School of Mathematics and Physical Science

Forecasting of GDP (Gross Domestic Product) and Inflation with Time Series Model and Machine Learning Model

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In making this submission I declare that the information contained on this cover sheet is correct and that the content of this dissertation is my own work.

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Abstract

This dissertation tackles the complex subject of "Time Series Forecasting of GDP and Inflation," focusing on the application of mathematical models and historical data to anticipate the economic futures of four significant economies: the United States, China, Japan, and the United Kingdom. The primary objective is to compare the performance of three prominent forecasting methods—ARIMA (Auto Regressive Integrated Moving Average), SARIMA (Seasonal Auto Regressive Integrated Moving Average), and Random Forest, a machine learning technique.

The literature review highlights a conspicuous gap in comparative studies directly evaluating ARIMA, SARIMA, and Random Forest models for GDP and inflation forecasting. Although extensive research exists on various forecasting methods, a direct comparison of these three approaches remains scarce. This dissertation aims to fill this gap, providing a comprehensive analysis of each model's effectiveness across different economic contexts.

The selection of the United States, China, Japan, and the United Kingdom for this study is deliberate, reflecting diverse economic landscapes. The United States and the United Kingdom represent developed economies, China is a rapidly emerging economy, and Japan, although developed, possesses unique economic characteristics. By including these varied economic profiles, the research seeks to evaluate the robustness and applicability of the forecasting models in different scenarios, akin to testing a recipe across multiple kitchens to ensure consistent results.

Data for GDP and inflation is sourced from the World Bank Database, recognized globally for its reliability and accuracy. This data serves as the foundation for analyzing the predictive capabilities of ARIMA, SARIMA, and Random Forest models, enabling a thorough examination of their performance.

Expected outcomes of this research include a detailed comparison of the models' accuracy in forecasting GDP and inflation. It is anticipated that ARIMA may excel in capturing short-term economic fluctuations, while SARIMA and Random Forest models might be more effective in identifying trends with seasonal patterns. The analysis aims to uncover insights into the strengths and limitations of each model, offering valuable guidance for policymakers and analysts in selecting the most appropriate forecasting tool for their needs.

By addressing the current research gap and providing a nuanced understanding of the comparative performance of ARIMA, SARIMA, and Random Forest models, this dissertation contributes significantly to the field of economic forecasting. The findings are expected to enhance the precision of economic predictions, thereby supporting more informed decision-making processes in government and business sectors.

Introduction

Forecasting economic indicators such as Gross Domestic Product (GDP) and inflation is a crucial task that plays a pivotal role in guiding policy decisions, business strategies, and financial planning. The ability to predict future economic conditions accurately allows governments and businesses to prepare for potential changes, allocate resources efficiently, and implement measures to stabilize the economy. Despite its importance, economic forecasting remains a complex challenge due to the myriad factors influencing economic activities and the dynamic nature of economies.

In recent years, advancements in statistical methods and machine learning algorithms have opened new avenues for improving the accuracy of economic forecasts. Traditional time series models like ARIMA (Auto Regressive Integrated Moving Average) and its seasonal counterpart, SARIMA (Seasonal Auto Regressive Integrated Moving Average), have long been used for economic forecasting due to their simplicity and effectiveness in capturing temporal dependencies in data. However, the emergence of machine learning techniques, particularly Random Forest, has introduced new possibilities for enhancing forecasting accuracy by leveraging complex, non-linear relationships within the data.

This dissertation aims to explore and compare the performance of ARIMA, SARIMA, and Random Forest models in forecasting GDP and inflation for four major economies: the United States, China, Japan, and the United Kingdom. These countries were chosen for their diverse economic profiles, providing a broad spectrum for analysis and comparison. The United States and the United Kingdom represent developed economies with stable economic environments, China is a rapidly emerging economy with significant growth potential, and Japan, while also a developed economy, faces unique economic challenges.

The study is motivated by the need to address a critical gap in the existing literature: the lack of direct comparative analyses of ARIMA, SARIMA, and Random Forest models in the context of GDP and inflation forecasting. Although each of these methods has been extensively studied individually, there is limited research that directly compares their performance using the same datasets and criteria. By filling this gap, this dissertation seeks to provide valuable insights into the relative strengths and weaknesses of these models, thereby aiding policymakers and analysts in selecting the most appropriate tools for economic forecasting.

In the following sections, the dissertation will delve into the background and motivation for the study, outline the specific research objectives, describe the datasets used, and present the overall structure of the dissertation. Each of these sections will provide a detailed foundation for understanding the context, methodology, and significance of the research.

Background and Motivation

Economic forecasting is a cornerstone of economic planning and decision-making. Accurate forecasts of GDP and inflation help in the formulation of fiscal and monetary policies, budget planning, investment strategies, and risk management. Traditional statistical methods like ARIMA and SARIMA have been widely used for their ability to model time series data effectively. However, the advent of machine learning has introduced powerful tools like Random Forest, which can capture complex patterns and interactions that traditional models might miss.

Despite the advancements in forecasting methods, there remains a significant gap in the literature regarding the direct comparison of ARIMA, SARIMA, and Random Forest models specifically for GDP and inflation forecasting. This dissertation seeks to address this gap by conducting a comprehensive comparative analysis of these methods using data from the United States, China, Japan, and the United Kingdom. The motivation behind this study is to enhance the understanding of which methods perform best under different economic conditions, thereby providing more reliable tools for economic forecasting.

Research Objectives

The primary objectives of this dissertation are designed to advance the understanding and application of forecasting models in predicting key economic indicators such as GDP and inflation. The specific objectives are as follows:

1. **Comparative Analysis of Forecasting Models**: Conduct an exhaustive comparative analysis of ARIMA (Auto Regressive Integrated Moving Average), SARIMA (Seasonal Auto Regressive Integrated Moving Average), and Random Forest models to forecast GDP and inflation for four major economies: the United States, China, Japan, and the United Kingdom. This objective involves a detailed examination of the mathematical and statistical properties of each model. By comparing these models head-to-head, the study aims to evaluate their performance in capturing economic trends and fluctuations, thereby providing a nuanced understanding of their strengths and weaknesses. This comparative analysis is critical for identifying which models are most effective under varying economic conditions and data characteristics.
2. **Identification of Strengths and Limitations**: Investigate the specific strengths and limitations of ARIMA, SARIMA, and Random Forest models in different economic contexts. Each country in the study—representing a mix of developed and emerging economies—offers unique economic environments, including different levels of economic stability, growth patterns, and seasonal effects. This objective aims to understand how these diverse economic characteristics influence the performance of each forecasting model. By dissecting the performance metrics and error rates across different scenarios, the study seeks to highlight where each model excels and where it falls short, providing deeper insights into their applicability and robustness.
3. **Recommendations for Appropriate Forecasting Models**: Develop practical recommendations on the most appropriate forecasting models for different types of economies based on empirical evidence. By synthesizing the findings from the comparative analysis, this objective aims to offer actionable guidance to policymakers, economists, and financial analysts. The recommendations will focus on identifying the optimal conditions for using ARIMA, SARIMA, or Random Forest models, ensuring that decision-makers can select the

most reliable and accurate forecasting tools for their specific economic contexts. This objective underscores the practical relevance of the research, aiming to enhance the decision-making processes in government and business sectors by providing evidence-based insights.

4. **Contribution to Existing Literature**: Make a significant contribution to the existing literature by addressing the research gap in the comparative analysis of ARIMA, SARIMA, and Random Forest models for GDP and inflation forecasting. Despite extensive research on individual forecasting methods, there is a noticeable lack of studies that directly compare these models using the same datasets and evaluation criteria. This dissertation aims to fill this gap by providing a comprehensive, empirical comparison of these models across multiple countries. By doing so, it will add valuable knowledge to the field of economic forecasting, offering a clearer understanding of the relative efficacy of traditional time series models versus advanced machine learning techniques.

Dataset Description

The data used in this study comprises GDP and inflation figures for the United States, China, Japan, and the United Kingdom. These data were sourced from the World Bank Database, which is widely recognized for its accuracy and comprehensiveness. The dataset includes historical GDP and inflation data, which will be used to train and test the forecasting models. The selection of these four countries provides a diverse range of economic environments, enabling a thorough evaluation of the forecasting models under different conditions.

Literature Review

Forecasting economic indicators such as GDP and inflation is a critical task for policymakers, economists, and business leaders. Accurate predictions enable better decision-making, strategic planning, and policy formulation. Various methods have been developed and employed for this purpose, including traditional statistical models and modern machine learning techniques. This literature review provides an overview of the key approaches and their comparative effectiveness, highlighting the gaps that this dissertation aims to address.

Traditional Statistical Models

The ARIMA (Auto-Regressive Integrated Moving Average) model, introduced by Box and Jenkins in the early 1970s, has been a cornerstone in time series forecasting. ARIMA models are particularly valued for their simplicity and effectiveness in capturing linear relationships within the data. They decompose a time series into auto-regressive (AR) terms, differencing (I) terms, and moving average (MA) terms. Numerous studies, such as those by Hyndman and Athanasopoulos (2018), have demonstrated ARIMA's utility in short-term economic forecasting. However, ARIMA models assume linearity and stationarity, which can limit their applicability to more complex, non-linear economic phenomena.

To address seasonality, the SARIMA (Seasonal ARIMA) model extends ARIMA by incorporating seasonal differencing and seasonal AR and MA terms. Studies like those by Franses (1991) have shown SARIMA's effectiveness in capturing periodic fluctuations in economic data, making it particularly useful for predicting seasonally affected indicators like inflation. Despite its enhanced capabilities, SARIMA's complexity increases with the addition of seasonal parameters, requiring larger datasets and more computational resources.

Machine Learning Approaches

In recent years, machine learning methods have gained prominence in economic forecasting due to their ability to model complex, non-linear relationships. Random Forest, an ensemble learning technique developed by Breiman (2001), constructs multiple decision trees and aggregates their predictions. This method is robust to overfitting and capable of handling high-dimensional data. Research by Varian (2014) and others has illustrated the potential of Random Forest in economic forecasting, particularly for capturing intricate patterns that traditional models may miss.

Despite their advantages, machine learning models like Random Forest can be less interpretable than statistical models. This lack of transparency poses challenges for policymakers who need to understand the underlying drivers of the forecasts. Furthermore, machine learning models typically require more data and computational power, which can be a limitation in certain contexts.

Comparative Studies

Comparative studies that evaluate the performance of ARIMA, SARIMA, and machine learning models in forecasting economic indicators are relatively sparse. While there is substantial research on each method independently, few studies directly compare these models' effectiveness across different economic contexts. For instance, Shumway and Stoffer (2017) and Makridakis et al. (2018) provide comprehensive overviews of time series analysis and forecasting methods, but their comparative analyses are limited in scope.

Identified Gaps

The literature reveals a significant gap in research that directly compares ARIMA, SARIMA, and Random Forest models for GDP and inflation forecasting. Most existing studies focus on a single method or compare a few models without considering a broad range of economic contexts. Moreover, there is a lack of head-to-head comparisons that evaluate these models' performance in both developed and emerging economies. This gap is critical because different economic environments may influence the models' accuracy and applicability. For instance, developed economies with stable growth patterns might favor traditional models, while emerging economies with volatile conditions might benefit more from machine learning approaches.

Objective of the Dissertation

This dissertation aims to fill this research gap by conducting a comprehensive comparative analysis of ARIMA, SARIMA, and Random Forest models for forecasting GDP and inflation. By applying these models to four diverse economies, the study seeks to identify the strengths and limitations of each method in different economic contexts. The selected countries represent a mix of developed and emerging economies, providing a robust framework for evaluating the models' effectiveness.

In conclusion, while ARIMA and SARIMA models have proven effective for linear and seasonal time series data, their limitations in handling non-linear patterns necessitate the exploration of machine learning alternatives like Random Forest. This dissertation's comparative analysis will contribute to the existing body of knowledge by providing insights into which forecasting methods are most suitable under various economic conditions, ultimately aiding in more informed economic planning and policymaking.

Methodology:

Introduction

The methodology section of this dissertation outlines the comprehensive research design and methods employed to forecast GDP and inflation for the United States, China, Japan, and the United Kingdom using ARIMA, SARIMA, and Random Forest models. This section aims to provide a detailed and transparent account of the research process, ensuring the study's reproducibility and reliability.

The primary objective of this section is to explain how the selected methods effectively address the research questions and objectives. Specifically, it describes the comparative analysis of traditional time series forecasting methods (ARIMA and SARIMA) and machine learning techniques (Random Forest) to determine the most accurate and robust approach for predicting economic indicators.

Research Design

The primary objective of this study is to evaluate and compare the accuracy of ARIMA, SARIMA, and Random Forest models in forecasting GDP and inflation for four diverse economies: the United States, China, Japan, and the United Kingdom. This comparison is crucial as each model has unique characteristics that may influence its performance depending on the economic context. ARIMA models, known for their simplicity and effectiveness in short-term forecasting, use past values and relationships to predict future points. However, they assume linearity and might not perform well with complex or non-linear patterns. SARIMA models extend ARIMA by incorporating seasonal elements, making them suitable for data with regular seasonal fluctuations, but their complexity increases due to additional parameters, requiring larger datasets to accurately model seasonality. Random Forest models, on the other hand, leverage machine learning to handle non-linear relationships and interactions, providing robustness against overfitting by averaging multiple decision trees. However, they are less interpretable and require more computational resources compared to ARIMA and SARIMA.

The study will utilize performance metrics such as Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE) to compare the accuracy of these models. By applying each model to the same dataset and computing these metrics, the study aims to identify under what conditions each model performs best. Statistical tests, like the Diebold-Mariano test, will be used to determine if the differences in accuracy are statistically significant. This comprehensive comparison will help in understanding which model provides the most precise forecasts for GDP and inflation.

Furthermore, the study aims to explore the strengths and limitations of each model in capturing economic trends and seasonal patterns. ARIMA models, while effective for short-term forecasts, may not handle seasonality well and are limited in capturing complex, non-linear patterns. SARIMA models address seasonality but can be prone to overfitting if not correctly specified. Random Forest models excel in capturing complex interactions and non-linear relationships but may struggle with interpretability and require substantial computational power. The study will conduct time series decomposition to separate trend, seasonal, and residual components, assessing how well each model captures these elements by comparing predicted trends and seasonal patterns against actual data.

Lastly, the study examines how the economic context, specifically the distinction between developed (United States, United Kingdom, Japan) and emerging economies (China), influences the performance of these forecasting models. Developed economies typically exhibit stable and mature economic systems with consistent growth rates, while emerging economies often experience rapid growth, higher volatility, and less predictable economic indicators. By comparing model performance separately for developed and emerging economies, the study aims to understand how economic stability and growth rates affect forecasting accuracy. Detailed case studies for each country will analyze specific economic events and their impact on model performance, determining if Random Forest models better capture complex relationships in volatile economies like China, while ARIMA or SARIMA might be more effective for stable economies like Japan.

This comprehensive approach will provide valuable insights into the applicability and robustness of ARIMA, SARIMA, and Random Forest models in different economic contexts, contributing to more informed decision-making in economic planning and policy formulation.

Data Collection

Data collection for this dissertation is a critical phase, aimed at ensuring that the analysis is based on reliable and comprehensive economic indicators. The primary source of data for this study is the World Bank Database, a reputable and widely used repository that provides up-to-date and accurate economic statistics for countries worldwide. Specifically, the data for Gross Domestic Product (GDP) and Inflation are extracted from this database for four selected countries: the United States, China, Japan, and the United Kingdom. These countries were chosen due to their diverse economic profiles, which include both developed economies (United States, United Kingdom, and Japan) and an emerging economy (China), allowing for a comprehensive analysis of the forecasting models under different economic conditions.

The GDP data used in this study covers the period from 1960 to 2022 for all four countries. This extensive dataset provides a robust foundation for time series analysis, capturing multiple economic cycles and significant historical events that could influence economic performance. The data is measured in constant 2015 US dollars, ensuring comparability over time by adjusting for inflation. Inflation data, on the other hand, represents the annual percentage change in consumer prices, reflecting the cost of living and purchasing power in each country.

An important consideration in the data collection process is the availability of inflation data for China. Unlike the other countries, where inflation data is available from 1960 to 2022, China's inflation data is only available from 1990 onwards. This discrepancy is due to the historical economic transformations in China, which affect the availability and consistency of earlier economic records. Consequently, the analysis for China regarding inflation forecasting is based on a shorter timeframe compared to the other countries. This limitation is acknowledged and accounted for in the modelling and comparative analysis to ensure that the results are interpreted correctly and contextually.

Data Preprocessing

Data preprocessing is a critical step in preparing the dataset for accurate and reliable time series forecasting. For this dissertation, we undertook a series of meticulous preprocessing steps to ensure that the data used for modelling GDP and inflation is clean, consistent, and suitable for analysis. Here's an elaborate explanation of the preprocessing steps:

- **Handling Null Values**

The first step in preprocessing was to address any missing values within the dataset. In time series data, null values can occur due to various reasons such as reporting errors or gaps in data collection. For this dissertation, we used Python libraries like Pandas to handle these null values. Specifically, we omitted rows with null values to maintain data integrity. This approach is straightforward and ensures that the models are trained on complete datasets without introducing potential biases or inaccuracies that could arise from imputation methods.

- **Splitting the Data into Training and Testing Sets**

To evaluate the performance of the forecasting models accurately, we split the dataset into training and testing sets. The training set, which constituted 70% to 80% of the total data, was used to train the models. The testing set, comprising 20% to 30% of the data, was reserved for validating the models' predictive accuracy. This split ensures that the models are trained on a substantial portion of the data while leaving enough data to test their performance on unseen data. The specific split ratio was chosen based on standard practices in time series forecasting, providing a balance between training and testing data.

- **Checking for Outliers**

Outliers are data points that deviate significantly from the overall pattern of the dataset. They can potentially skew the results of the forecasting models if not addressed properly. We checked for outliers in the training data using statistical methods and visualizations such as box plots. Interestingly, the presence of outliers was minimal across the datasets for all countries. Given their sparse occurrence, these outliers did not create any significant impact on the forecasting models. As such, we decided not to remove them, thereby retaining the integrity of the original data.

- **Decomposition to Find Trend and Seasonality**

Time series decomposition is a technique used to separate a time series into its underlying components: trend, seasonality, and residuals. This step is crucial for understanding the patterns within the data and for selecting appropriate forecasting models. We applied seasonal decomposition using techniques to identify the long-term trend and seasonal patterns in the GDP and inflation data. This decomposition helped in understanding how the data behaves over time and was particularly useful for models like SARIMA, which require an understanding of seasonality.

