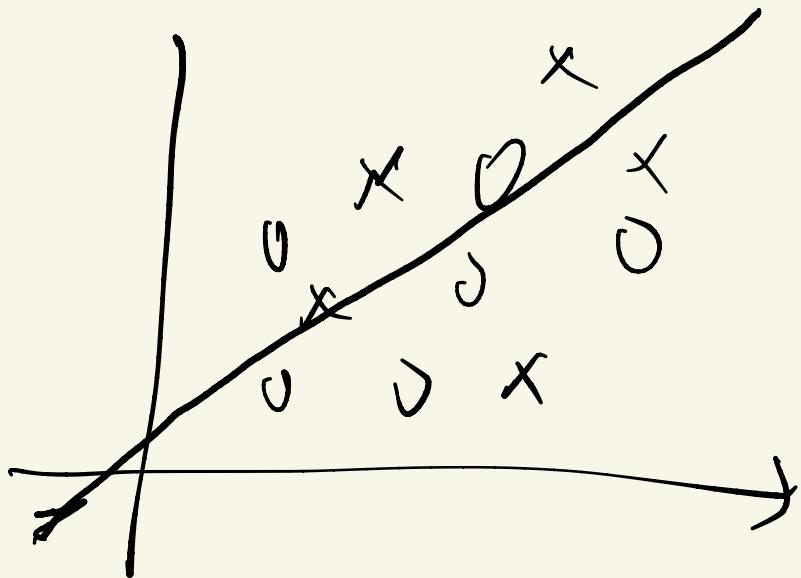
$$\hat{V} = 1$$



$$\frac{\hat{V}^a}{\|\omega\|}$$

$$\frac{1}{\|\omega\|}$$

$$\arg \max_w \frac{1}{\|\omega\|} = \arg \min_w \frac{1}{2} \|\omega\|^2$$



$$y^{(i)} \leftarrow w^T x^{(i)} + b$$

$\leftarrow 0$

$$L(cw, \beta) = f(cw) + \sum_{i=1}^L \underbrace{\beta_i}_{\gamma_i} h_i(cw)$$

$$\frac{\partial L}{\partial \beta_i} = h_i(cw) =_0 \neq 0$$

$$\underbrace{f(cw)}_{\gamma_0} + \sum_{i=1}^L \underbrace{\alpha_i g_i}_{\gamma_i}(cw) \leq 0$$

$$g_i(cw) > 0$$

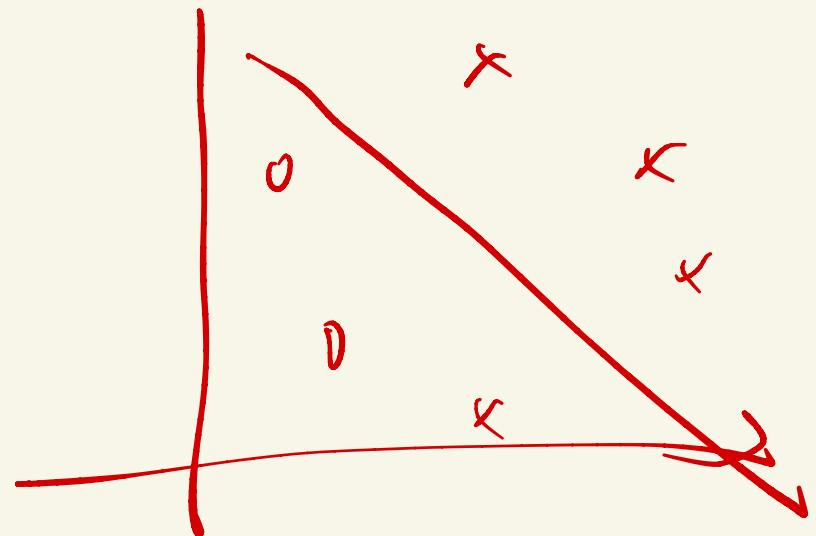
$$\underbrace{\alpha_i}_{\gamma} \geq 0$$

