Truncation Error in Euler's Method

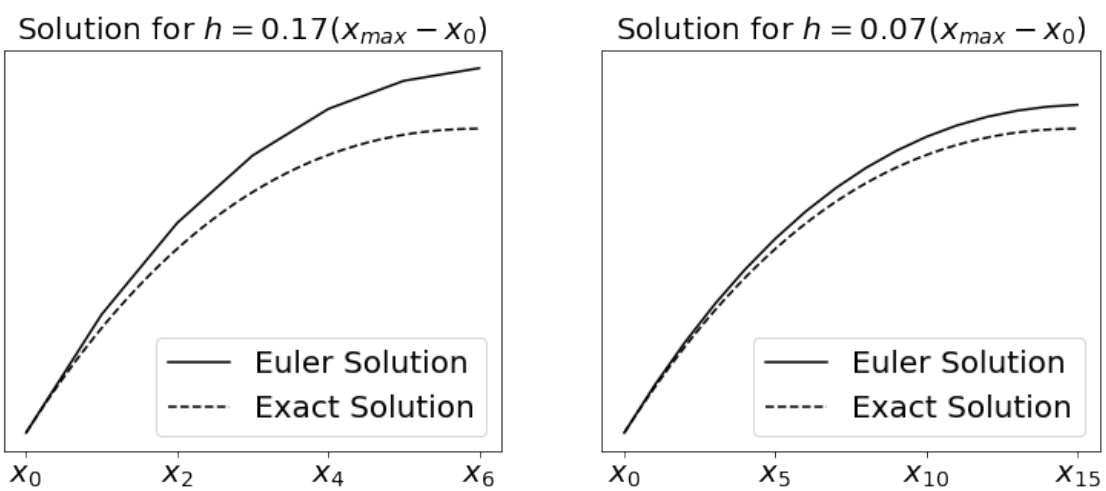
Euler's Method: Truncation Error

Like all numerical methods, Euler's method has systemic error. This is introduced when we discard the higher order terms in the Taylor expansion. The **local** truncation error is thus:

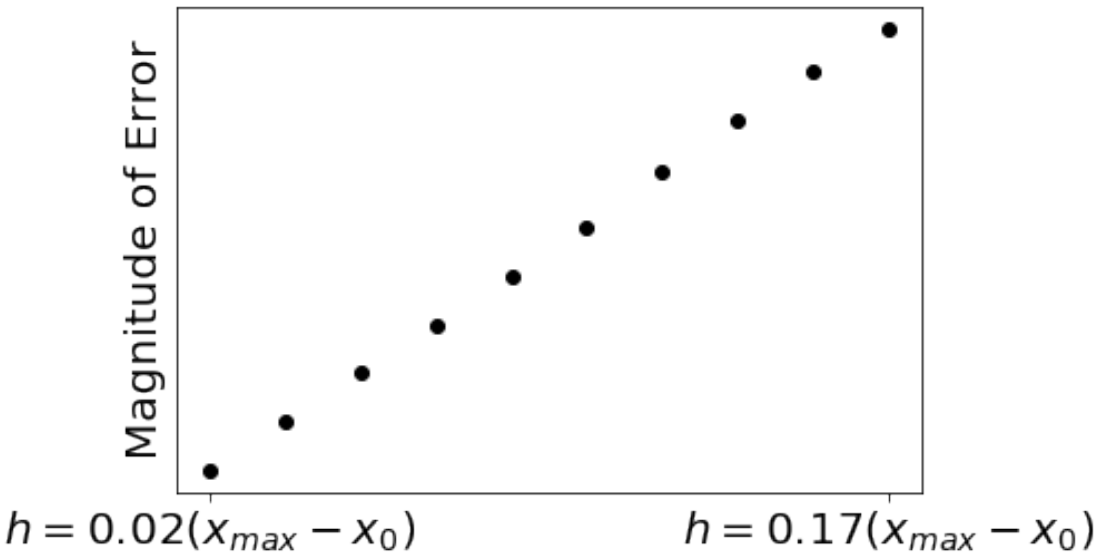
$$E_{n+1} = \frac{1}{2}y''(x_n)h^2 + (h^3)$$

If you are unfamiliar with the notation for (h^3) (big O notation), in this case it stands for all the terms where the lowest order of h is 3. The h^3 term is more relevant than higher order terms for $0 < h < 1$.

The **local** truncation error is associated with a single integration step. It is far more useful, however, to consider the **global** truncation error, which is the error accumulated over multiple integration steps. The global truncation error is (h) {% cite efferson-numerical-methods %}. The derivation for the bounds of the error are beyond the scope of the course. As this error approximately scales linearly with h, reducing the size of h will generally reduce the global error:



We can illustrate the relationship between the global error and h directly by looking at the magnitude of error at the same final x value for different h values:



There is a limit to how much reducing h will help you. If h is too small you could introduce floating point errors, that is when operations require more precision than afforded by the float data type. Reducing the size of h also means that you will have more steps to integrate to a final x, which increases the computational time.

References

{% bibliography –cited %}