- **Checking for Stationarity through Augmented Dickey-Fuller (ADF) Test**

Stationarity is a key assumption for ARIMA and SARIMA models, meaning that the statistical properties of the time series (mean, variance, autocorrelation) are constant over time. To check for stationarity, we employed the Augmented Dickey-Fuller (ADF) test. This test helps determine whether a time series is stationary by testing the null hypothesis that a unit root is present (indicating non-stationarity). For the datasets used, we applied the ADF test to both the raw and differenced data. If the data was found to be non-stationary, differencing was applied to make it stationary, a necessary step before fitting ARIMA and SARIMA models. This step ensures that the models accurately capture the underlying patterns without being influenced by trends or seasonality that vary over time.

In summary, the preprocessing steps for this dissertation involved a systematic approach to clean and prepare the time series data. By omitting null values, splitting the data into appropriate training and testing sets, checking and addressing outliers, decomposing the data to identify trends and seasonality, and ensuring stationarity through the ADF test, we established a robust foundation for accurate and reliable forecasting. These steps are critical to ensuring that the ARIMA, SARIMA, and Random Forest models perform optimally and provide meaningful insights into GDP and inflation forecasting for the selected countries.

Model Implementation

The model implementation part of this dissertation involves the application of ARIMA, SARIMA, and Random Forest models to forecast GDP and inflation for the United States, China, Japan, and the United Kingdom. This section provides a comprehensive overview of how each model is developed, trained, validated, and used for forecasting. The goal is to rigorously evaluate and compare the performance of these models in different economic contexts.

ARIMA Model Implementation

a. Data Preparation:

- **Loading Data:** Begin by loading the relevant time series data for GDP and inflation. The data is typically sourced from a reliable database and structured to have a time index.
- **Stationarity Check:** Check if the time series data is stationary, which means its statistical properties do not change over time. This can be done through visual inspections of plots and statistical tests like the Augmented Dickey-Fuller (ADF) test. If the data is not stationary, apply differencing to transform it into a stationary series.
- **Determine Differencing Order (d):** The number of differences needed to achieve stationarity is identified based on the ADF test results.

b. Parameter Identification:

- **Auto-Regressive (p) and Moving Average (q) Orders:** Determine the values for ‘p’ (the number of lagged observations) and ‘q’ (the number of lagged forecast errors) using Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots. These plots help in identifying the extent of autocorrelation and partial autocorrelation in the differenced series.

c. Model Estimation:

- Fit the ARIMA model to the training data using statistical software. The model parameters are estimated based on the chosen p, d, and q values. The fitting process involves optimizing the model parameters to best represent the underlying data patterns.

d. Model Diagnostics:

- After fitting the model, it is crucial to check the residuals (the differences between the actual and predicted values). The residuals should resemble white noise, meaning they have no autocorrelation and are normally distributed. This indicates a good model fit. Additional diagnostic tests, such as the Ljung-Box test, can confirm the adequacy of the model.

e. Forecasting:

- Once the model is validated, it can be used to forecast future values of the time series. The ARIMA model generates predictions based on the historical data and the identified patterns in the series.

SARIMA Model Implementation

a. Seasonal Decomposition:

- Decompose the time series data to identify and separate the trend, seasonal, and residual components. This step helps in understanding the seasonal patterns within the data, which are crucial for SARIMA modelling.

b. Identifying Seasonal Parameters:

- Determine the seasonal differencing order (D) and the seasonal AR (P) and MA (Q) terms along with the seasonal period (s). This is done using seasonal ACF and PACF plots, which help in identifying the seasonal lags and patterns in the data.

c. Model Estimation:

- Fit the SARIMA model to the training data by incorporating the seasonal parameters identified in the previous step. The model is specified with both non-seasonal (p, d, q) and seasonal (P, D, Q, s) components.

d. Model Diagnostics:

- Similar to the ARIMA model, check the residuals to ensure they are white noise. This step ensures that the seasonal components are adequately captured by the model. Perform additional diagnostic tests to validate the model fit.

e. Forecasting:

- Use the fitted SARIMA model to make seasonal forecasts. The model accounts for both non-seasonal and seasonal patterns, providing more accurate predictions for data with strong seasonal components.

Random Forest Model Implementation

a. Feature Selection:

- Identify and select relevant features (predictors) that may influence GDP and inflation. These features could include various economic indicators such as interest rates, trade volumes, and other macroeconomic variables.

b. Data Splitting:

- Split the dataset into training and testing sets to evaluate the model's performance. Typically, a certain percentage of the data is used for training the model, while the remaining data is reserved for testing and validation.

c. Model Training:

- Train the Random Forest model using the training data. Random Forest is an ensemble learning method that constructs multiple decision trees and averages their predictions to improve accuracy and robustness. The model parameters, such as the number of trees and depth of each tree, are optimized during the training process.

d. Model Evaluation:

- Evaluate the model's performance using metrics such as Mean Absolute Error (MAE), Mean Squared Error (MSE), and R-squared. These metrics help in assessing the accuracy and reliability of the model's predictions. Out-of-bag (OOB) error estimates can also be used to evaluate the model without needing a separate validation set.

e. Forecasting:

- Use the trained Random Forest model to make future predictions. The model leverages the selected features to generate forecasts for GDP and inflation, capturing complex, non-linear relationships and interactions in the data.

The implementation of ARIMA, SARIMA, and Random Forest models involves a series of well-defined steps to ensure accurate and reliable forecasting of GDP and inflation. The process begins with data preparation, including stationarity checks and differencing for ARIMA and SARIMA models, and feature selection for the Random Forest model. Parameter identification follows, with the use of ACF and PACF plots for ARIMA and SARIMA, and model training and optimization for Random Forest. After fitting the models, residual diagnostics are conducted to ensure the adequacy of the fit. Finally, the models are used to make forecasts, providing valuable insights into future economic trends. Each model's unique strengths and capabilities are leveraged to capture different aspects of the time series data, contributing to a comprehensive comparative analysis of their forecasting performance.

Model Evaluation

Model evaluation is a critical part of the forecasting process. It involves assessing the accuracy and reliability of the models' predictions by comparing them against actual observed values. This section provides a detailed explanation of the methods and metrics used to evaluate the ARIMA, SARIMA, and Random Forest models implemented for forecasting GDP and inflation.

1. Evaluation Metrics

a. Mean Absolute Error (MAE):

- **Definition:** MAE measures the average magnitude of errors in a set of predictions, without considering their direction. It is calculated as the average of the absolute differences between predicted and actual values.
- **Formula:**

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i|$$

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Where y_i is the actual value, $\hat{y}_i$ is the predicted value, and 'n' is the number of observations.

- **Interpretation:** Lower MSE values indicate better model performance. The squaring of errors penalizes larger errors more than smaller ones.

c. Root Mean Squared Error (RMSE):

- **Definition:** RMSE is the square root of the MSE. It provides the error in the same units as the data, making it more interpretable.
- **Formula:**

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$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

- **Interpretation:** Lower MAPE values indicate better model performance. MAPE is scale-independent and useful for comparing models on different datasets.

e. R-squared (Coefficient of Determination):

- **Definition:** R-squared measures the proportion of the variance in the dependent variable that is predictable from the independent variables. It indicates how well the model explains the variability of the data.
- **Formula:**

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2}$$

Where $\bar{y}$ is the mean of the actual values.

- **Interpretation:** R-squared values range from 0 to 1. Higher values indicate better model performance, with 1 indicating that the model perfectly explains the variability of the data.

2. Model Evaluation Process

a. Splitting Data into Training and Testing Sets:

- The dataset is split into training and testing sets to evaluate model performance. Typically, 70-80% of the data is used for training, and the remaining 20-30% is used for testing. This split allows the model to be trained on historical data and tested on unseen data to assess its predictive accuracy.

b. Training the Model:

- The ARIMA, SARIMA, and Random Forest models are trained on the training dataset. The models learn the underlying patterns and relationships in the data during this phase.

c. Making Predictions:

- Once the models are trained, they are used to make predictions on the testing dataset. These predictions are compared against the actual observed values to evaluate the models' accuracy.

d. Calculating Evaluation Metrics:

- The evaluation metrics (MAE, MSE, RMSE, MAPE, and R-squared) are calculated using the actual and predicted values from the testing dataset. These metrics provide a quantitative assessment of the models' performance.

e. Residual Analysis:

- Residuals (the differences between actual and predicted values) are analyzed to check for patterns that the model may have missed. For ARIMA and SARIMA models, residuals should resemble white noise, indicating a good fit. Plots such as residual plots, ACF, and PACF of residuals are used to diagnose model performance.

f. Statistical Tests:

- Additional statistical tests, such as the Ljung-Box test, are conducted to confirm that residuals are uncorrelated. For comparing predictive accuracy between models, the Diebold-Mariano test can be used to test if the differences in forecast errors are statistically significant.

g. Sensitivity Analysis:

- Sensitivity analysis involves testing the robustness of the models under different scenarios, such as varying economic conditions or data perturbations. This analysis helps in understanding how changes in input data affect the model's predictions and stability.

h. Cross-Validation:

- Cross-validation techniques, such as k-fold cross-validation, can be applied to further validate the models. In k-fold cross-validation, the dataset is divided into k subsets, and the model is trained and tested k times, each time using a different subset as the testing set and the remaining as the training set. This approach ensures that the model's performance is consistent across different subsets of data.

3. Model Comparison

a. Comparative Analysis:

- The performance metrics of ARIMA, SARIMA, and Random Forest models are compared to identify which model performs best under various conditions. The strengths and limitations of each model are analyzed based on their performance metrics and residual analysis.

b. Visualization:

- Visualization techniques, such as plotting the actual vs. predicted values, are used to visually compare the models' performance. These plots help in understanding the models' accuracy and any patterns or biases in the predictions.

c. Reporting Results:

- The evaluation results are documented, highlighting the best-performing model for each scenario. Insights and recommendations for future forecasting are provided based on the evaluation and comparative analysis.

Model evaluation is a comprehensive process that involves calculating performance metrics, analyzing residuals, conducting statistical tests, and performing sensitivity analysis. By following these steps, the dissertation rigorously assesses the accuracy and reliability of ARIMA, SARIMA, and Random Forest models in forecasting GDP and inflation. This thorough evaluation helps in identifying the most suitable model for different economic contexts, thereby enhancing the robustness and credibility of the forecasting results.

Data Analysis

In this dissertation, data analysis serves as the cornerstone for predicting the future economic performance of four major economies: the United States, China, Japan, and the United Kingdom. The goal is to forecast Gross Domestic Product (GDP) and Inflation using a combination of traditional time series models (ARIMA and SARIMA) and modern machine learning techniques (Random Forest). This comprehensive approach not only aims to identify the most accurate forecasting methods for each economic indicator but also seeks to provide a deeper understanding of the underlying economic trends and patterns in these diverse economies.

The data analysis process begins with **data collection** from the World Bank Database, a reliable and widely recognized source of economic statistics. This ensures that the analysis is grounded in accurate and up-to-date information, which is critical for making reliable predictions. The collected data includes historical GDP and Inflation figures, which form the basis for the subsequent steps of the analysis.

Following data collection, **data preprocessing** is carried out to prepare the dataset for analysis. This involves cleaning the data to handle any missing values or outliers, which could skew the results. It also includes transforming the data into a suitable format for time series analysis and machine learning models, ensuring consistency and accuracy.

The next phase, **Exploratory Data Analysis (EDA)**, is crucial for understanding the characteristics of the data. By visualizing trends, seasonality, and correlations between GDP and Inflation, we can identify key patterns and anomalies. This step lays the groundwork for selecting the appropriate forecasting models and fine-tuning them to capture the nuances of each country's economic data.

In the **time series forecasting** stage, models such as ARIMA and SARIMA are employed. These models are well-suited for capturing temporal dependencies and seasonality in the data. The process includes testing for stationarity, differencing the data if necessary, and fitting the models to the historical data. This approach allows us to generate forecasts that account for the inherent temporal structure of the economic indicators.

Parallelly, the **machine learning forecasting** approach leverages the power of algorithms like Random Forest. This involves feature engineering to create relevant predictors, splitting the data into training and testing sets, and training the models on the training data. The machine learning models are then evaluated on their ability to predict the testing data accurately, using metrics such as RMSE and MAE.

The **model comparison and selection** phase are critical for determining the best forecasting method. By comparing the performance of ARIMA, SARIMA, and Random Forest models across different metrics, we can identify which model provides the most accurate predictions for each country and economic indicator. This comparative analysis highlights the strengths and limitations of each approach, offering valuable insights into their applicability in different economic contexts.

Finally, the **documentation and reporting** phase involves summarizing the findings, visualizing the forecasts, and providing actionable insights. This includes discussing the implications of the forecasts, making recommendations for policymakers and researchers, and documenting the methodology and results comprehensively.

ARIMA & SARIMA:

ARIMA (Autoregressive Integrated Moving Average) and SARIMA (Seasonal Autoregressive Integrated Moving Average) are two widely used models in time series forecasting. ARIMA is a versatile model that can handle a variety of data patterns by incorporating aspects of autoregression (AR), differencing (I), and moving averages (MA). It is particularly effective for non-seasonal data, where it can capture trends and patterns to make accurate forecasts. On the other hand, SARIMA extends ARIMA by including seasonal components to account for seasonal effects in the data. This makes SARIMA ideal for datasets exhibiting regular, repeating patterns over specific intervals, such as monthly sales data with annual peaks. Understanding the distinction between these models is crucial for selecting the appropriate method based on the characteristics of the data under analysis.

Time Series Analysis & Explanation:

Japan GDP

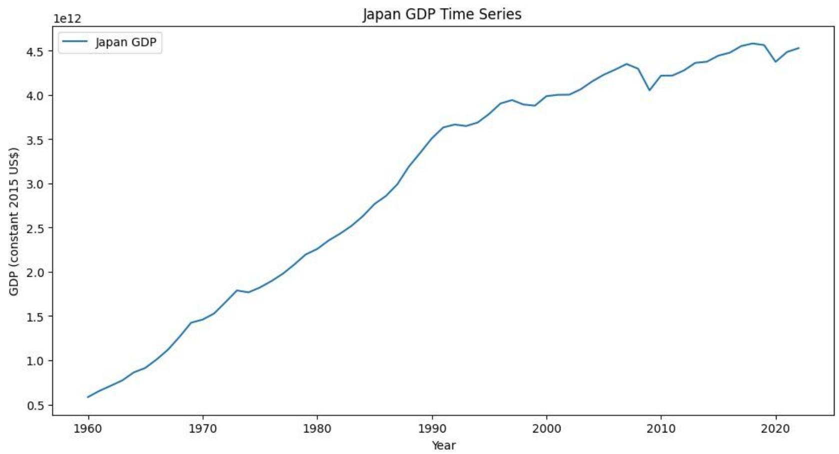

Figure 1: Time series analysis of Japan's GDP

The GDP time series for Japan, spanning from 1960 to the present, illustrates a consistent and significant growth trend over the decades. Starting from a low base in the early 1960s, Japan’s GDP shows a rapid increase, particularly during the 1960s and 1970s, a period known as the Japanese economic miracle. This era was marked by rapid industrialization and technological advancement, supported by government policies, education reforms, and a strong work ethic. The GDP continued to grow steadily through the 1980s, albeit at a slightly slower pace. However, the growth rate decelerated in the 1990s, corresponding to Japan's economic stagnation, often referred to as the "Lost Decade," following the burst of the asset price bubble in 1991. From the 2000s onwards, the GDP growth appears relatively stable, with minor fluctuations, reflecting Japan’s mature economy with slow growth rates.

United Kingdom GDP

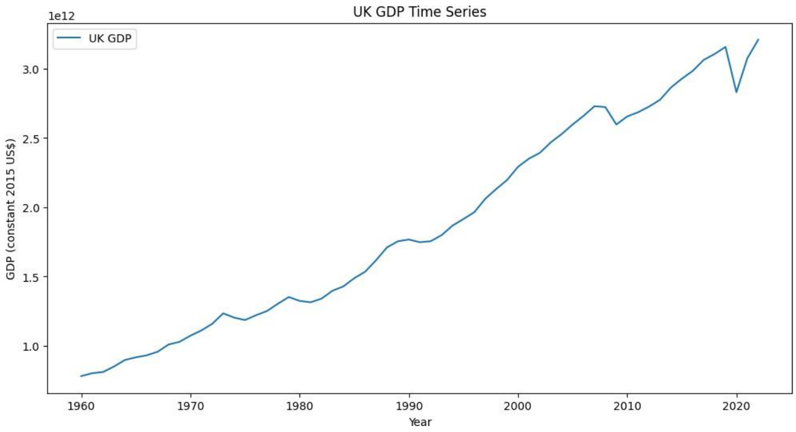

Figure 2: Time series analysis of United Kingdom's GDP

The GDP time series for the United Kingdom from 1960 onwards demonstrates steady growth with some noticeable fluctuations. The initial growth phase from the 1960s to the mid-1970s reflects post-war economic expansion and modernization. However, the 1970s exhibit some volatility, likely due to economic challenges such as the oil crisis, inflation, and industrial strife. Moving into the 1980s and 1990s, the UK economy shows robust growth, driven by financial sector expansion, deregulation, and privatization under the Thatcher government. The upward trend continues into the 2000s, although the global financial crisis of 2008-2009 caused a significant dip in GDP. The recovery period post-2009 shows a gradual increase in GDP, with a notable drop in 2020, reflecting the economic impact of the COVID-19 pandemic. The subsequent rise in GDP towards the end of the series suggests a recovery phase.

United States GDP

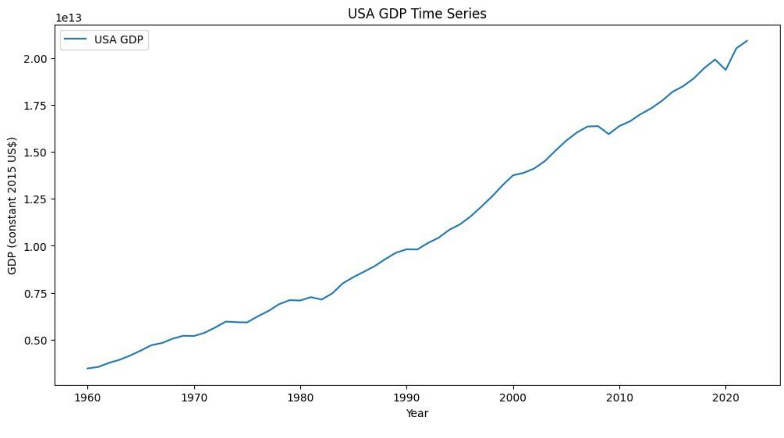

Figure 3: Time series analysis of United States of America's GDP

The GDP time series for the United States from 1960 to the present displays a continuous and steady growth trajectory, indicating the robustness and resilience of the U.S. economy. The early years show a gradual increase, reflecting post-war prosperity and industrial growth. The GDP growth accelerates through the 1980s and 1990s, driven by technological innovation, globalization, and increased productivity. This period includes the dot-com boom of the late 1990s. The early 2000s show continued growth, though the 2008 financial crisis caused a significant downturn. Post-crisis recovery is evident, with the GDP rebounding and continuing its upward trend, showcasing the economy’s ability to recover from financial shocks. The graph shows a noticeable dip in 2020 due to the COVID-19 pandemic, followed by a strong recovery, highlighting the economic resilience and adaptability of the United States.