$\theta_{pcw} =$
 f(w) if w satisfies
 constraints
 ∞ otherwise

$\boxed{\min_w \theta_{pcw}} \equiv \min_w f(w)$
 under the
 primal const

$p^* = \min_w f(w)$
 under the --

$$\begin{aligned} w^T x \\ w^T \phi(x) \end{aligned}$$



$$\phi(x) = \begin{bmatrix} x_1 \\ x_1^2 x_2 \\ x_2^2 \\ x_2^3 \end{bmatrix} \quad \leftarrow n_{\phi} \text{ need}$$

$$k(x_1, x_2) = \langle \phi(x_1), \phi(x_2) \rangle$$

$$k(x, z) = (x^T z)^d \quad \text{and} \quad \phi(x) \rightarrow d^*$$

$\Theta_P(\omega)$ func of ω

$\Theta_D(\alpha, \beta)$ func of α, β

$$d^* \leq p^*$$

$$\max_x \min_y f(x, y) \leq \min_y \max_x$$

convex function def.

$$\underline{f(x)}$$

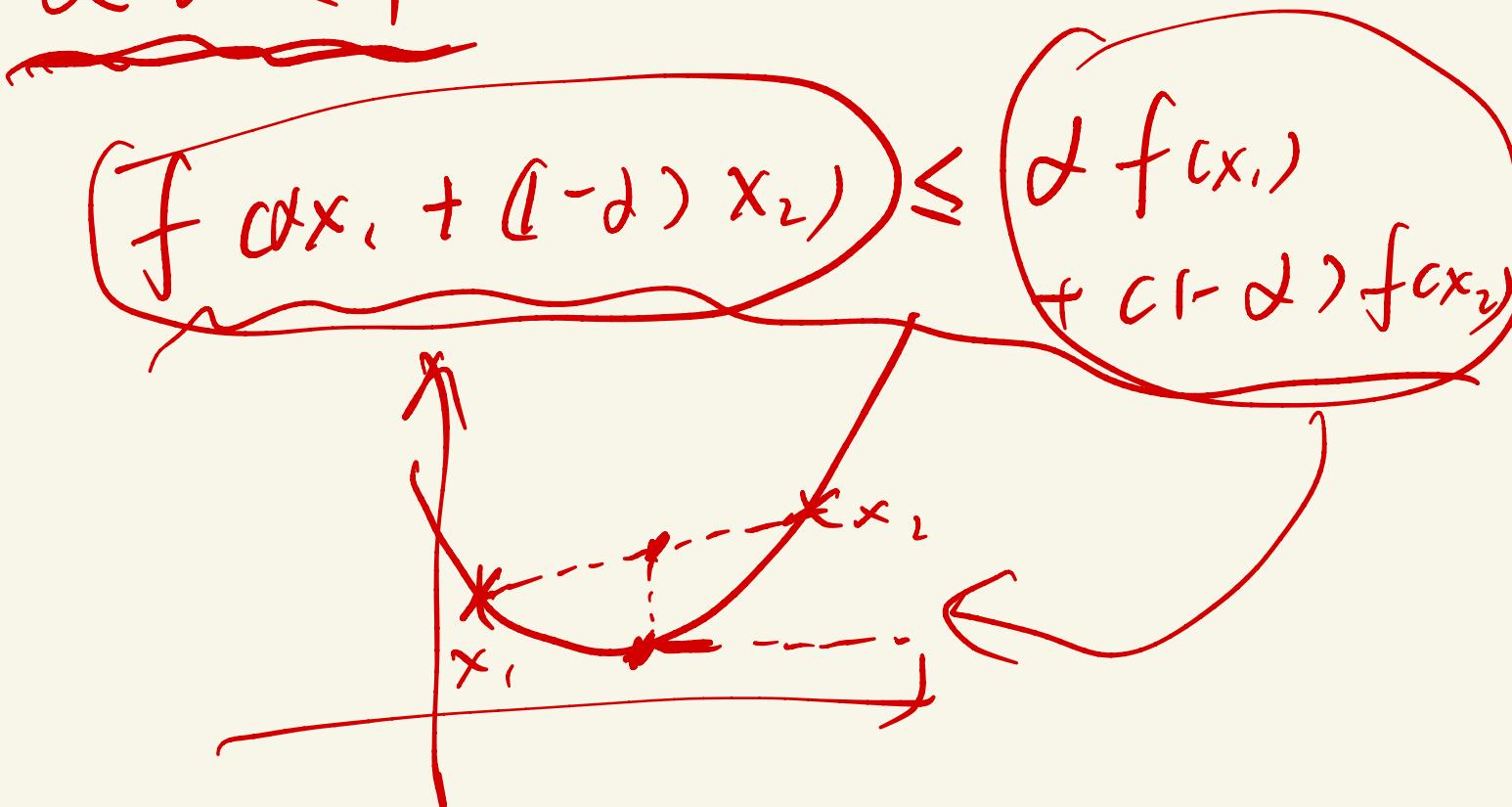
$$f(x, y)$$

$$\alpha \beta \leq 1$$

$$(f(\alpha x_1 + (-\beta) x_2)) \leq$$

$$\alpha f(x_1)$$

$$+ (1-\alpha) f(x_2)$$



$$g_i(cw) < 0$$

SVM :

$$f(w) = \underbrace{\frac{1}{2} \|w\|^2}$$

