



SPRING - MASS - DAMPER  
SYSTEM

EQUATION OF MOTION

$$M\ddot{x} + c\dot{x} + kx + \underbrace{k_3 x^3}_{3^{\text{rd}} \text{ order nonlinearity}} = F \sin(\omega t + \varphi)$$

The solution to the equation above is assumed to be:

$$x = A \sin(\omega t)$$

$$x^3 = A^3 \sin^3(\omega t)$$

$$= \frac{3}{4} A^3 \sin \omega t - \frac{1}{4} A^3 \sin(3\omega t)$$

$$\dot{x} = A\omega \cos(\omega t)$$

$$\ddot{x} = -A\omega^2 \sin(\omega t)$$

$\varphi$  reflects the phase required  
to have this solution  
To make things easier

Sub

For the equations to have solutions, the co-efficients of  $\sin(\omega t)$  &  $\cos(\omega t)$  have to be equal on LHS & RHS.

$$\sin \omega t: -MA\omega^2 + kA + \frac{3}{4} k_3 A^3 = F \cos \varphi$$

$$\cos \omega t: cA\omega = F \sin \varphi$$

EQUATIONS  
IN THE CODE

$$\begin{cases} \omega = x \\ A = y \end{cases}$$