China GDP

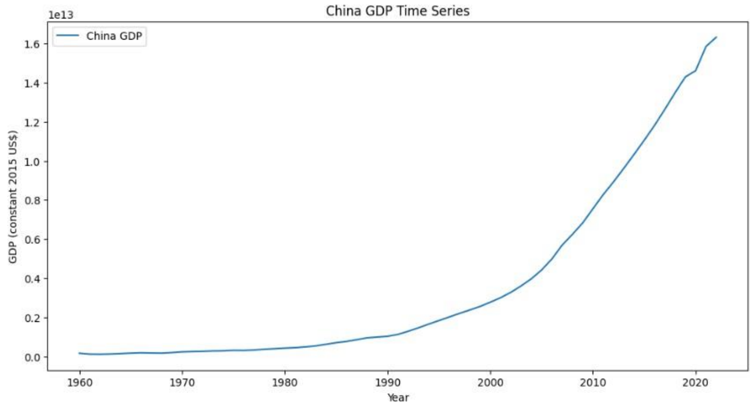

Figure 4: Time series analysis of China's GDP

The GDP time series for China from 1960 onwards presents a dramatic and steep growth curve, particularly from the late 1970s onwards. The early part of the series shows a relatively flat growth, reflecting the economic conditions under Mao Zedong's leadership, characterized by limited industrialization and several economic disruptions, including the Cultural Revolution. The sharp upward trajectory beginning in the late 1970s corresponds to the economic reforms introduced by Deng Xiaoping, marking the transition from a planned economy to a market-oriented one. These reforms included the introduction of the "Open Door" policy, economic liberalization, and the establishment of Special Economic Zones (SEZs). The rapid industrialization, foreign investment, and export-led growth have led to China becoming the second-largest economy in the world. The graph illustrates the exponential growth, especially post-2000, highlighting China’s emergence as a global economic powerhouse. The recent years show a continued upward trend, though the growth rate slightly moderates, indicating a transition to more sustainable and quality-focused economic growth.

United States Inflation

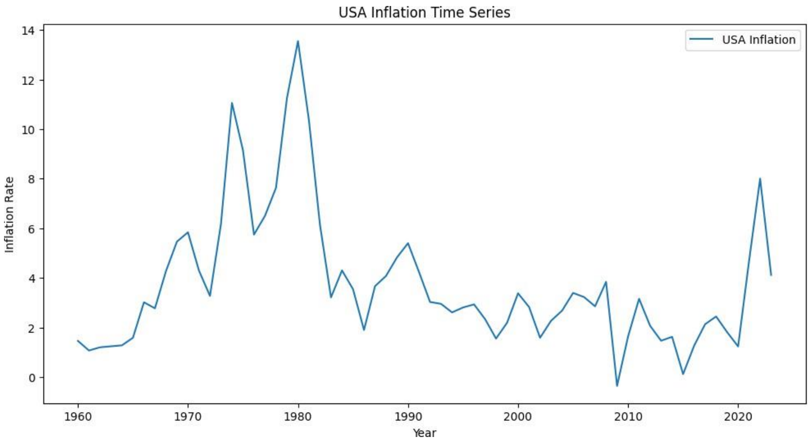

Figure 5: Time series analysis of United States of America's inflation.

The inflation time series for the United States from 1960 to the present provides a detailed view of inflationary trends and significant economic events over several decades. In the 1960s, the inflation rate started at relatively low levels but began to climb steadily towards the end of the decade. The 1970s were marked by severe inflationary pressures, often referred to as the "Great Inflation" period. This era saw inflation rates soaring, reaching peaks above 10%, primarily driven by factors such as the oil price shocks of 1973 and 1979, which were triggered by geopolitical tensions and oil embargoes. These spikes in inflation were exacerbated by expansive fiscal policies and wage-price spirals. The early 1980s marked the peak of inflation, with rates exceeding 13%. This prompted the Federal Reserve, under Chairman Paul Volcker, to implement aggressive monetary tightening policies, which successfully brought inflation under control but at the cost of inducing a severe recession. Throughout the 1990s and 2000s, the inflation rate in the United States remained relatively stable, hovering around the 2-4% range, reflecting effective monetary policy and economic stability. However, the graph shows a notable spike around 2008, corresponding to the global financial crisis, followed by a sharp decline. More recently, the COVID-19 pandemic in 2020 led to significant economic disruptions, contributing to a notable spike in inflation, which peaked and then started to decline as the economy began to recover.

Japan Inflation

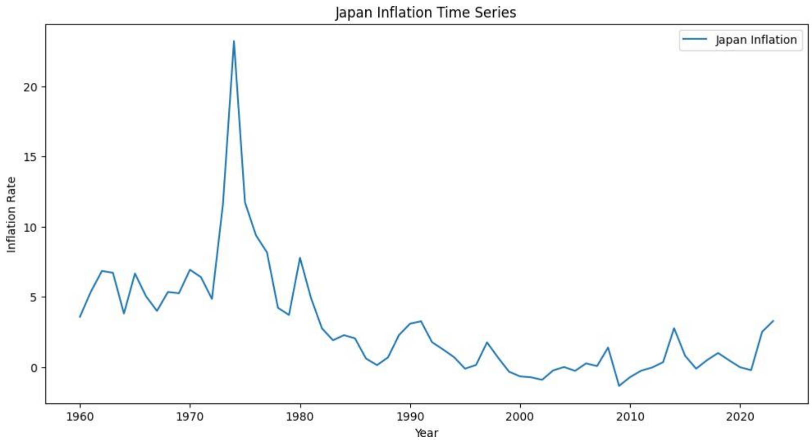

Figure 6: Time series analysis of Japan's inflation.

The inflation time series for Japan from 1960 to the present shows a distinctive pattern, reflecting the unique economic history of the country. In the early years, Japan experienced moderate inflation, which increased during the 1970s, reaching a significant peak of over 20% in the mid-1970s. This peak was largely due to the oil price shocks and subsequent increases in import prices, combined with expansive economic policies aimed at rapid industrial growth. However, the late 1970s and early 1980s saw a significant reduction in inflation rates, driven by a combination of tight monetary policies and structural economic changes. From the mid-1980s onwards, Japan's inflation rate remained remarkably low, even entering periods of deflation (negative inflation) in the 1990s and 2000s. This deflationary period is associated with the bursting of the asset price bubble in the early 1990s, leading to a prolonged economic stagnation often referred to as the "Lost Decade." Despite various policy measures to stimulate the economy and combat deflation, including ultra-low interest rates and quantitative easing, inflation has remained low. The recent years show minor fluctuations with slight increases, reflecting Japan's ongoing struggle to achieve stable inflation and robust economic growth.

United Kingdom Inflation

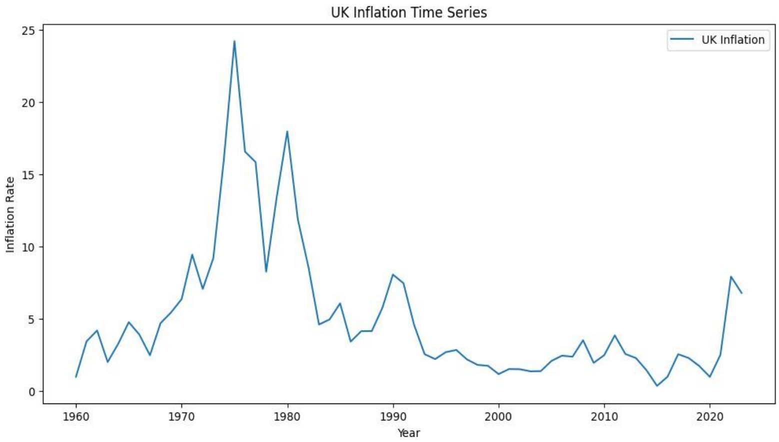

Figure 7: Time series analysis of United Kingdom's inflation.

The inflation time series for the United Kingdom from 1960 onwards highlights several key periods of economic turbulence and stability. In the 1960s, the UK experienced moderate inflation, which escalated significantly during the 1970s. This period saw inflation rates reaching dramatic highs, peaking above 25% in the mid-1970s. The sharp rise in inflation was driven by several factors, including the oil price shocks, industrial unrest, and expansive fiscal policies. The UK government and Bank of England implemented stringent monetary and fiscal policies in the late 1970s and early 1980s to control inflation, leading to a steep decline in rates but also contributing to economic recessions and high unemployment. Throughout the 1980s and 1990s, the inflation rate stabilized at lower levels, reflecting improved economic management and the impact of global economic trends. The early 2000s saw relatively low and stable inflation, aligning with global economic stability and effective monetary policies. However, the 2008 financial crisis caused a spike in inflation, followed by a sharp decline. The graph also shows a notable spike around 2020, corresponding to the economic disruptions caused by the COVID-19 pandemic, followed by a subsequent decline as the economy began to recover. The overall trend indicates that while the UK has experienced periods of high inflation, particularly in the 1970s, effective policy measures have generally maintained inflation at moderate levels in recent decades.

Decomposition:

Japan GDP Decomposition

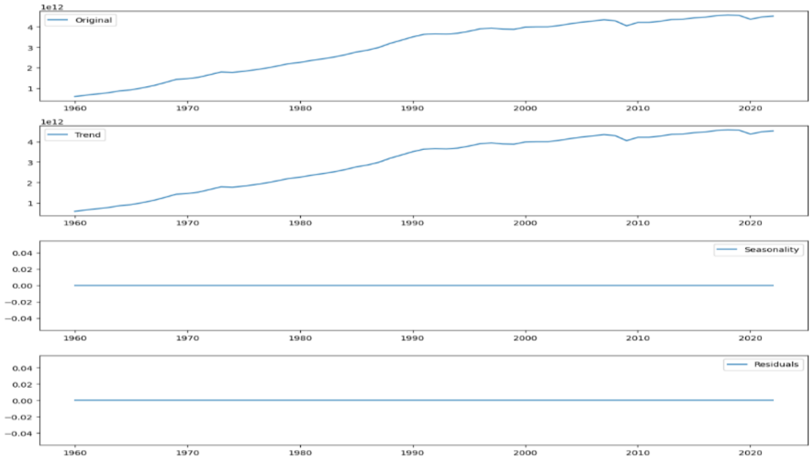

Figure 8: Decompositions of Japan's GDP

The decomposition of Japan's GDP data shows a steady growth trend in the original time series, albeit with some fluctuations, particularly in the late 1990s and early 2000s. The trend component captures this overall growth, showing a consistent upward trajectory with minor dips. The seasonality component, like in the previous cases, remains flat, indicating no seasonal effects in the GDP data. This suggests that Japan's GDP does not follow a predictable seasonal pattern. The residuals component is also flat, reinforcing the absence of seasonality and indicating that any variations in the GDP data are due to irregular factors rather than seasonal influences.

UK GDP Decomposition

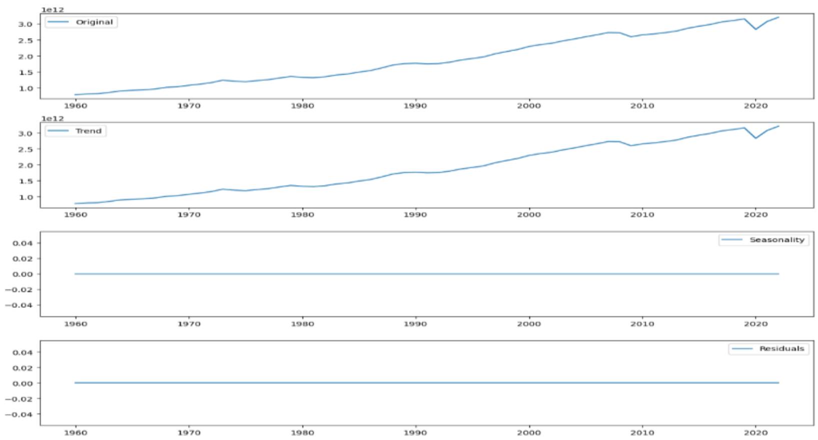

Figure 9: Decompositions of United Kingdom's GDP

The decomposition of the UK GDP data illustrates a clear upward trend in the original time series, indicating consistent economic growth over the period from 1960 to 2023. The trend component mirrors this, showing a smooth, increasing trajectory without significant fluctuations. Interestingly, the seasonality component remains flat, suggesting no apparent seasonal effects in the GDP data. This lack of seasonality indicates that the GDP values do not follow a predictable, repeating pattern at regular intervals, such as quarterly or annually. The residuals component, which captures the random noise or irregular fluctuations after removing the trend and seasonality, also shows a flat line, reinforcing the absence of seasonality and indicating that any variations in the GDP data are likely due to irregular factors rather than seasonal patterns.

USA GDP Decomposition

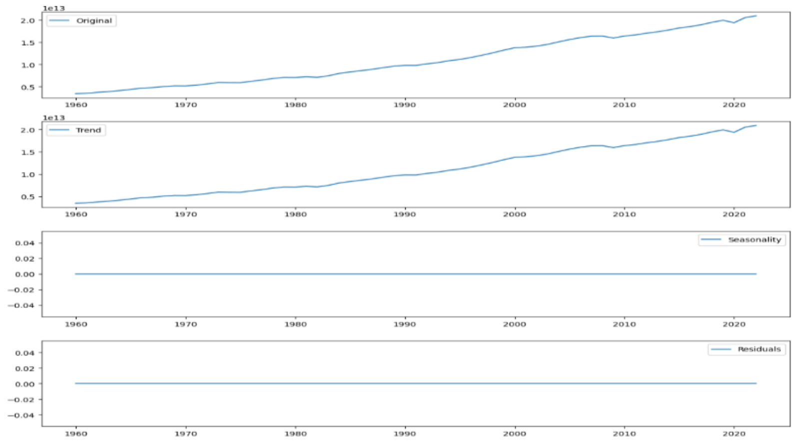

Figure 10: Decompositions of United States of America's GDP

The USA GDP data decomposition also shows a steady upward trend in the original time series, indicative of consistent economic growth. The trend component reflects this with a smooth, rising curve. Similar to the UK and China, the seasonality component is flat, indicating no discernible seasonal patterns in the GDP data. This lack of seasonality means that the GDP values do not exhibit predictable fluctuations at regular intervals. The residuals component shows a flat line, suggesting that the variations in the GDP data are primarily irregular and not influenced by seasonal factors.

China GDP Decomposition

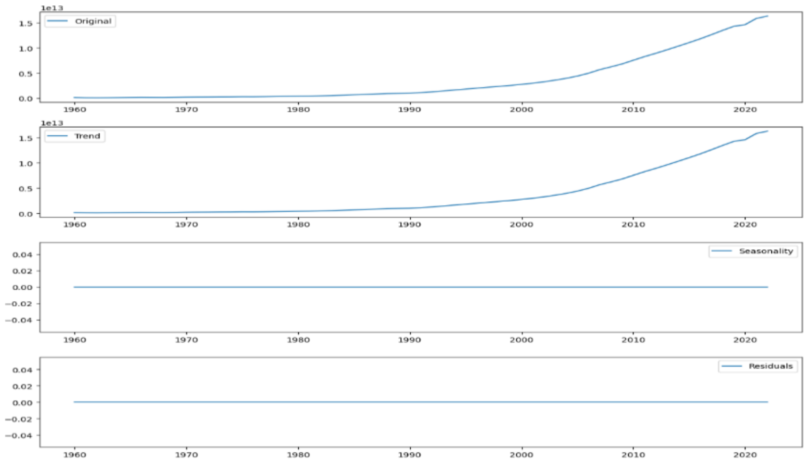

Figure 11: Decompositions of China's GDP

The decomposition of China's GDP data reveals a similar upward trend in the original time series, though more pronounced compared to the UK, reflecting China's rapid economic growth over the same period. The trend component shows a sharp and accelerating increase, particularly from the late 1990s onwards, consistent with China's economic boom. The seasonality component again remains flat, indicating no seasonal effects in the GDP data. This suggests that the significant growth observed in China's GDP is primarily driven by long-term factors rather than seasonal variations. The residuals component shows minimal fluctuations, further confirming the lack of seasonality and suggesting that the observed variations are due to non-seasonal, irregular factors.

United States' Inflation Decomposition

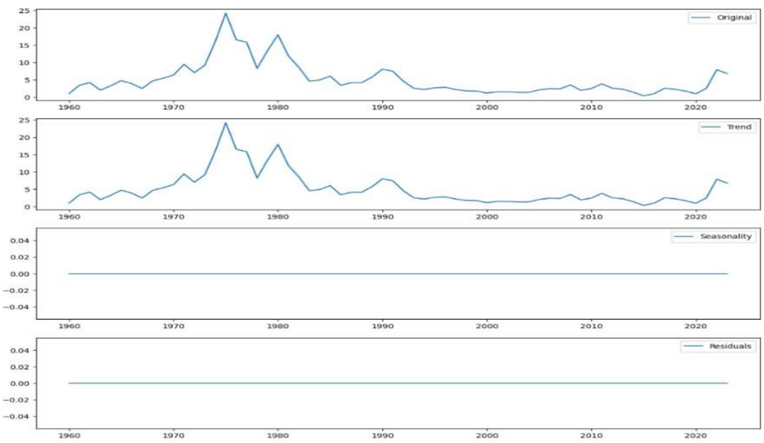

Figure 12: Decompositions of United States of America's inflation.

The decomposition of the time series for the United States reveals several key insights. The original series displays notable peaks, particularly around the 1970s, suggesting significant events or trends during that period. The trend component closely mirrors the original series, highlighting a long-term increase and decrease pattern with a prominent peak in the 1970s, indicating a major trend rather than seasonal fluctuations or random noise. Interestingly, the seasonality component is a flat line at zero, which implies that there is no repeating seasonal pattern in this dataset. This suggests that the fluctuations observed in the original series are not due to regular, periodic factors. Similarly, the residuals component is also a flat line at zero, indicating no residual error after extracting the trend and seasonality components. This absence of residuals might suggest that the decomposition method used may not have captured the seasonality or residuals properly, or there might be issues with the data itself.