$$g(w) = 1 - y(w^T x + b)$$

$$y(w^T x + b) \geq 1$$

$$g(w) \leq 0$$

$$\rho^* = d^*$$

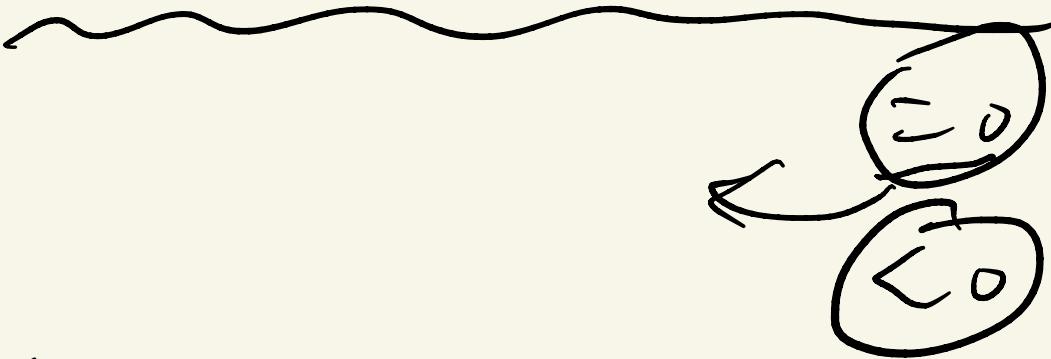
Slater condition \Rightarrow strong duality

strong duality \Leftrightarrow KKT

$\alpha_i^* g_i(cw^*)$ solution to $\Theta_{DFX, \beta}$ w^* $\alpha_i^* \beta_i^*$ solution to $\Theta_P(cw)$ $\boxed{\alpha_i^* g_i(cw^*) = 0}$ $g_i(cw^*) \leq 0$ $g_i(cw^*) < 0 \rightarrow \alpha_i = 0$ g

$$y^{(c_i)} \langle w^\top x^{(c_i)} + b \rangle \geq 1$$

$$\underbrace{g_i(w^*)}_{=} = 1 - y^{(c_i)} \langle w^\top x^{(c_i)} + b \rangle \leq 0$$



$$\underbrace{\partial_i g_i(w^*)}_{=} = 0$$

$$\left| \sum_{i=1}^n \alpha_i g^{(c_i)} = 0 \right|$$

$L(\omega, b, \alpha)$

$$\theta_D(\alpha)$$

$$\max_{\omega} \frac{1}{2} \|\omega\|^L \rightarrow \max_{\alpha}$$

$$\langle \underbrace{x^{(i)}, x^{(j)}}_{\downarrow}, \rangle = \underbrace{x^{(i)\top} x^{(j)}}_{K(x^{(i)}, x^{(j)})}$$

