MATH324 (Statistics) – Lecture Notes
McGill University

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3.1 Markov's Inequality

Let X be a random variable and h be a **non-negative** function; ie:

$$h : R \rightarrow R^{+} \ \cup \ \{0\} \quad = \quad [0, \infty)$$

Suppose $E(h(x)) \ < \infty$,then for some $\lambda > 0$, we have:

$$P(h(x) \geq \lambda) \ \leq \ \frac{E[h(x)]}{\lambda} \tag{1}$$

Proof. Suppose X is a continuous random variable:

$$\begin{aligned}
E[h(x)] &= \int_x h(x) d_x(x) dx \\
&= \Big(\int_{x:h(x)\geq\lambda} h(x) f_x(x) dx + \int_{x:h(x)<\lambda} h(x) f_x(x) dx \Big) \\
&\geq \int_{x:h(x)\geq\lambda} h(x) f_x(x) dx \qquad\qquad \underline{since}\ h \geq 0 \\
&\geq \lambda \int_{x:h(x\geq\lambda} f_x(x) dx = \lambda \ P(h(x) \geq \lambda)
\end{aligned}$$

$$\implies P(h(x) \geq \lambda) \leq \frac{E(h(x))}{\lambda}$$

The proof for the discrete case is similar. □

Now consider $h(x) = (x - \mu)^2$, then:

$$\begin{aligned}
P(|x - \mu| \geq \lambda) &= P((x-\mu)^2 \geq \lambda^2) \\
&\leq \frac{E[(x-\mu)^2]}{\lambda^2} \qquad\qquad if \ E[(x-\mu)^2] < \infty
\end{aligned}$$

Let $\mu = E(X)$, then $E[(x-\mu)^2] = Var(X)$ denoted by σ_x^2. We therefore have:

$$P(|x - \mu_x| \geq \lambda) \leq \frac{\sigma_x^2}{\lambda^2} \qquad\qquad where \ \mu_x = E(x) \tag{2}$$

Now consider $\lambda = K\sigma_x$ where K is a known number. Then:

$$P(|x - \mu_x| \geq K\sigma_x) \geq \frac{\sigma_x^2}{K^2\sigma_x^2} = \frac{1}{K^2} \tag{3}$$

3.2 Chebyshev's Inequality

Chebyshev's Inequality is a special case of Markov's Inequality.

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