Japan's Inflation Decomposition

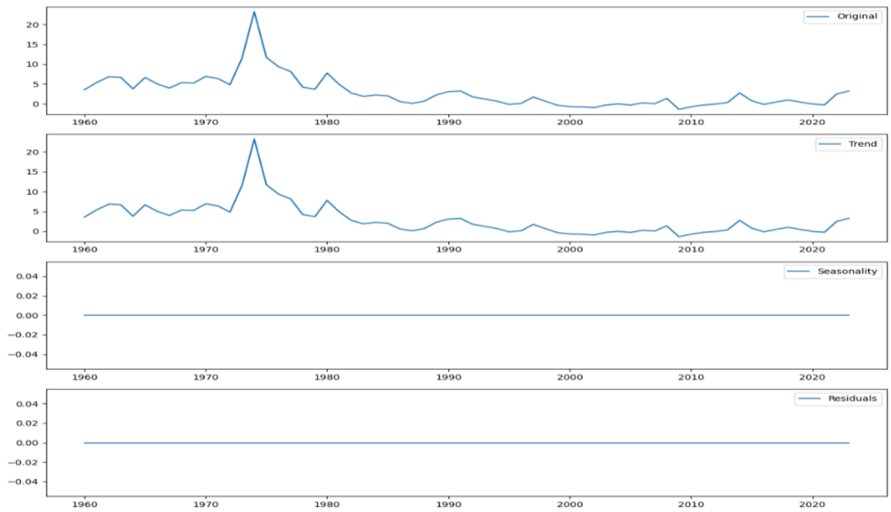

Figure 13: Decompositions of Japan's inflation.

The decomposition of the time series for Japan exhibits similar characteristics to that of the United States. The original series shows noticeable peaks around the 1970s, suggesting significant events or trends during that period. The trend component for Japan closely follows the original series, emphasizing long-term trends rather than short-term fluctuations, with the peak in the 1970s being particularly prominent. This highlights a significant long-term event or trend during that period. Like the United States, the seasonality component for Japan is a flat line at zero, indicating no regular seasonal patterns in the data. The residuals component is also flat at zero, suggesting no residual errors after extracting the trend and seasonality components. As with the United States, this could imply a lack of proper decomposition of seasonality and residuals or issues with the dataset.

United Kingdom's Inflation Decomposition

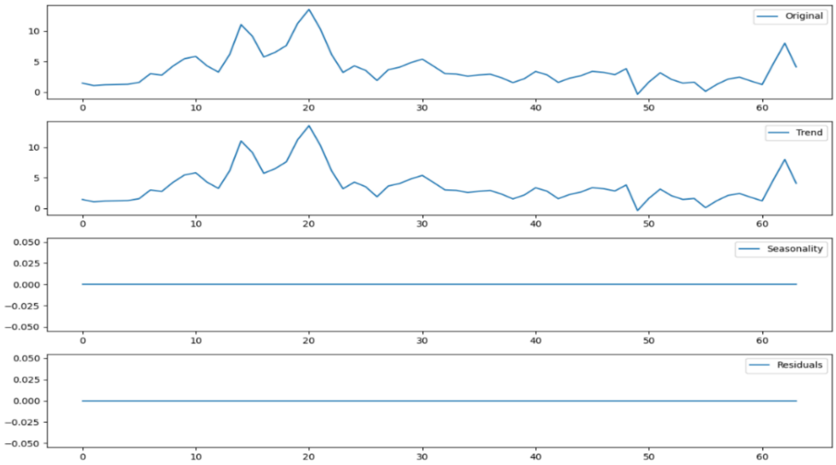

Figure 14: Decompositions of United Kingdom's inflation.

The decomposition of the time series for the United Kingdom also reveals similar patterns to those observed in the United States and Japan. The original series shows peaks, particularly around the 1970s, suggesting significant events or trends during that period. The trend component for the United Kingdom closely follows the original series, highlighting long-term trends with a significant peak in the 1970s, like the other countries. The seasonality component for the United Kingdom is a flat line at zero, indicating no repeating seasonal patterns. The residuals component is flat at zero, suggesting no residual errors after removing the trend and seasonality components. As with the other countries, this could indicate issues with the decomposition process or the data itself.

Seasonality refers to regular and predictable patterns that recur over specific intervals within a time series dataset, such as monthly sales peaking during holiday seasons or temperature variations over different seasons of the year. When data exhibits these repeating cycles, seasonal components can be extracted and modelled to improve forecasting accuracy. SARIMA models extend the ARIMA model by incorporating terms that account for seasonality, making them particularly useful for data with strong seasonal effects.

However, GDP and inflation data typically do not display strong seasonal patterns. GDP data represents the overall economic output of a country over a period, and while it may show long-term trends and cyclical fluctuations, these do not usually repeat at regular seasonal intervals. Instead, GDP growth or contraction can be influenced by a wide range of factors such as policy changes, economic shocks, technological advancements, and global economic conditions, none of which follow a predictable seasonal pattern.

Similarly, inflation data reflects the rate at which the general level of prices for goods and services is rising, eroding purchasing power. Inflation can be influenced by diverse factors such as changes in monetary policy, supply chain disruptions, commodity price changes, and global economic events. These factors do not necessarily follow a seasonal pattern either. While there might be some short-term fluctuations, these are not regular or predictable enough to be considered seasonal.

Given the absence of clear seasonality in both GDP and inflation data, using a SARIMA model for forecasting would not be appropriate or beneficial. SARIMA models rely on the presence of

seasonality to provide accurate forecasts, and without this component, the model may not perform well and could lead to misleading results. Therefore, for datasets like GDP and inflation, where seasonality is not a defining characteristic, SARIMA does not offer a significant advantage.

<u>Stationarity:</u>

Data stationarity is a crucial concept in time series analysis, particularly when using ARIMA (Auto Regressive Integrated Moving Average) models. Stationarity refers to the property of a time series where its statistical characteristics, such as mean, variance, and autocorrelation, remain constant over time. This stability in the data is essential for ARIMA models to make accurate and reliable predictions. Non-stationary data, which exhibits trends, seasonality, or varying variances, can lead to misleading results and poor model performance. Therefore, transforming a time series to achieve stationarity, often through differencing or detrending, is a fundamental step in the ARIMA modelling process, ensuring that the underlying patterns in the data are consistent and can be effectively captured by the model's parameters.

China’s GDP Stationary Data

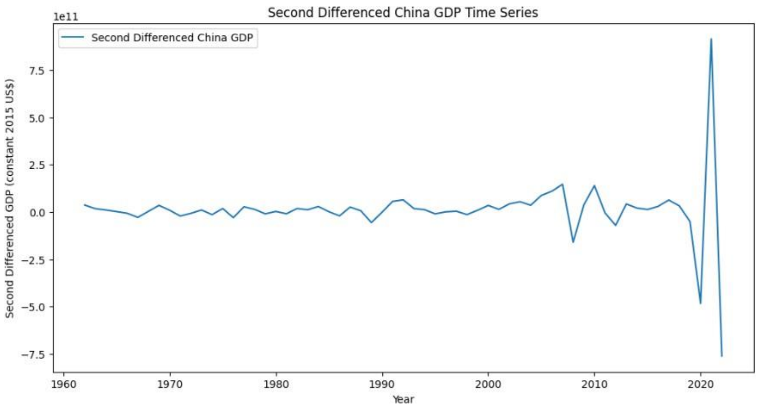

Figure 15: Time series analysis of China's stationary GDP data.

Starting with China, the second differenced GDP time series plot reveals that the data, once differenced twice, fluctuates around a stable mean near zero. This indicates that the series has achieved stationarity, a crucial step for reliable time series analysis and forecasting. The plot shows relatively consistent fluctuations with a significant spike around 2020, which is likely due to the economic impact of the COVID-19 pandemic. This spike highlights how extraordinary events can cause abrupt changes in GDP, affecting the stationarity of the series temporarily.

United Kingdom's GDP Stationary Data

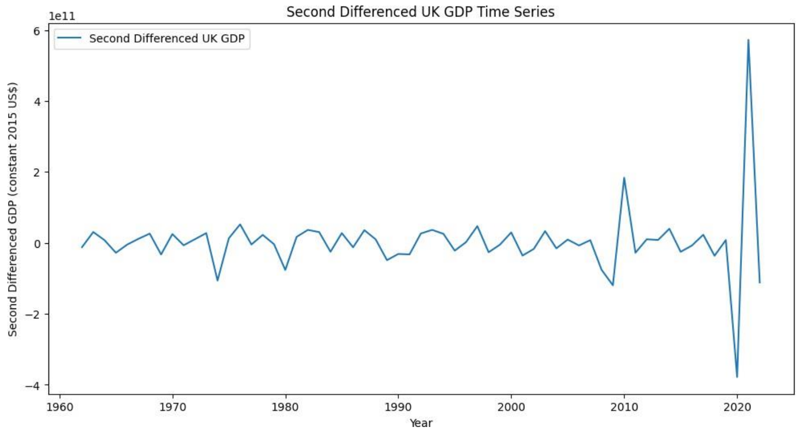

Figure 16: Time series analysis of United Kingdom's stationary GDP data.

Moving to the UK, the second differenced GDP time series plot also shows a similar stabilization around a zero mean after differencing. The consistent fluctuations around this mean indicate that long-term trends and seasonality have been effectively removed, achieving stationarity. However, notable spikes during significant events such as the 2008 financial crisis and the 2020 COVID-19 pandemic are visible. These events caused substantial deviations from the norm, which are crucial to consider in economic modelling and forecasting.

Japan's GDP Stationary Data

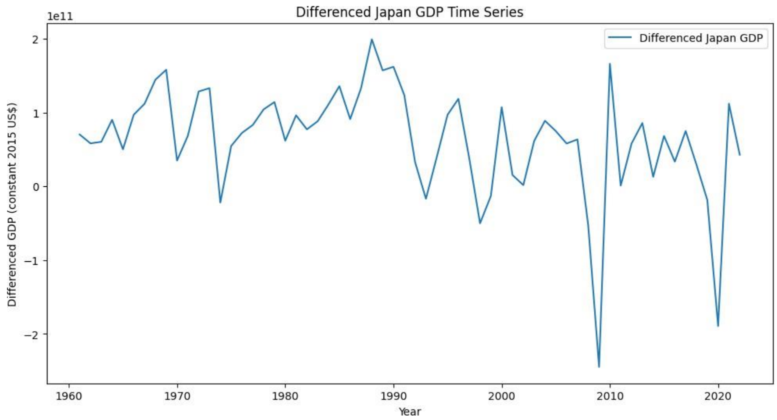

Figure 17: Time series analysis of Japan's stationary GDP data.

The GDP time series for Japan, after the first differencing, demonstrates stabilization around a zero mean, indicating that the series has achieved stationarity. The plot reveals periods of economic instability, particularly during the late 1990s and around 2010. These fluctuations are attributed to various economic factors, including the bursting of the asset price bubble in the early 1990s and the global financial crisis. Despite these periods of volatility, the overall series remains stationary, which is essential for accurate ARIMA modelling.

United States’ GDP Stationary Data

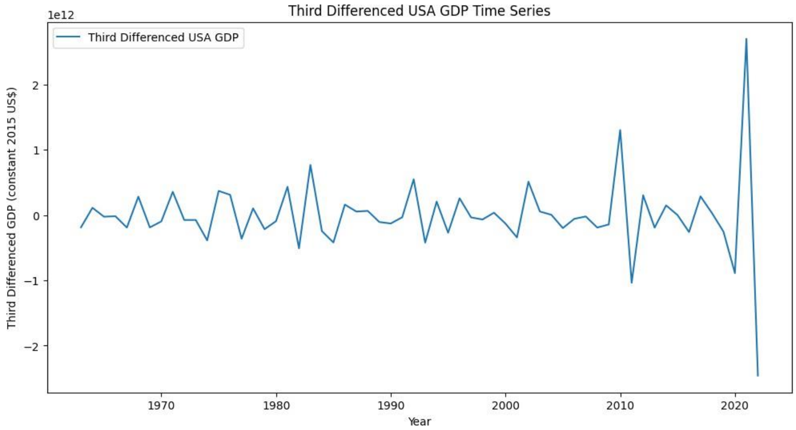

Figure 18: Time series analysis of United States of America's stationary GDP data.

For the USA, the third differenced GDP time series plot is shown. The data, after being differenced three times, fluctuates around a zero mean, indicating that the series has achieved stationarity. The plot highlights consistent fluctuations with occasional spikes, such as those seen around 2020, again likely due to the economic impacts of the COVID-19 pandemic. This transformation removes trends and seasonality, making the data appropriate for ARIMA modelling and allowing for more accurate forecasting.

In summary, the transformation of GDP time series data through differencing has successfully achieved stationarity for China, the UK, Japan, and the USA. This process removes trends and seasonality, stabilizing the data around a constant mean, which is crucial for accurate and reliable ARIMA modelling. The visual representation of the stationary data highlights the impact of significant economic events, such as financial crises and the COVID-19 pandemic, on the GDP, which is vital for comprehensive economic analysis and forecasting.

United Kingdom’s Inflation Stationary Data

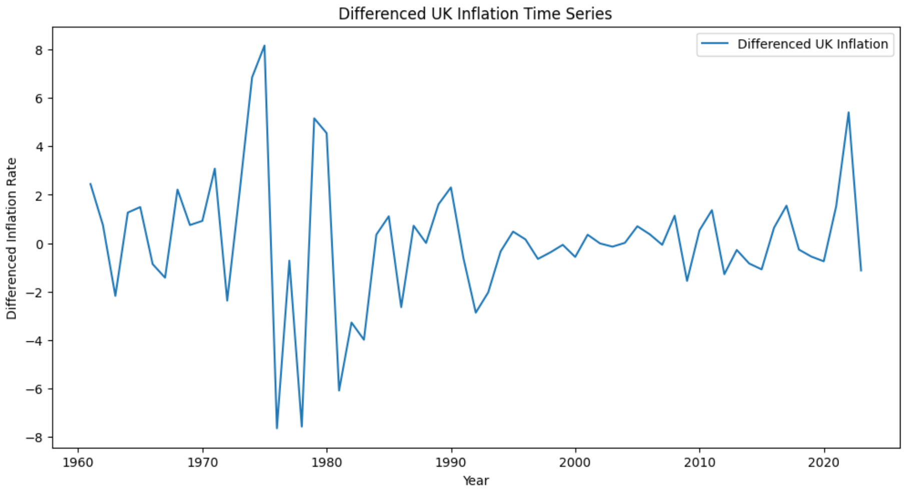

Figure 19: Time series analysis of United Kingdom's stationary inflation data.

In the above diagram of United Kingdom inflation data, the differenced inflation time series illustrates considerable volatility over the approximately 60-year period. In the initial years, the data reveal significant fluctuations with large spikes and drops, indicative of substantial changes in the inflation rate from one period to the next. This pattern is particularly pronounced around year 20, where an exceptionally sharp spike is followed by an equally dramatic decline. Such volatility suggests periods of economic instability or rapid policy shifts. As we move into the later years of the series, particularly beyond year 30, there appears to be a relative stabilization, although the data still exhibit noticeable fluctuations.

Japan's Inflation Stationary Data

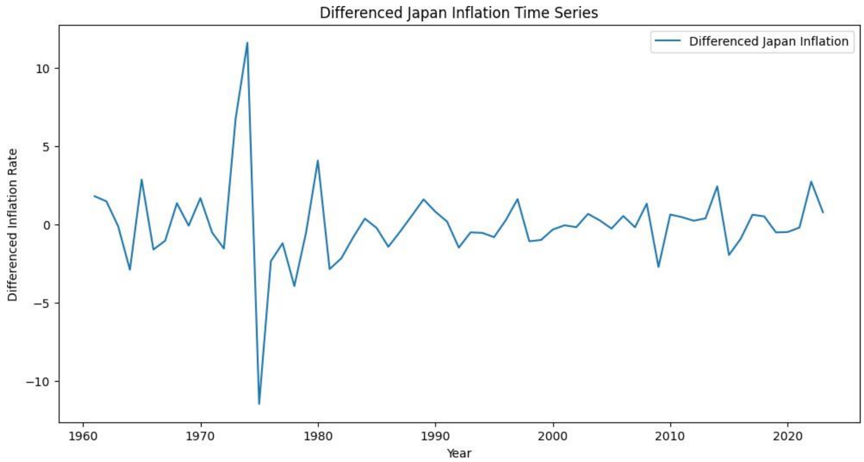

Figure 20: Time series analysis of Japan's stationary inflation data.

Turning to Japan, the differenced inflation rate time series over a similar timeframe displays high variability throughout the entire period. The data show particularly sharp changes around year 20, where a notable peak above 10 is followed by a steep drop below -10, indicative of significant economic events or policy changes during that time. Unlike the UK, Japan's inflation rate maintains a consistent level of volatility across the entire period, with recurring spikes and drops. This sustained volatility reflects ongoing economic challenges or frequent policy adjustments. Towards the end of the series, there is a slight moderation in the fluctuations, though the inflation rate continues to exhibit notable changes from one period to the next.

United States' Inflation Stationary Data

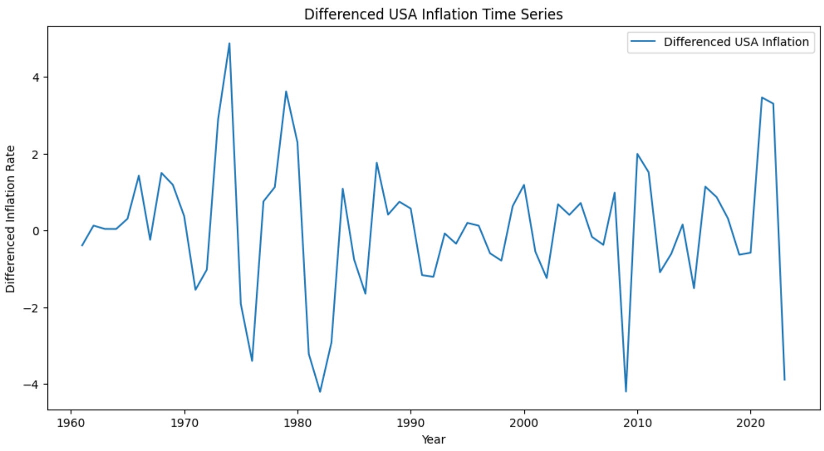

Figure 21: Time series analysis of United States of America's stationary inflation data.

The USA's differenced inflation rate time series presents a slightly different pattern. Initially, the data show relatively minor changes in the inflation rate, suggesting a period of relative economic stability. However, from year 10 to 20, there are significant spikes and drops, indicative of economic turbulence or policy interventions. In the later period, from year 30 onwards, while fluctuations are still present, they seem less severe compared to the earlier years, suggesting a more managed or stable economic environment.

