Newton-Raphson Method

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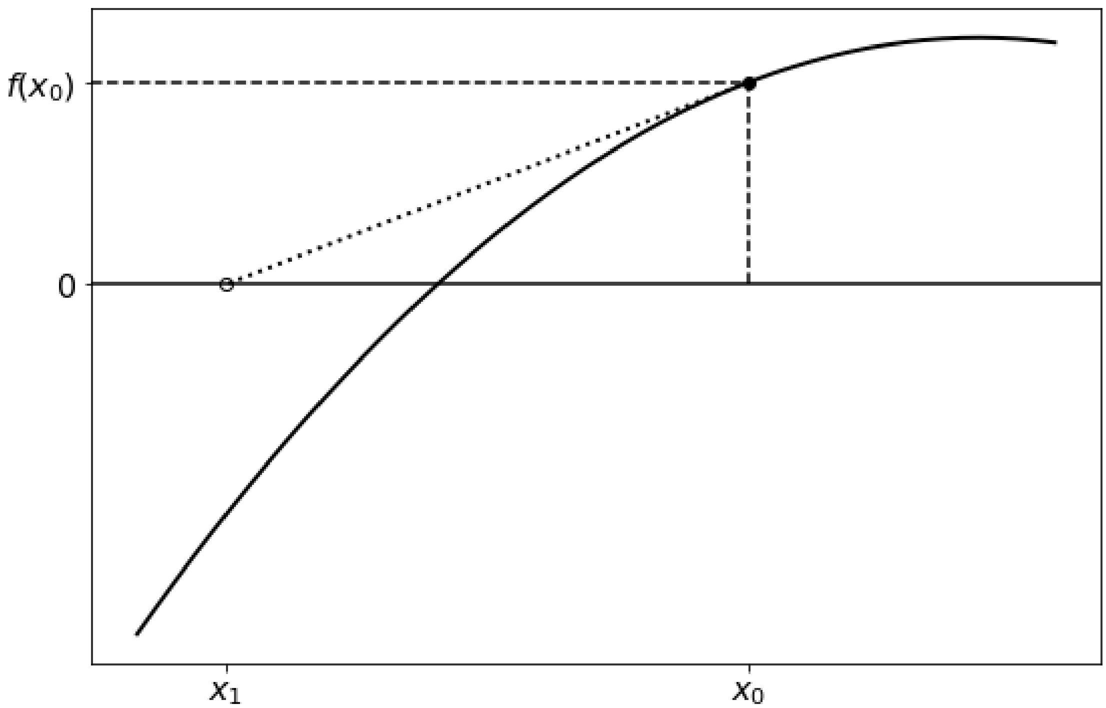
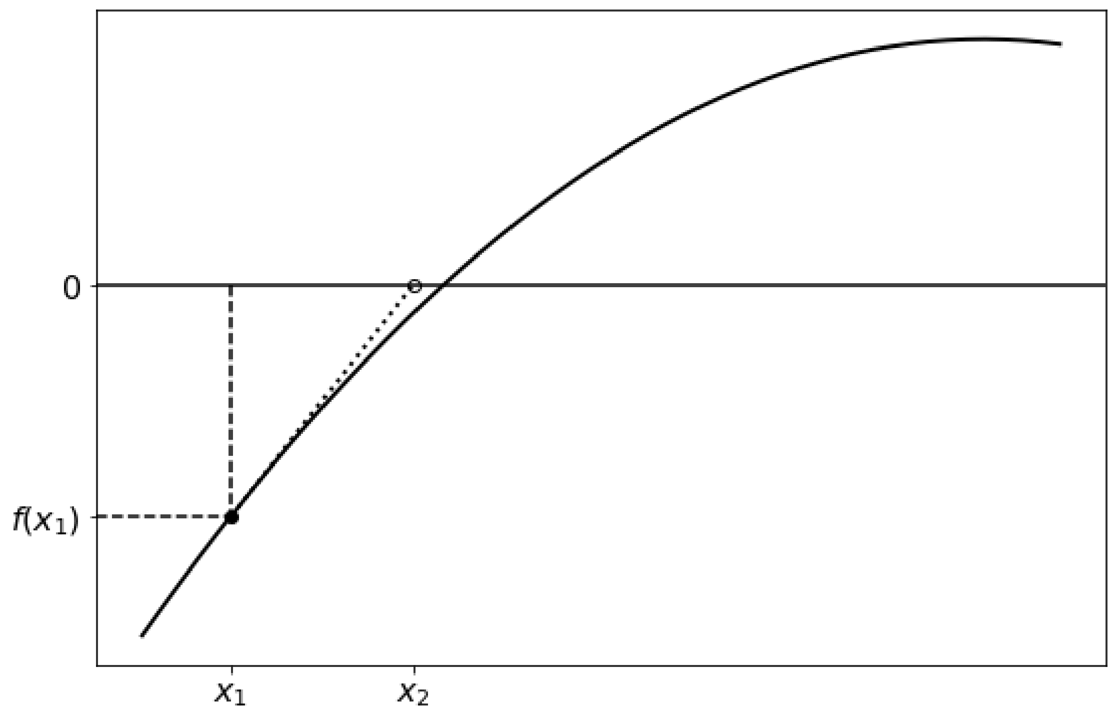
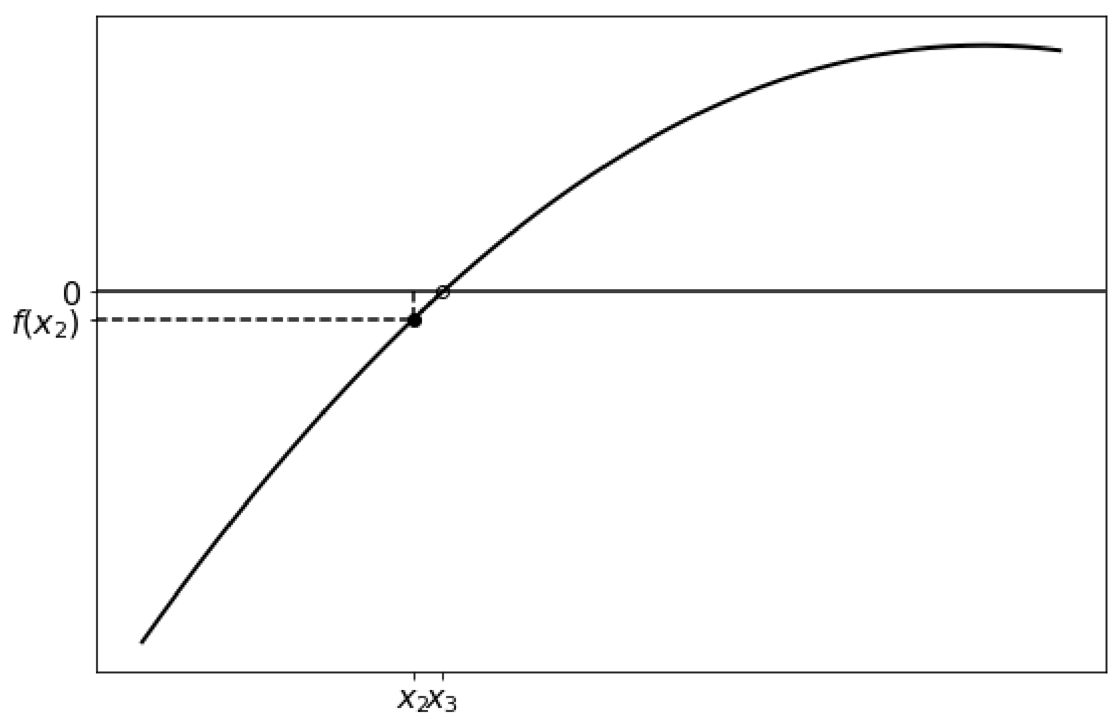
The Newton-Raphson method is similar to the secant method, except here we construct a straight line that passes through a point $(x_0, f(x_0))$ with a gradient of $f'(x_0)$, the tangent of $f(x)$ at that point. The next point, x_1, is the intersection of this line with the x-axis:



As before, this process can be repeated with x_1, and the rest of the points after it, converging closer to the root. Further iterations are illustrated in the following figures:





To calculate the point x_n using the previous point x_{n-1}, we start by constructing the line running through $(x_{n-1}, f(x_{n-1}))$:

$$\frac{y - f(x_{n-1})}{x - x_{n-1}} = f'(x_{n-1})$$

at the x-intercept, $y = 0$ and $x = x_n$:

$$\frac{0 - f(x_{n-1})}{x_n - x_{n-1}} = f'(x_{n-1})$$

$$\therefore x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

Precision

Similarly to the secant method, the precision for the Newton-Raphson method can be set for a given tolerance by finding n such that:

$$|x_n - x_{n-1}| < \text{tolerance}$$

Instability

The Newton-Raphson method suffers from much the same issues as the secant method.