$$\min y^{(c_i)} w^*{}^\top x^{(c_i)} + b^* = 1$$

~~w^{*}~~

$$y^{(c_i)} = 1$$

$$\min w^*{}^\top x$$

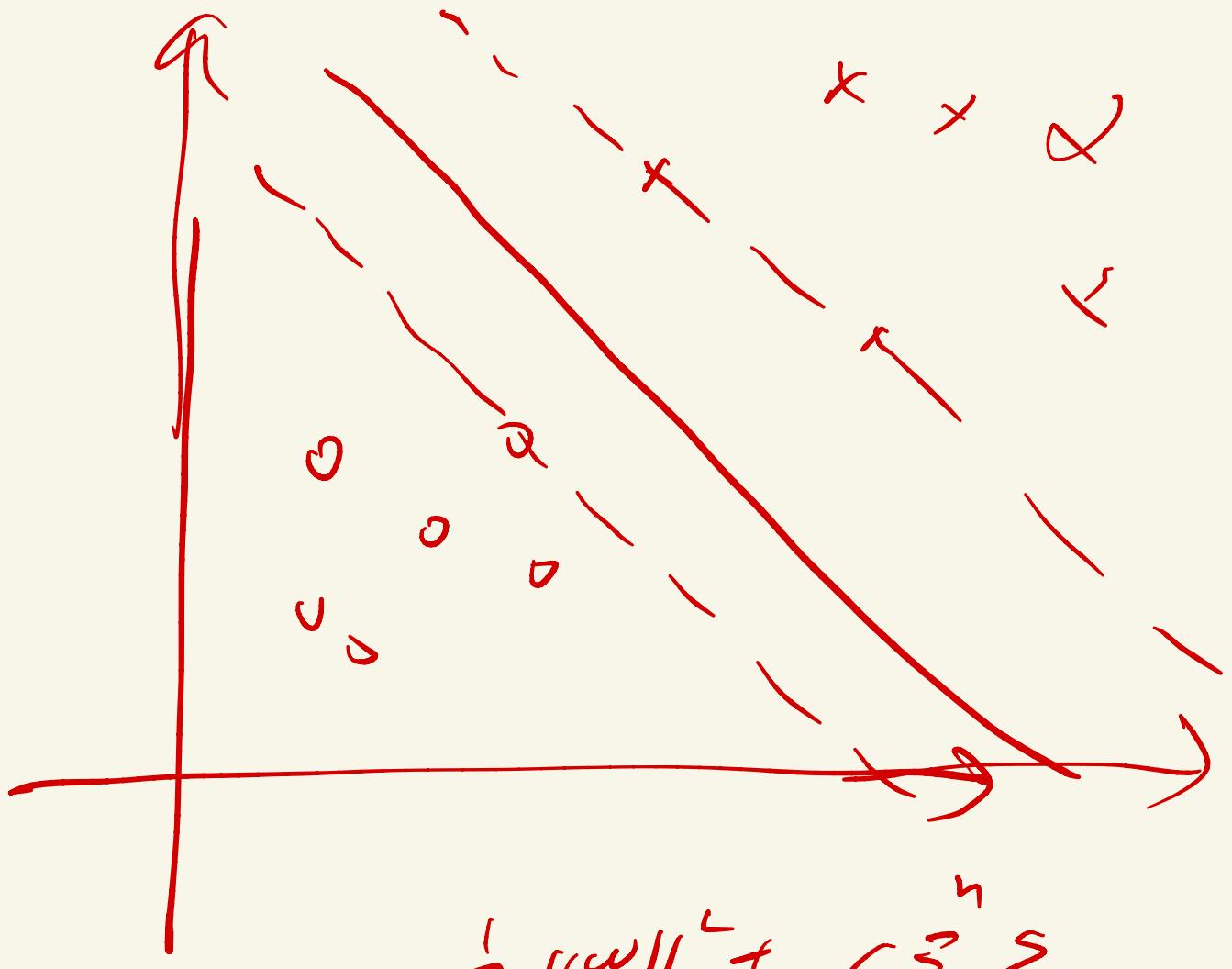
$$y^{(c_j)} = -1$$

$$\max w^*{}^\top x$$

$$\phi(x) = \sum_{i=1}^n \alpha_i y^{c_{i1}} (cx^{a_{i1}}, x) + b$$

Diagram illustrating the components of the function $\phi(x)$:

- The term $\sum_{i=1}^n \alpha_i y^{c_{i1}}$ is circled in red and labeled "3 items".
- The term $(cx^{a_{i1}}, x)$ is enclosed in a black brace and labeled "3 vectors".
- The constant term b is shown below the main equation.



$$\frac{1}{2} \|w\|^2 + C \sum_i \xi_i$$

ξ_i

$$1 - \xi_i$$

$$\boxed{\xi_i \geq 0}$$