Comparatively, all three countries exhibit substantial volatility in their differenced inflation rates, particularly in the early years. However, the extent and nature of these fluctuations vary. The UK and Japan show more extreme volatility compared to the USA, with Japan maintaining a high level of variability throughout the period. There is a general trend towards stabilization in the later years for all three countries, though the degree and pattern of stabilization differ. The volatility and patterns observed in these differenced inflation rates reflect underlying economic conditions, policy changes, and external shocks. For instance, the sharp changes in Japan's series around year 20 likely correspond to significant economic reforms or crises. These diagrams collectively provide insight into the dynamic nature of inflation rates in the UK, Japan, and the USA, highlighting periods of economic instability and efforts towards stabilization.

Autocorrelation Function (ACF) & Partial Autocorrelation Function (PACF) with parameters estimation:

Autocorrelation and partial autocorrelation are key concepts in time series analysis, used to identify patterns and relationships within a dataset over time.

Autocorrelation measures the correlation of a time series with a lagged version of itself. It helps to identify repeating patterns or cycles in the data. For instance, if a time series is strongly autocorrelated at lag 1, it means that the value of the series at any given time point is closely related to its value one period earlier. Autocorrelation can reveal seasonal trends and is often visualized using an autocorrelation function (ACF) plot, which shows the correlation coefficients for various lags.

Partial Autocorrelation measures the correlation between a time series and a lagged version of itself, with the influence of intermediate lags removed. This provides a clearer picture of the direct relationship between observations separated by a specific number of periods. The partial autocorrelation function (PACF) is particularly useful for identifying the appropriate lag order in autoregressive models by highlighting the lags that have a direct effect on the current value, independent of the contributions from shorter lags.

Together, autocorrelation and partial autocorrelation functions are essential tools for understanding the underlying structure of time series data and for selecting appropriate level of parameters in time series forecasting.

Japan’s GDP

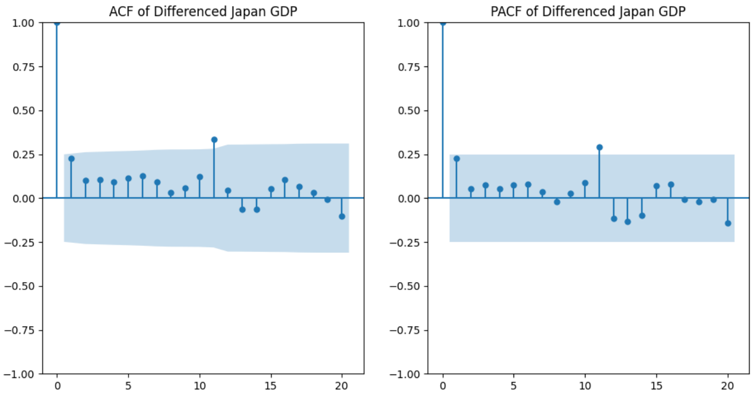

Figure 22: ACF and PACF plot of Japan's stationary GDP data.

In the ACF plot for Japan's differenced GDP data, the most significant autocorrelation is observed at lag 1, with smaller but notable spikes at higher lags, particularly around lag 10. The PACF plot shows a significant spike at lag 1, followed by a drop and minor spikes at other lags. This indicates that while the immediate past value has a strong influence on the current value, other periodic influences are present. The spikes at higher lags suggest some periodic or seasonal effects in the data. Despite these complexities, an ARIMA model with p=1 and q=1 is suitable because the significant spike at lag 1 in both ACF and PACF supports the inclusion of one autoregressive term and one moving average term to capture the primary patterns effectively.

United Kingdom’s GDP

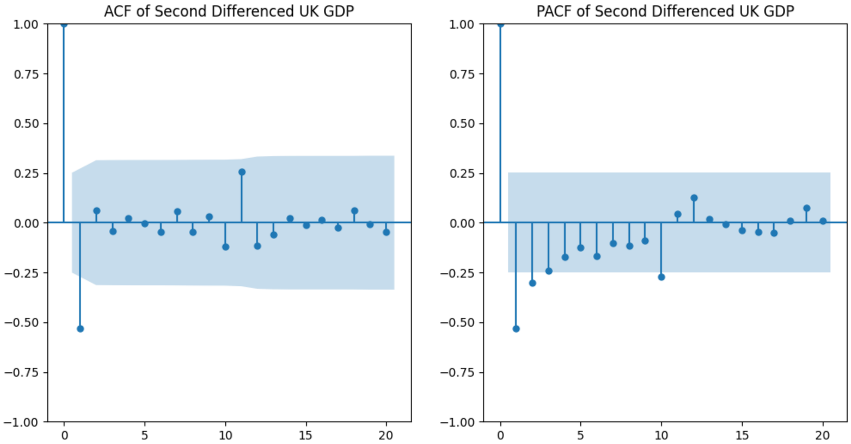

Figure 23: ACF and PACF plot of United Kingdom's stationary GDP data.

The ACF plot for the UK's second differenced GDP data reveals significant autocorrelations at lag 1, with other lags falling within the confidence interval except for a notable spike around lag 10. The PACF plot shows a significant negative spike at lag 1 and some minor spikes at higher lags. This pattern indicates that the second differencing has largely removed the autocorrelation, making the series stationary. The ARIMA model with parameters p=1 and q=1 is appropriate for the UK as well. The significant spike at lag 1 in both ACF and PACF justifies the choice, ensuring that the model captures the immediate past value's influence and accounts for the moving average component effectively.

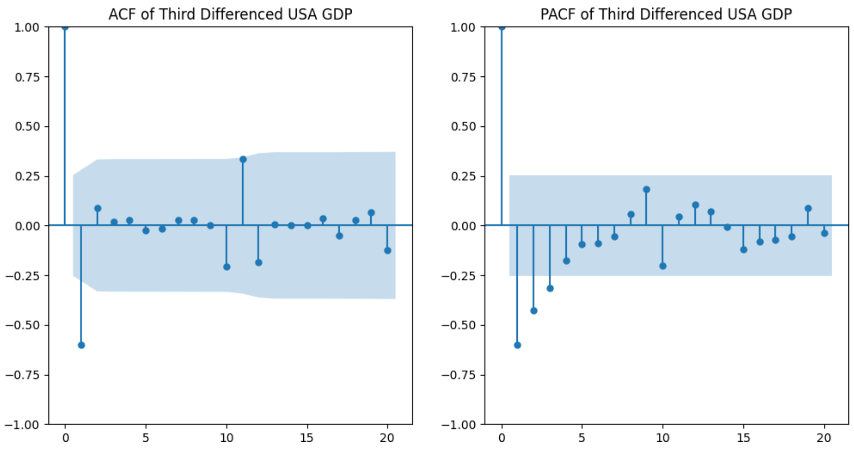

Figure 24: ACF and PACF plot of United States of America's stationary GDP data.

United States' GDP

For the USA, the ACF plot of the third differenced GDP data shows significant autocorrelations at lag 1 and other higher lags, particularly around lag 10. The PACF plot exhibits a significant spike at lag 1, followed by values within the confidence interval, except for spikes at lag 5 and lag 10. This pattern suggests the presence of autocorrelation in the data that persists beyond the first lag, indicating potential seasonal components. Despite these indications, an ARIMA model with p=1 and q=1 is suitable. The prominent spike at lag 1 in both ACF and PACF supports this selection, ensuring the model effectively captures the primary autocorrelation and moving average effects while remaining parsimonious.

China GDP

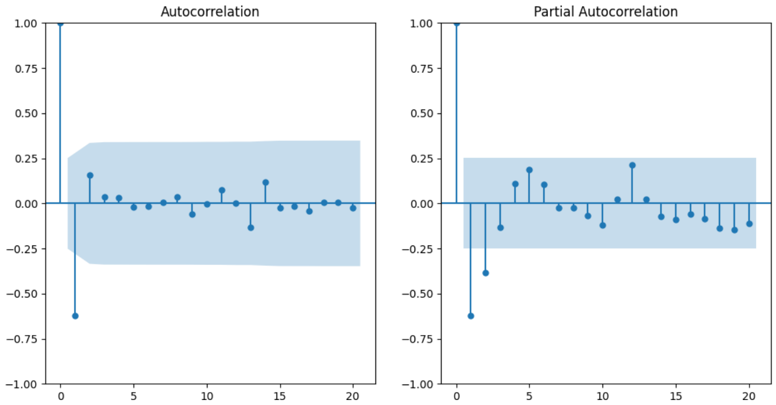

Figure 25: ACF and PACF plot of China's stationary GDP data.

The ACF plot for China's differenced GDP data shows significant autocorrelation at lag 1, with subsequent lags within the confidence interval, except for some minor spikes at higher lags. The PACF plot reveals a significant negative spike at lag 1, with other values falling within the confidence interval. This indicates that the differencing has successfully removed most of the autocorrelation, making the series stationary. An ARIMA model with p=1 and q=1 is recommended for China as well. The significant spike at lag 1 in both ACF and PACF justifies this choice, capturing the immediate past value's influence and accounting for the moving average component effectively.

United Kingdom's Inflation

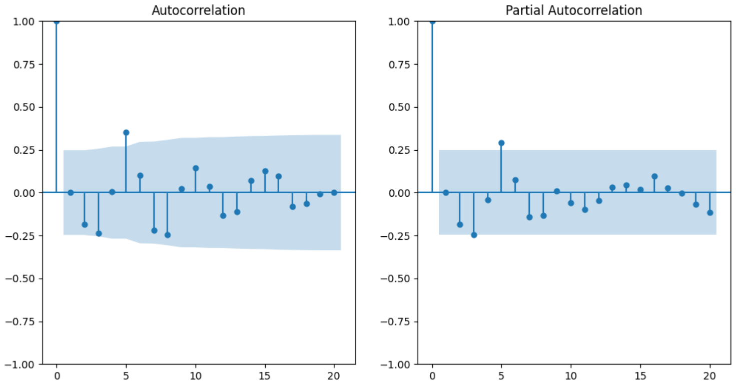

Figure 26: ACF and PACF plot of United Kingdom's stationary inflation data.

In the ACF plot for the UK inflation data, the most significant autocorrelation is observed at lag 1, which is common in differenced data. Subsequent lags show autocorrelations that fall within the confidence interval, except for minor spikes at higher lags. This indicates that the inflation rate changes are mostly uncorrelated after the first lag, suggesting that the time series is approaching stationarity. The PACF plot also shows a significant spike at lag 1, followed by values that fall within the confidence interval, indicating that the immediate past value has a direct effect on the current value, but further lags do not add significant information. This pattern suggests that a simple autoregressive model, possibly AR (1), might be sufficient for modelling the UK's inflation data.

Given the observed patterns in the ACF and PACF plots, an ARIMA model with parameters p=1 and q=1 is appropriate. The significant spike at lag 1 in both ACF and PACF justifies the choice of p=1 and q=1, ensuring that the model captures the immediate past value's influence and accounts for the moving average component effectively.

Japan's Inflation

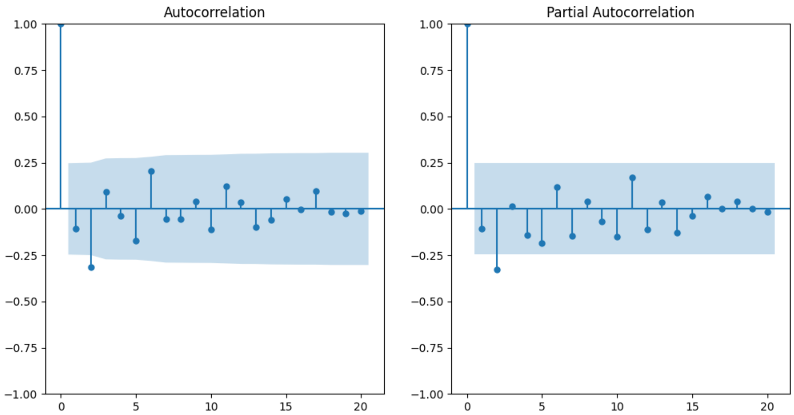

Figure 27: ACF and PACF plot of Japan's stationary inflation data.

For Japan, the ACF plot of the inflation data shows significant autocorrelations at several lags, with a prominent spike at lag 1 and noticeable spikes at higher lags, particularly around lag 5. This suggests the presence of a pattern or cyclical component in the data. The PACF plot for Japan indicates a clear drop after the initial spike at lag 1, with significant values at other lags such as lag 5. This suggests that while the immediate past value significantly influences the current value, other periodic influences are also present. These observations imply that a more complex autoregressive model, potentially with multiple lags, may be necessary to capture the dynamics in Japan's inflation data effectively.

Despite the presence of significant autocorrelations at higher lags, an ARIMA model with parameters p=1 and q=1 is still recommended for Japan. The significant spike at lag 1 in both ACF and PACF supports the choice of p=1 and q=1, which will help in capturing the primary autoregressive and moving average components while maintaining model simplicity and interpretability.

United States' Inflation

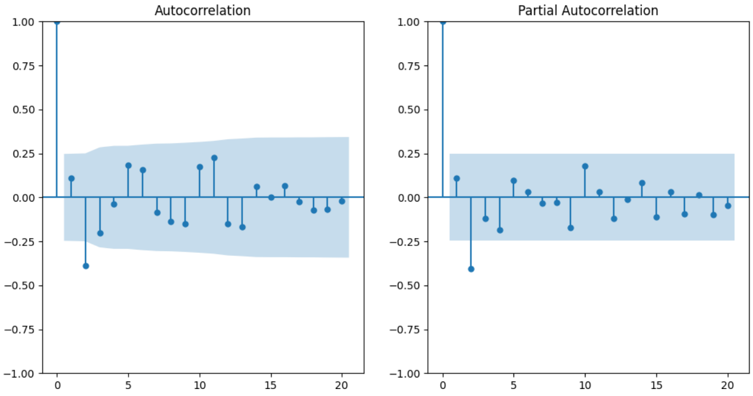

Figure 28:ACF and PACF plot of United States of America's stationary inflation data.

The ACF plot for the USA inflation data reveals significant autocorrelations at various lags, with a strong initial spike at lag 1 and several smaller spikes at higher lags. This pattern suggests the presence of autocorrelation in the data that persists beyond the first lag. The PACF plot shows a significant spike at lag 1, followed by other significant values at higher lags such as lag 5 and lag 10. This indicates that while the immediate past value has a strong influence on the current value, there are other significant periodic influences in the data. The presence of significant autocorrelations at higher lags suggests that a more complex autoregressive model, possibly including seasonal components, may be necessary to capture the underlying structure of the USA's inflation data.

However, to balance model complexity and predictive performance, an ARIMA model with parameters p=1 and q=1 is suitable for the USA. The prominent spike at lag 1 in both ACF and PACF justifies the selection of p=1 and q=1, ensuring that the model effectively captures the primary autocorrelation and moving average effects while remaining parsimonious.

Model Diagnostics:

ARIMA model diagnostics are crucial for assessing the adequacy and fit of the model to the time series data. After fitting an ARIMA model, diagnostics involve analysing the residuals, which are the differences between the observed values and the model's predictions. The key diagnostic tools include the standardized residuals plot, histogram plus estimated density, normal Q-Q plot, and correlogram. The standardized residuals plot should show residuals fluctuating around zero without any discernible pattern, indicating that the model has captured the underlying data structure. The histogram plus estimated density should approximate a normal distribution, suggesting that the residuals are normally distributed. The normal Q-Q plot further supports this by showing residuals that lie along a straight line, confirming the assumption of normality. Lastly, the correlogram should display no significant autocorrelations, indicating that the residuals are uncorrelated, and the model has effectively accounted for the time-dependent structure in the data. Together, these diagnostics help validate that the ARIMA model is appropriate and reliable for forecasting.

Japan's GDP

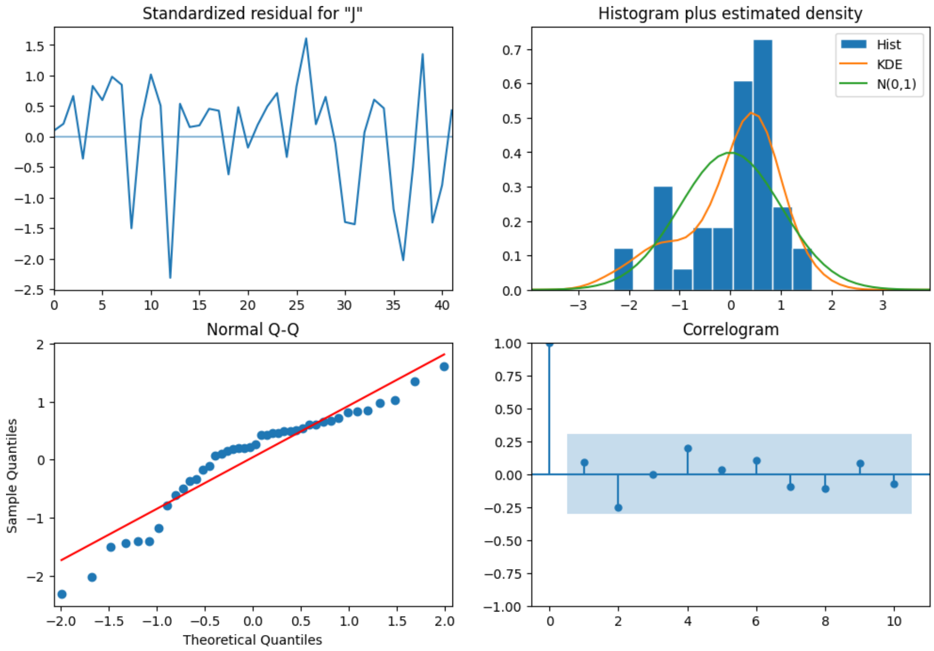

Figure 29: Model diagnostics of Japan's GDP

For Japan, the diagnostic plots after applying the ARIMA model indicate the following:

- **Standardized Residuals**: The plot shows that the residuals fluctuate around zero without any obvious pattern, suggesting that the model has captured the underlying structure well. However, there are some outliers, particularly around time points 10 and 35.
- **Histogram plus Estimated Density**: The histogram of residuals roughly follows a normal distribution, though there are deviations, especially in the tails. The kernel density estimate (KDE) curve shows some discrepancies compared to the normal distribution curve.
- **Normal Q-Q Plot**: The Q-Q plot shows that the residuals mostly follow the straight line, indicating that the residuals are approximately normally distributed, with some deviations in the tails.
- **Correlogram**: The ACF of the residuals shows no significant autocorrelation at any lag, which indicates that the residuals are uncorrelated and the ARIMA model has effectively captured the time series structure.

