## Truncation Error in Euler's Method

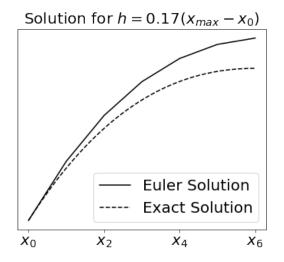
## **Euler's Method: Truncation Error**

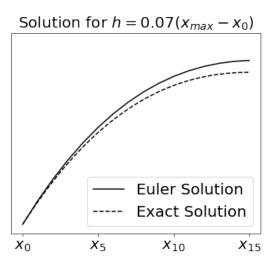
Like all numerical methods, Euler's method has systemic error. This is introduced when we discard the higher order terms in the Taylor expansion. The **local** truncation error is thus:

$$E_{n+1} = \frac{1}{2}y''(x_n)h^2 + (h^3)$$

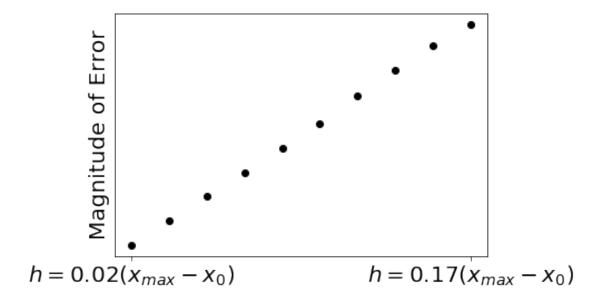
If you are unfamiliar with the notation for  $(h^3)$  (big O notation), in this case it stands for all the terms where the lowest order of h is 3. The  $h^3$  term is more relevant than higher order terms for 0 < h < 1.

The **local** truncation error is associated with a single integration step. It is far more useful, however, to consider the **global** truncation error, which is the error accumulated over multiple integration steps. The global truncation error is (h) {% cite efferson-numerical-methods %}. The derivation for the bounds of the error are beyond the scope of the course. As this error approximately scales linearly with h, reducing the size of h will generally reduce the global error:





We can illustrate the relationship between the global error and h directly by looking at the magnitude of error at the same final x value for different h values:



There is a limit to how much reducing h will help you. If h is too small you could introduce floating point errors, that is when operations require more precision than afforded by the float data type. Reducing the size of h also means that you will have more steps to integrate to a final x, which increases the computational time.

## References

 ${\% \text{ bibliography } -\text{cited } \%}$