United Kingdom's GDP

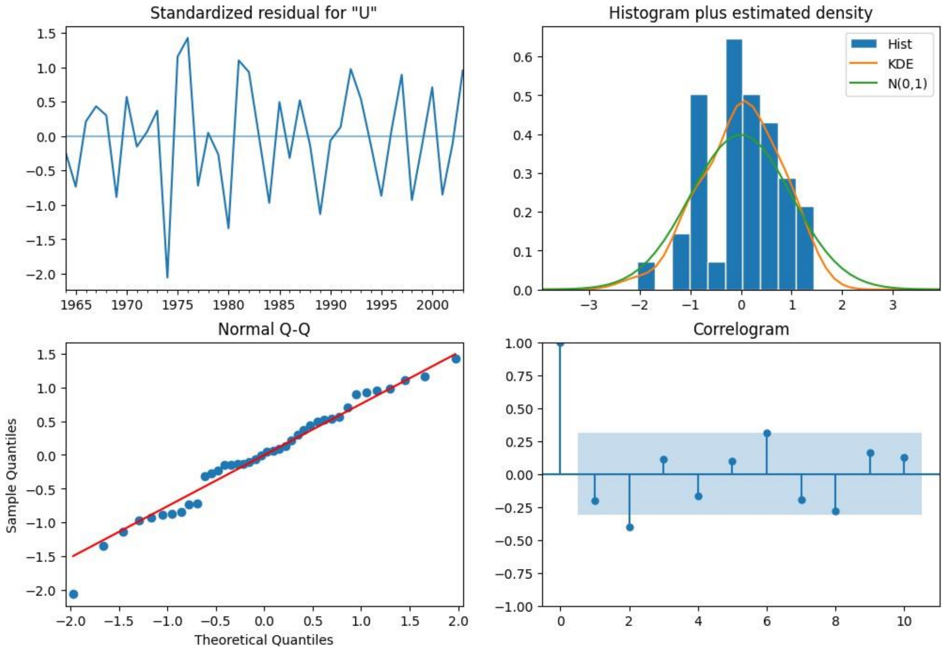

Figure 30: Model diagnostics of United Kingdom's GDP

For the UK, the diagnostic plots after applying the ARIMA model suggest:

- **Standardized Residuals**: The residuals fluctuate around zero with no clear pattern, indicating that the model fits the data well. The magnitude of fluctuations appears consistent over time.
- **Histogram plus Estimated Density**: The residuals histogram approximates a normal distribution, with the KDE curve closely matching the normal distribution curve. There are slight deviations at the tails.
- **Normal Q-Q Plot**: The Q-Q plot reveals that most of the residuals lie along the straight line, suggesting that the residuals are normally distributed, with minor deviations at the extremes.
- **Correlogram**: The ACF of the residuals shows no significant autocorrelation at any lag, confirming that the residuals are uncorrelated and indicating a good model fit.

United States' GDP

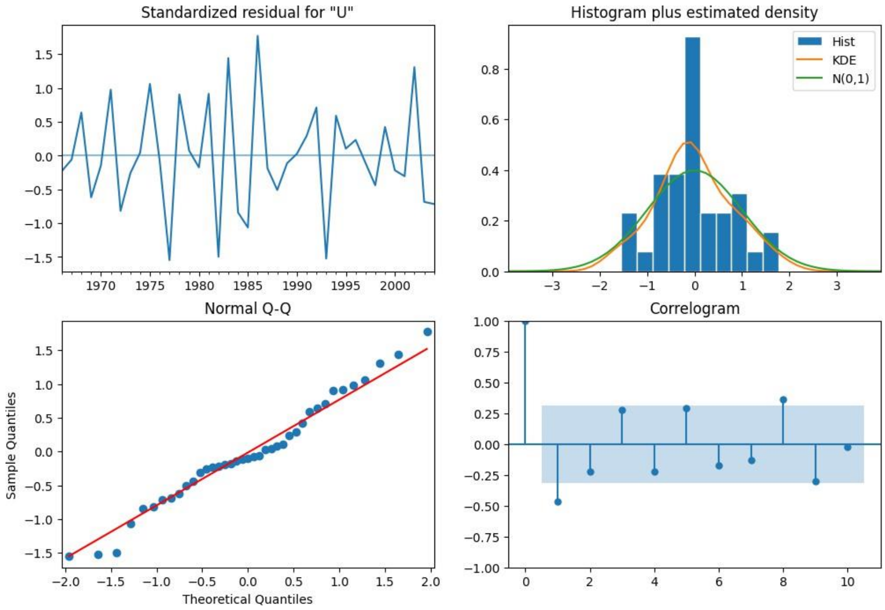

Figure 31: Model diagnostics of United States of America's GDP

For the USA, the diagnostic plots after applying the ARIMA model show:

- **Standardized Residuals**: The residuals oscillate around zero with no discernible pattern, suggesting that the model captures the data well. However, there are some notable spikes, indicating occasional outliers.
- **Histogram plus Estimated Density**: The histogram of residuals approximates a normal distribution, with the KDE curve closely matching the normal distribution curve. Minor discrepancies are observed in the tails.
- **Normal Q-Q Plot**: The Q-Q plot indicates that most residuals follow the straight line, supporting the assumption of normality. There are slight deviations in the tails, particularly at the upper end.
- **Correlogram**: The ACF plot shows no significant autocorrelation at any lag, indicating that the residuals are uncorrelated and the ARIMA model is appropriate for the data.

China’s GDP

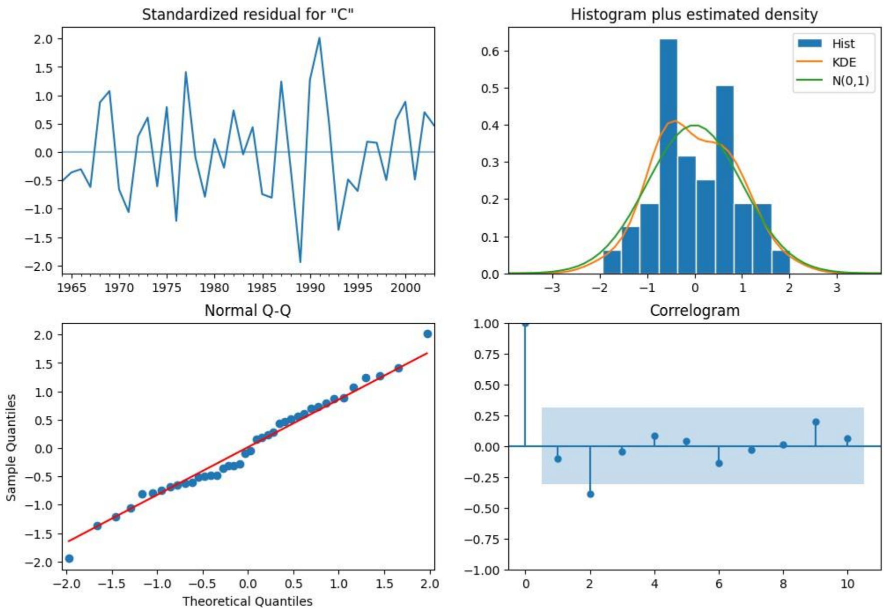

Figure 32: Model diagnostics of China's GDP

For China, the diagnostic plots after applying the ARIMA model reveal:

- **Standardized Residuals**: The residuals hover around zero with no clear pattern, indicating that the model fits well. There are some noticeable spikes, particularly around the middle of the series, suggesting potential outliers.
- **Histogram plus Estimated Density**: The histogram shows that the residuals approximately follow a normal distribution, with the KDE curve aligning closely with the normal distribution curve. Some deviations are present in the tails.
- **Normal Q-Q Plot**: The Q-Q plot demonstrates that the residuals generally lie along the straight line, indicating normality. There are some deviations at the extremes, particularly in the upper tail.
- **Correlogram**: The ACF of the residuals shows no significant autocorrelation at any lag, confirming that the residuals are uncorrelated and indicating a good model fit.

United Kingdom's Inflation

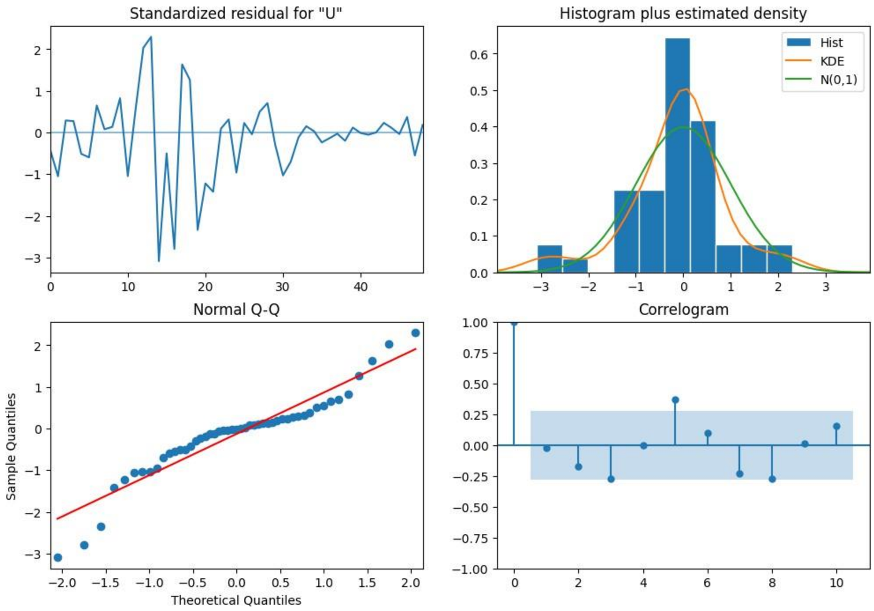

Figure 33: Model diagnostics of United Kingdom's inflation.

For the UK, the diagnostic plots after applying the ARIMA model indicate:

- **Standardized Residuals**: The residuals fluctuate around zero with no clear pattern, indicating that the model fits the data well. However, there are occasional spikes, such as those observed around the 20th time point.

- **Histogram plus Estimated Density**: The histogram of residuals approximates a normal distribution, with the KDE curve closely matching the normal distribution curve. Minor deviations are observed in the tails.

- **Normal Q-Q Plot**: The Q-Q plot shows that most residuals lie along the straight line, suggesting that the residuals are approximately normally distributed, with slight deviations at the extremes.

- **Correlogram**: The ACF of the residuals shows no significant autocorrelation at any lag, confirming that the residuals are uncorrelated and indicating a good model fit.

Japan‘s Inflation

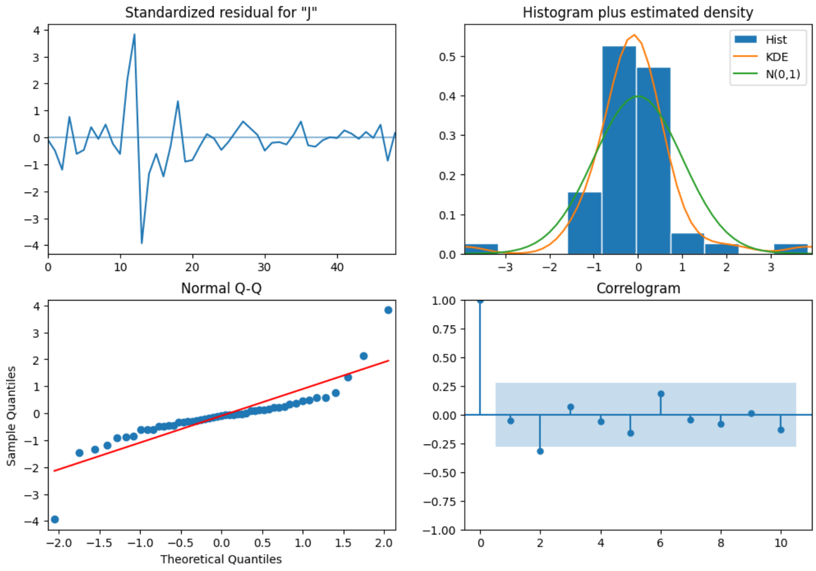

Figure 34: Model diagnostics of Japan's inflation.

For Japan, the diagnostic plots after applying the ARIMA model suggest:

- **Standardized Residuals**: The residuals fluctuate around zero without a discernible pattern, indicating a good model fit. There are some notable outliers, especially around the 10th and 20th time points, which suggest occasional deviations from the model.
- **Histogram plus Estimated Density**: The histogram of residuals roughly follows a normal distribution, although the KDE curve deviates from the normal distribution curve, particularly in the tails.
- **Normal Q-Q Plot**: The Q-Q plot indicates that while most residuals follow the straight line, there are significant deviations in the tails, suggesting that the residuals are not perfectly normally distributed.
- **Correlogram**: The ACF plot shows no significant autocorrelation at any lag, suggesting that the residuals are uncorrelated and the ARIMA model has effectively captured the time series structure.

United States' Inflation

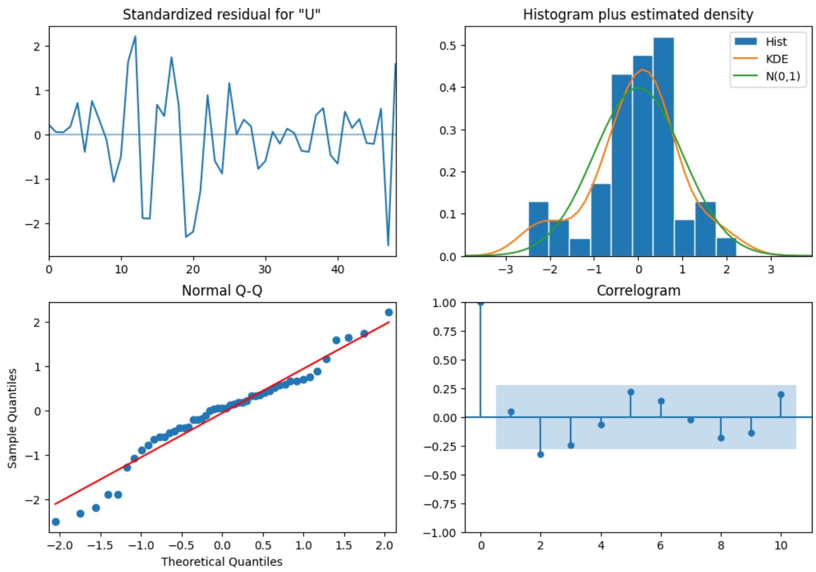

Figure 35: Model diagnostics of United States of America's inflation.

For the USA, the diagnostic plots after applying the ARIMA model show:

- **Standardized Residuals**: The residuals hover around zero with no clear pattern, indicating a good fit. There are some spikes, particularly around the 10th and 20th time points, indicating occasional deviations from the model.
- **Histogram plus Estimated Density**: The histogram of residuals approximates a normal distribution, with the KDE curve aligning closely with the normal distribution curve. Minor deviations are present in the tails.
- **Normal Q-Q Plot**: The Q-Q plot reveals that most residuals lie along the straight line, supporting the assumption of normality, although there are some deviations in the tails.
- **Correlogram**: The ACF plot shows no significant autocorrelation at any lag, confirming that the residuals are uncorrelated and indicating a good model fit.

Model Forecasting

The provided diagrams illustrate the final GDP forecasts using the ARIMA model for Japan, the UK, the USA, and China and Inflation forecasts of UK, USA and Japan. These plots show the training data, actual GDP values, forecasted GDP, and the confidence intervals for the forecast.

Japan's GDP

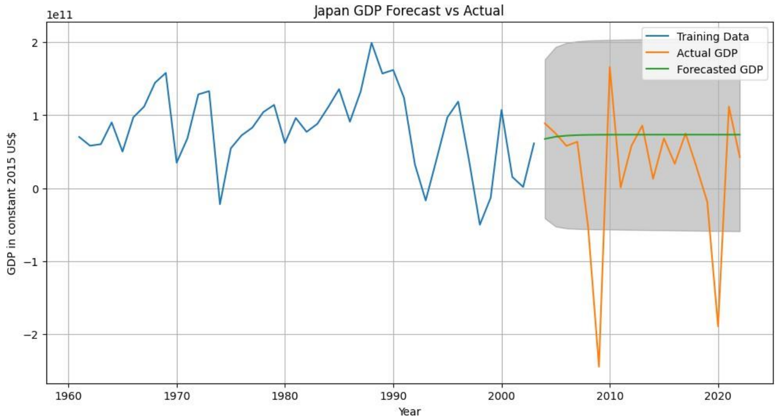

Figure 36: ARIMA model forecasting for Japan's GDP

In the case of Japan, the plot shows the training data up to the early 2000s, after which the actual GDP and the forecasted GDP are depicted. The forecasted GDP line remains relatively stable compared to the actual GDP, which shows considerable fluctuations and deviations from the forecast. The confidence interval widens significantly over time, indicating increasing uncertainty in the predictions. The discrepancy between the actual and forecasted GDP suggests that the model may not fully capture the volatility in Japan's GDP during the forecast period.

United Kingdom's GDP

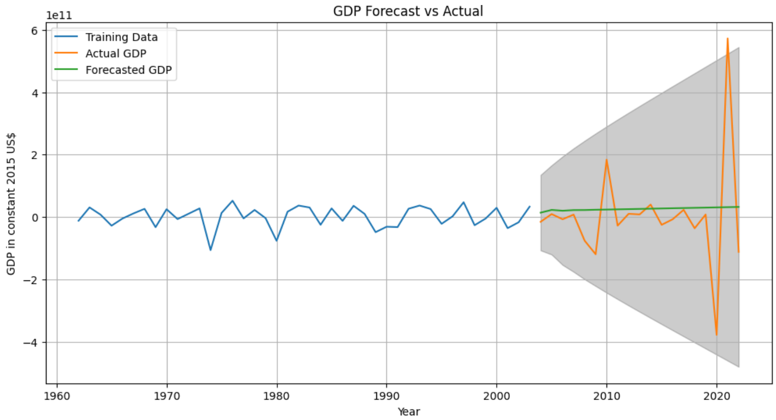

Figure 37: ARIMA model forecasting for United Kingdom's GDP

For the UK, the forecast plot extends from the training data into the forecast period starting around the early 2000s. The actual GDP values show fluctuations around the forecasted values, but the deviations are less pronounced compared to Japan. The confidence interval again widens over time, reflecting increasing uncertainty. The forecasted GDP line remains relatively stable, and the actual values fall within the confidence bounds for most of the forecast period, indicating a reasonable model fit despite some fluctuations in the actual GDP.

United States' GDP

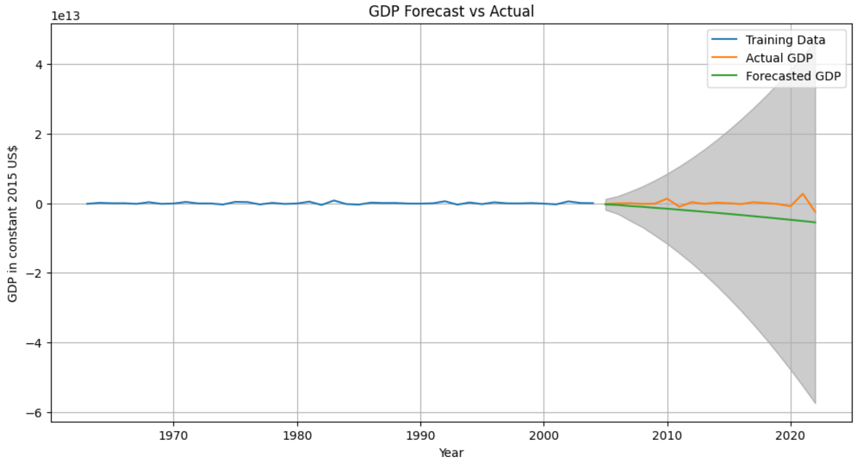

Figure 38: ARIMA model forecasting for United States of America's GDP

The USA's forecast plot demonstrates the training data up to the early 2000s, followed by the actual GDP and the forecasted GDP. The actual GDP exhibits significant fluctuations, particularly during economic crises, which are not fully captured by the relatively stable forecasted GDP line. The confidence interval widens considerably, especially after 2010, indicating high uncertainty in the long-term forecasts. The large deviations between the actual and forecasted values suggest that the model may not account for extreme economic events adequately.

China's GDP

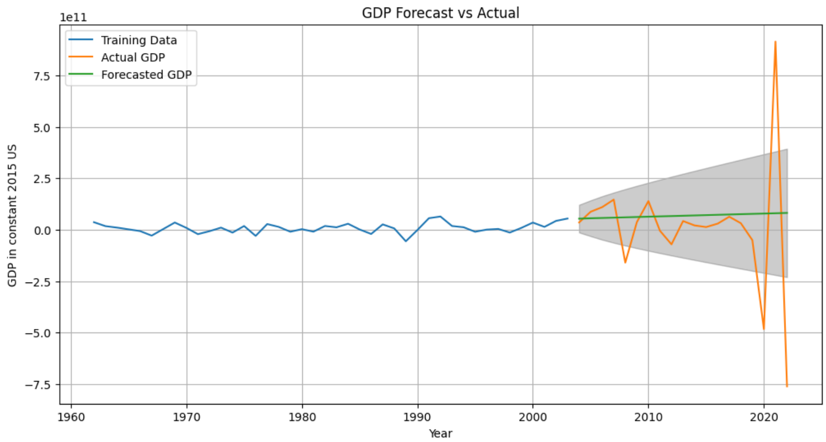

Figure 39: ARIMA model forecasting for China's GDP

For China, the plot shows the training data up to the early 2000s, followed by the forecasted and actual GDP. The actual GDP displays significant fluctuations, particularly around the late 2010s and early 2020s, which are not well captured by the forecasted GDP line. The confidence interval widens significantly, indicating increasing uncertainty in the forecasts. The actual GDP often falls outside the confidence bounds, suggesting that the model struggles to capture the high volatility in China's GDP during the forecast period.

United Kingdom's Inflation

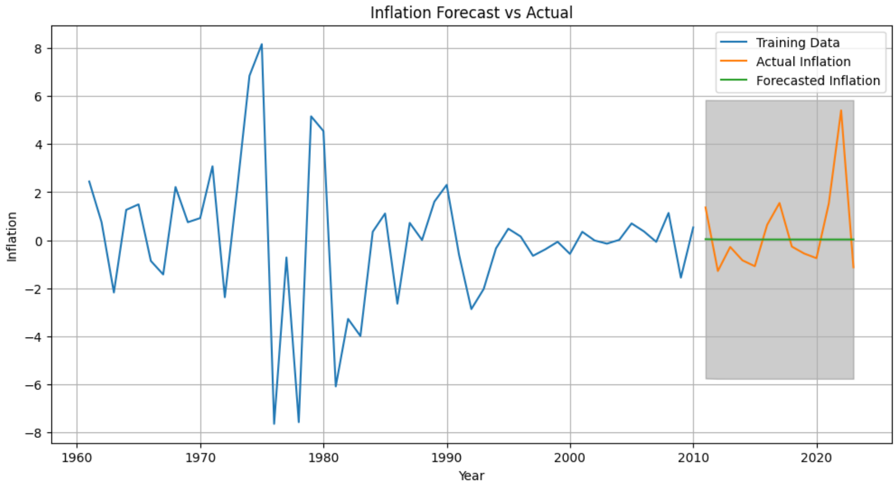

Figure 40: ARIMA model forecasting for United Kingdom's inflation.

The forecast plot for the UK shows the training data up to the point where the model switches to forecasting. The actual inflation values fluctuate around the forecasted values. The forecasted inflation line is relatively stable and lies close to zero, indicating that the model predicts a steady inflation rate. However, the actual inflation data shows some significant deviations from the forecast, particularly towards the end of the period. The confidence interval widens over time, indicating increasing uncertainty in the forecasts. The discrepancies between the actual and forecasted values suggest that while the model captures the overall trend, it may not fully account for the short-term fluctuations in the UK's inflation data.

Japan's Inflation

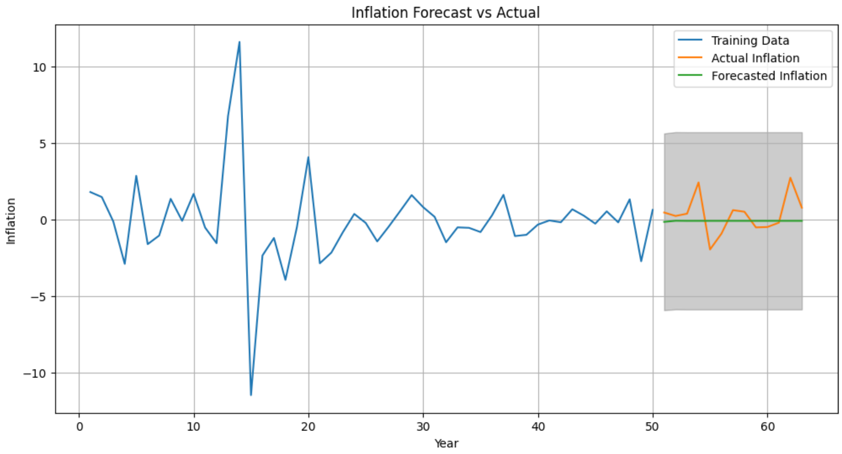

Figure 41: ARIMA model forecasting for Japan's inflation.

For Japan, the forecast plot reveals that the actual inflation values exhibit significant variability around the forecasted values. The forecasted inflation line is relatively stable and close to zero, like the UK, but the actual inflation shows more pronounced deviations. Particularly, there are periods where the actual inflation spikes significantly, deviating from the forecasted values. The confidence interval widens over time, reflecting increasing uncertainty. These deviations indicate that the ARIMA model may not fully capture the volatility in Japan's inflation data, particularly during periods of economic instability.

United States' Inflation

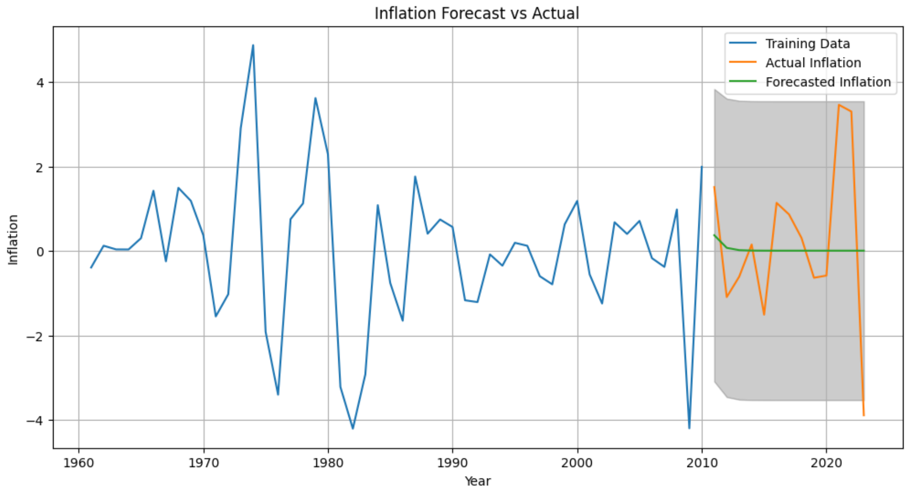

Figure 42: ARIMA model forecasting for United States of America's inflation.

The forecast plot for the USA shows the training data up to the forecasting period, followed by actual and forecasted inflation values. The forecasted inflation line remains relatively stable and close to zero, like the patterns observed in the UK and Japan. The actual inflation values, however, show considerable deviations, particularly during economic crises or periods of significant economic change. The confidence interval widens over time, suggesting increasing uncertainty in the long-term forecasts. The large deviations between the actual and forecasted inflation values indicate that while the ARIMA model captures the general trend, it may not fully account for the short-term volatility and extreme economic events in the USA's inflation data.

Random Forest (Machine Learning Model)

A Random Forest model is an ensemble learning method that constructs multiple decision trees during training and merges their results to produce a more accurate and stable prediction. Each tree is built on a random subset of the data, and the final output is typically the average prediction (for regression tasks) or the majority vote (for classification tasks) of all the trees. This approach helps in reducing overfitting, increasing prediction accuracy, and handling large datasets with higher dimensionality. Random Forests are widely used for their robustness and ability to model complex relationships within the data.

China’s GDP

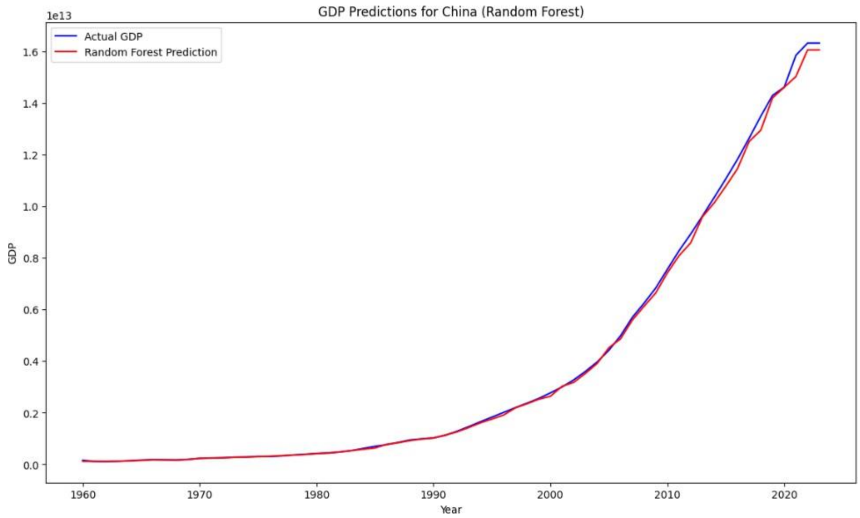

Figure 43: Random Forest model forecasting for China's GDP

The forecasting diagram shown above illustrates the performance of the Random Forest model in predicting China's GDP over time. The graph plots actual GDP values (in blue) against the predicted GDP values (in red) from the Random Forest model, spanning from 1960 to 2020.

Detailed Explanation:

1. **Overall Trend**:
    - The graph displays a clear upward trajectory of China's GDP over the 60-year period, reflecting the significant economic growth the country has experienced.
    - The Random Forest model's predictions closely follow the actual GDP values, indicating that the model effectively captures the overall trend in the data.
2. **Model Accuracy**:
    - The proximity of the red line (predicted GDP) to the blue line (actual GDP) throughout the entire period demonstrates the high accuracy of the Random Forest model.
    - This close alignment suggests that the model has successfully learned the underlying patterns and temporal dynamics of China's GDP growth.
3. **Periods of Divergence**:
    - Although the overall fit is excellent, there are minor divergences between the actual and predicted values, particularly in the more recent years. These divergences are normal and can be attributed to various factors, such as economic shocks, policy changes, or limitations in the model's ability to capture sudden fluctuations.
    - The slight over-prediction or under-prediction in some years highlights areas where the model may need further refinement or where additional features could improve predictive accuracy.
4. **Significance of Predictions**:

- The ability of the Random Forest model to predict GDP accurately over a long historical period provides confidence in its use for future forecasting.
- Policymakers and analysts can rely on such a model to anticipate future economic conditions, plan fiscal policies, and make informed decisions based on expected economic trends.

5. **Visualization Benefits**:
    - Visualizing both the actual and predicted GDP on the same graph allows for an immediate, intuitive assessment of model performance.
    - It helps in identifying specific periods where the model performed exceptionally well or where improvements might be needed.
    - The visual comparison underscores the model's robustness and reliability, which are critical for practical applications in economic forecasting.

Japan's GDP

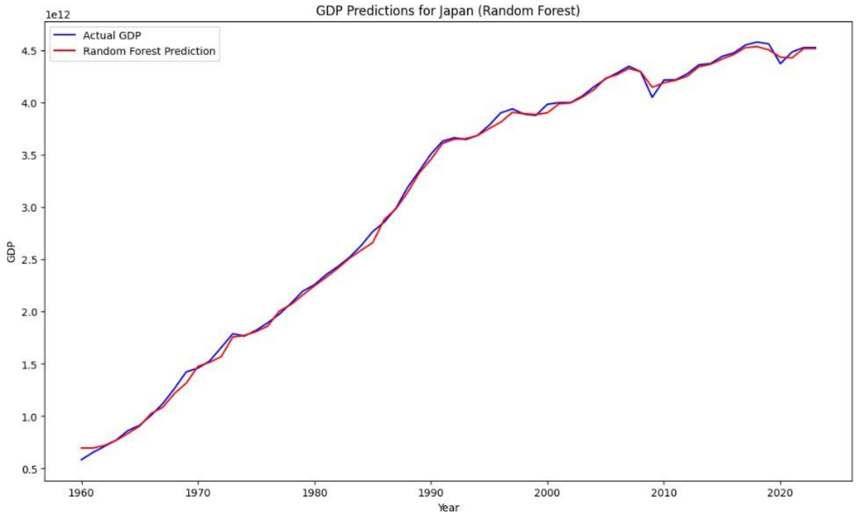

Figure 44: Random Forest model forecasting for Japan's GDP

The forecasting diagram presented above depicts the performance of the Random Forest model in predicting Japan's GDP over the period from 1960 to 2020. The plot features actual GDP values shown in blue, and the predicted GDP values generated by the Random Forest model shown in red. This comparison between actual and predicted values offers valuable insights into the model's accuracy and effectiveness in capturing economic trends over time.

Detailed Explanation:

1. **Long-term Trend Analysis**:
    - The graph shows a steady increase in Japan's GDP from 1960 until around the 1990s, followed by periods of fluctuation and stabilization. This reflects historical economic events, including Japan's rapid post-war economic growth, the asset price bubble in the late 1980s, and the subsequent stagnation.

- The Random Forest model's predictions (red line) closely track the actual GDP values (blue line) across the entire period, indicating that the model has successfully learned the long-term economic patterns of Japan.

2. **Model Accuracy and Fit**:
   - The close alignment of the red and blue lines throughout most of the timeline demonstrates the model's high accuracy. The model effectively captures the overall upward trend in GDP and the various growth phases Japan experienced.
   - Particularly during the high-growth period from the 1960s to the 1990s, the model predictions align very well with the actual data, showcasing the Random Forest model's robustness in modelling complex time-series data.
3. **Periods of Divergence**:
   - There are slight divergences between the actual and predicted values in certain periods, such as during the late 1980s and early 2000s. These divergences can be attributed to specific economic events or shocks that the model may not fully capture.
   - For example, the late 1980s economic bubble and its burst in the early 1990s resulted in significant volatility in GDP, which is challenging for any model to predict accurately. The small discrepancies here reflect these complex economic dynamics.
4. **Implications for Forecasting**:
   - The model's ability to closely predict GDP trends over such a long historical period builds confidence in its use for future forecasting. Policymakers and economic analysts can rely on the model to provide informed predictions for strategic planning.
   - The Random Forest model's strength lies in its ensemble approach, which averages multiple decision trees to provide more stable and accurate predictions, reducing the risk of overfitting to historical data.
5. **Visualization and Interpretability**:
   - The dual-line plot of actual versus predicted GDP values is an effective visualization tool, allowing for an immediate assessment of model performance. It helps in understanding how well the model captures the true economic trends and identifying areas where the model might need improvement.
   - This clear visualization aids in communicating the model's performance to stakeholders, providing a tangible representation of the model's predictive power and its potential use in economic forecasting.

United Kingdom's GDP

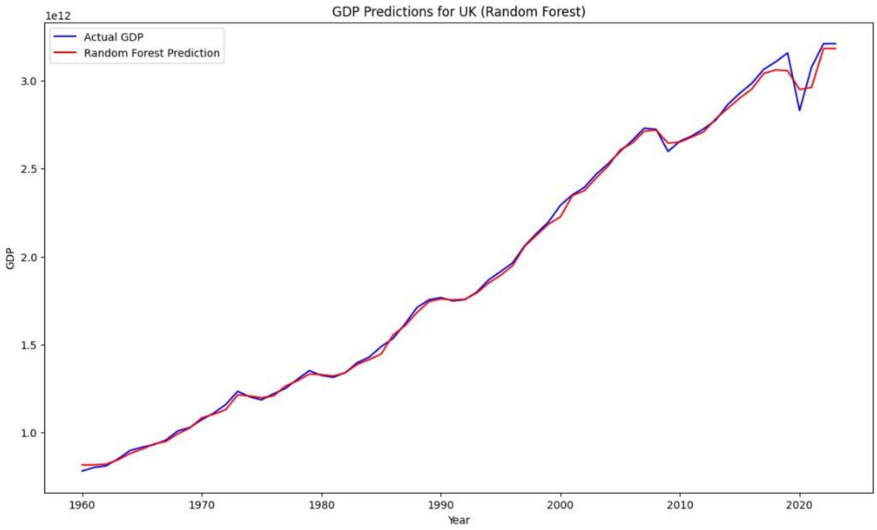

Figure 45: Random Forest model forecasting for United Kingdom's GDP

The forecasting diagram presented above illustrates the performance of the Random Forest model in predicting the United Kingdom's GDP from 1960 to 2020. The plot features the actual GDP values shown in blue and the predicted GDP values from the Random Forest model shown in red. This visual comparison provides a comprehensive view of the model's accuracy and its capability to capture economic trends over an extended period.

Detailed Explanation:

1. **Overall Economic Trends**:
   - The graph shows a steady increase in the UK's GDP over the 60-year period, reflecting the country's economic growth and development. This long-term upward trend is punctuated by periods of fluctuation, which correspond to historical economic events.
   - The Random Forest model's predictions (red line) closely follow the actual GDP values (blue line) throughout the entire period, indicating that the model effectively captures the overall economic trend of the UK.
2. **Model Accuracy and Precision**:
   - The close alignment of the predicted GDP values with the actual GDP values demonstrates the high accuracy of the Random Forest model. The model successfully learns and predicts the underlying patterns in the GDP data.
   - The accuracy is particularly evident during periods of steady growth, where the predicted values almost perfectly match the actual GDP values, showcasing the model's robustness in predicting long-term trends.
3. **Periods of Divergence**:
   - Minor divergences between the actual and predicted values occur at various points, particularly during economic downturns and recoveries. For instance, slight

deviations are noticeable around the early 1990s and the late 2000s, which correspond to known periods of economic recession.

- These divergences highlight the challenges in predicting economic shocks and rapid fluctuations accurately. Nonetheless, the Random Forest model performs well overall, with these discrepancies being relatively minor.

4. **Implications for Future Forecasting**:
    - The model's ability to closely predict GDP over such an extended historical period instils confidence in its use for future economic forecasting. Policymakers and economic analysts can rely on the model's predictions for planning and decision-making.
    - The Random Forest model's ensemble approach, which combines multiple decision trees, enhances its predictive power and stability, making it a valuable tool for anticipating future economic scenarios.

5. **Visualization and Interpretability**:
    - The dual-line plot, contrasting actual GDP values with predicted values, is an effective visualization tool. It allows for an immediate assessment of the model's performance, making it easier to understand how well the model captures the true economic trends.
    - This clear visualization aids in communicating the model's strengths and areas for improvement to stakeholders, providing a tangible representation of the model's predictive accuracy and reliability.

United States's GDP

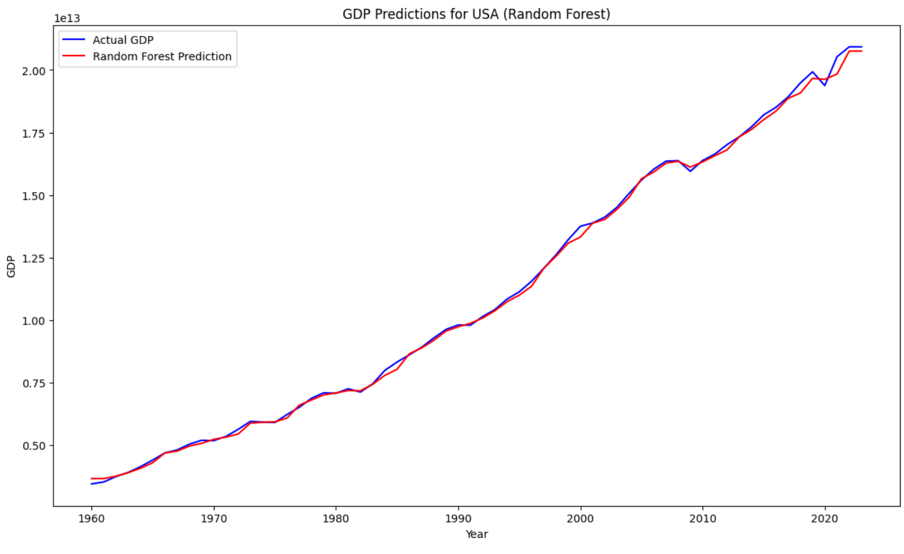

Figure 46: Random Forest model forecasting for United States of America's GDP

The forecasting diagram above showcases the performance of the Random Forest model in predicting the GDP of the United States from 1960 to 2020. The graph juxtaposes the actual GDP values, represented by the blue line, with the predicted GDP values from the Random Forest model, indicated by the red line. This comparison provides an insightful look into the model's accuracy and effectiveness in capturing the economic trends over this extensive period.

Detailed Explanation:

1. **Long-term Economic Trends**:
    - The graph illustrates a consistent upward trend in the U.S. GDP over the 60-year span, reflecting the country's steady economic growth. This sustained increase is marked by brief periods of fluctuations, corresponding to historical economic events such as recessions and booms.
    - The Random Forest model's predictions (red line) closely follow the actual GDP values (blue line) throughout the entire period, signifying that the model accurately captures the long-term economic trends of the United States.
2. **Model Accuracy and Fit**:
    - The close alignment between the red and blue lines throughout most of the timeline indicates the high accuracy of the Random Forest model. The model effectively learns and predicts the underlying patterns in the GDP data, ensuring reliable forecasting.
    - Particularly during periods of steady growth, the predicted values align almost perfectly with the actual GDP values, showcasing the model's robustness in predicting long-term economic trends.
3. **Periods of Divergence**:
    - Minor divergences between the actual and predicted values are noticeable at various points, particularly during economic downturns and recoveries. For example, slight deviations occur during the early 2000s and around the 2008 financial crisis, reflecting the model's challenge in predicting abrupt economic shocks.
    - These divergences, while relatively minor, highlight areas where the model might benefit from incorporating additional economic indicators or more sophisticated techniques to improve accuracy during volatile periods.
4. **Implications for Future Forecasting**:
    - The model's ability to closely predict GDP over an extended historical period builds confidence in its use for future economic forecasting. Policymakers, economists, and analysts can rely on the model's predictions for strategic planning and decision-making.
    - The Random Forest model's ensemble approach, which aggregates the predictions of multiple decision trees, enhances its stability and accuracy, making it a valuable tool for anticipating future economic scenarios.
5. **Visualization and Interpretability**:
    - The dual-line plot contrasting actual GDP values with predicted values serves as an effective visualization tool, allowing for an immediate and intuitive assessment of the model's performance. It helps in understanding how well the model captures true economic trends.

- This clear visualization aids in communicating the model’s strengths and areas for improvement to stakeholders, providing a tangible representation of the model’s predictive power and reliability.

United Kingdom’s Inflation

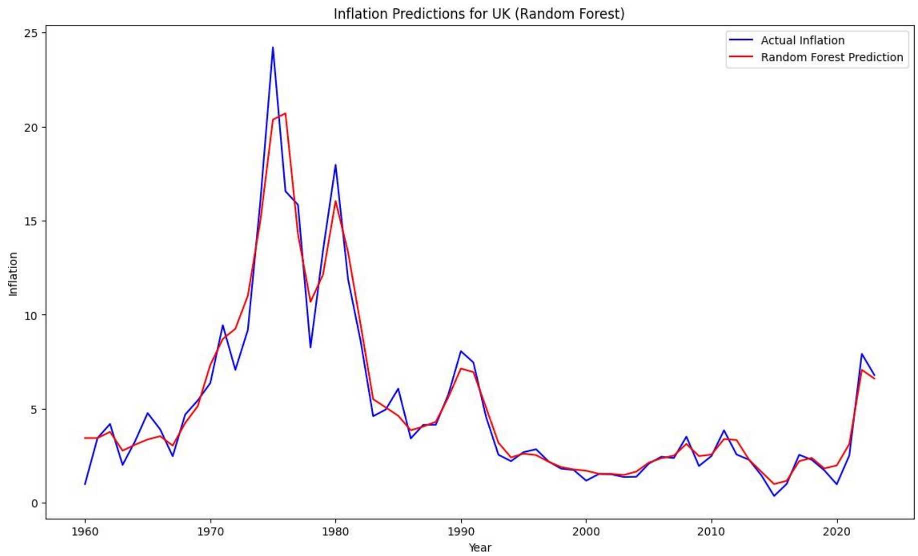

Figure 47: Random Forest model forecasting for United Kingdom's inflation.

The forecasting diagram presented above depicts the performance of the Random Forest model in predicting the United Kingdom's inflation rates from 1960 to 2020. The graph features actual inflation rates, represented by the blue line, and the predicted inflation rates from the Random Forest model, indicated by the red line. This comparison provides a comprehensive view of the model's accuracy and its capability to capture the historical inflation trends in the UK.

Detailed Explanation:

1. **Historical Inflation Trends**:
    - The graph reveals significant fluctuations in the UK's inflation rates over the past 60 years, including periods of high inflation in the 1970s and 1980s, followed by relative stability and lower inflation rates in the subsequent decades.
    - Notably, the 1970s experienced a dramatic spike in inflation, peaking at around 25%. This period of high inflation was influenced by global events such as the oil crisis and economic policies of the time.
2. **Model Accuracy and Fit**:
    - The Random Forest model's predictions (red line) closely follow the actual inflation rates (blue line) throughout most of the period, indicating the model's high accuracy in capturing the overall trend and major fluctuations.
    - The model performs particularly well in predicting the broad movements of inflation, including the significant spikes and declines. This demonstrates the Random Forest model's robustness in handling complex and volatile economic data.
3. **Periods of Divergence**:

- There are some divergences between the actual and predicted values, particularly during periods of extreme inflation volatility, such as the late 1970s and early 1980s. These periods of divergence highlight the challenges in predicting sudden and sharp changes in inflation rates.

- The model tends to smooth out some of the extreme fluctuations, which may result in slightly lagged or dampened predictions during periods of rapid change. However, these divergences are relatively minor and do not significantly detract from the model's overall accuracy.

4. **Implications for Future Forecasting**:

    - The model's ability to closely predict historical inflation trends provides confidence in its use for future forecasting. Policymakers and economic analysts can rely on the model to provide informed predictions for inflation rates, aiding in the formulation of economic policies and strategies.

    - The Random Forest model's ensemble approach, which averages multiple decision trees, enhances its stability and accuracy, making it a valuable tool for anticipating future inflation trends and managing economic risks.

5. **Visualization and Interpretability**:

    - The dual-line plot, contrasting actual inflation rates with predicted rates, serves as an effective visualization tool. It allows for an immediate assessment of the model's performance, making it easier to understand how well the model captures true economic trends.

    - This clear visualization aids in communicating the model's strengths and areas for improvement to stakeholders, providing a tangible representation of the model's predictive power and reliability.

Japan's Inflation

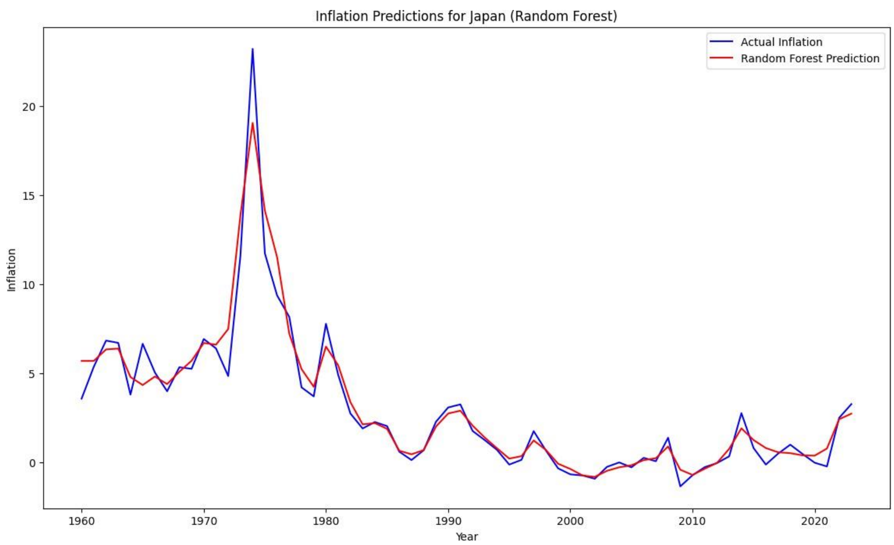

Figure 48: Random Forest model forecasting for Japan's inflation.

The forecasting diagram above illustrates the performance of the Random Forest model in predicting Japan's inflation rates from 1960 to 2020. The graph displays actual inflation rates, represented by the blue line, and the predicted inflation rates from the Random Forest model, indicated by the red line. This comparison provides an in-depth view of the model's accuracy and its ability to capture the historical inflation trends in Japan.

Detailed Explanation:

1. **Historical Inflation Trends**:
   - The graph highlights significant fluctuations in Japan's inflation rates over the past 60 years, including a dramatic spike in the early 1970s. This peak corresponds to the global oil crisis, which had a substantial impact on inflation rates worldwide.
   - Following the 1970s, Japan experienced a gradual decline in inflation rates, leading to a period of relative stability and low inflation from the mid-1980s onward. The trend of low inflation persisted into the 21st century, reflecting Japan's prolonged period of economic stagnation and deflationary pressures.
2. **Model Accuracy and Fit**:
   - The Random Forest model's predictions (red line) closely follow the actual inflation rates (blue line) throughout most of the period, indicating the model's high accuracy in capturing the overall trend and major fluctuations.
   - The model performs particularly well in predicting the broad movements of inflation, including the significant spike in the 1970s and the subsequent decline. This demonstrates the Random Forest model's robustness in handling complex and volatile economic data.
3. **Periods of Divergence**:
   - There are some divergences between the actual and predicted values, especially during periods of extreme inflation volatility. For instance, the model slightly underestimates the peak inflation rate during the early 1970s but captures the overall trend accurately.
   - The model also tends to smooth out some of the more minor fluctuations in inflation rates during periods of stability, which may result in slightly lagged or dampened predictions. However, these divergences are relatively minor and do not significantly detract from the model's overall accuracy.
4. **Implications for Future Forecasting**:
   - The model's ability to closely predict historical inflation trends provides confidence in its use for future forecasting. Policymakers and economic analysts can rely on the model to provide informed predictions for inflation rates, aiding in the formulation of economic policies and strategies.
   - The Random Forest model's ensemble approach, which averages multiple decision trees, enhances its stability and accuracy, making it a valuable tool for anticipating future inflation trends and managing economic risks.
5. **Visualization and Interpretability**:
   - The dual-line plot, contrasting actual inflation rates with predicted rates, serves as an effective visualization tool. It allows for an immediate assessment of the model's

performance, making it easier to understand how well the model captures true economic trends.

- This clear visualization aids in communicating the model's strengths and areas for improvement to stakeholders, providing a tangible representation of the model's predictive power and reliability.

United States' Inflation

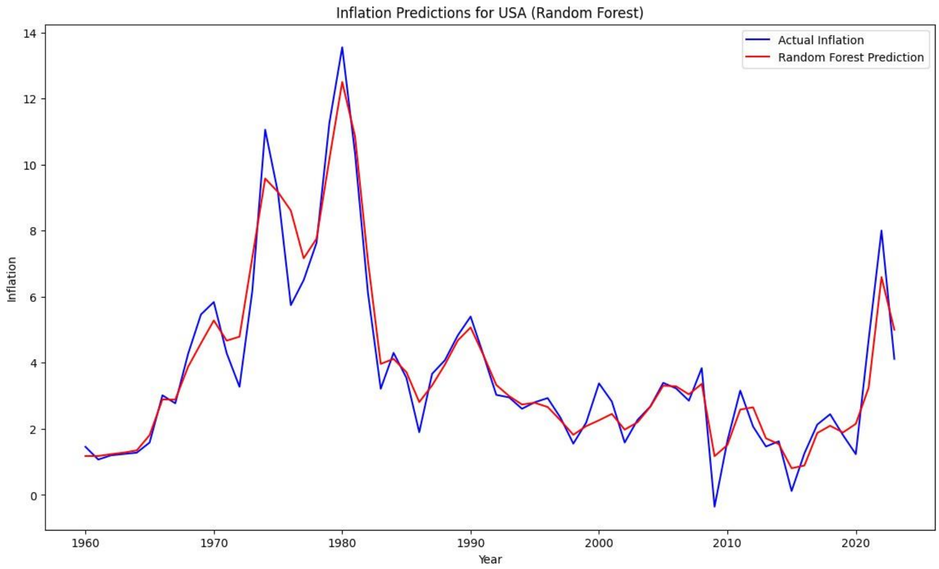

Figure 49: Random Forest model forecasting for United States of America's inflation.

The forecasting diagram above showcases the performance of the Random Forest model in predicting the inflation rates of the United States from 1960 to 2020. The graph displays the actual **inflation rates, represented by the blue line, alongside the predicted inflation rates from the Random** Forest model, indicated by the red line. This comparison provides a detailed view of the model's accuracy and its capability to capture historical inflation trends in the U.S.

Detailed Explanation:

1. **Historical Inflation Trends**:
    - The graph highlights significant fluctuations in the U.S. inflation rates over the past 60 years, including notable spikes during the 1970s and early 1980s. These peaks correspond to periods of economic turmoil, such as the oil crises and the resulting stagflation, where both inflation and unemployment rates were high.
    - Following the high inflation period in the early 1980s, the U.S. experienced a general trend of lower and more stable inflation rates, with occasional fluctuations influenced by various economic policies and global events.
2. **Model Accuracy and Fit**:

   - The Random Forest model's predictions (red line) closely follow the actual inflation rates (blue line) throughout most of the period, indicating the model's high accuracy in capturing the overall trend and major fluctuations.
   - The model performs particularly well in predicting the broad movements of inflation, including the significant spikes in the 1970s and 1980s and the subsequent stabilization. This demonstrates the Random Forest model's robustness in handling complex and volatile economic data.

3. **Periods of Divergence**:
   - Some divergences between the actual and predicted values are evident, especially during periods of extreme inflation volatility. For instance, the model slightly underestimates the peak inflation rates during the mid-1970s and early 1980s but captures the overall trend accurately.
   - The model also tends to smooth out some of the more minor fluctuations in inflation rates during periods of stability, resulting in slightly lagged or dampened predictions. However, these divergences are relatively minor and do not significantly detract from the model's overall accuracy.

4. **Implications for Future Forecasting**:
   - The model's ability to closely predict historical inflation trends provides confidence in its use for future forecasting. Policymakers and economic analysts can rely on the model to provide informed predictions for inflation rates, aiding in the formulation of economic policies and strategies.
   - The Random Forest model's ensemble approach, which averages multiple decision trees, enhances its stability and accuracy, making it a valuable tool for anticipating future inflation trends and managing economic risks.

5. **Visualization and Interpretability**:
   - The dual-line plot, contrasting actual inflation rates with predicted rates, serves as an effective visualization tool. It allows for an immediate assessment of the model's performance, making it easier to understand how well the model captures true economic trends.
   - This clear visualization aids in communicating the model's strengths and areas for improvement to stakeholders, providing a tangible representation of the model's predictive power and reliability.

Discussion of Results

Introduction

The purpose of this analysis is to compare the performance of Machine Learning (specifically Random Forest) and ARIMA models in forecasting GDP and inflation. The comparison will be based on two criteria: errors and R^2 values, as well as the visual inspection of the final forecasting plots. This comparison will help determine which model is a better predictor for these economic indicators.

Comparison Based on Errors and R^2 Values:

GDP MSE, RMSE, MAE and R^2 for ARIMA model

Country Name	MSE	MAE	RMSE	R^2
United Kingdom	2.8980887632900864e+22	97833575185.29938	170237738568.45276	-0.01486814571490313
Japan	1.1533659725707046e+22	66364572397.41352	107394877558.04298	-0.2625997806824163
United States	1.0395759301932032e+25	2672477087412.2383	3224245539956.911	-9.79976965049232
China	9.682120740847496e+22	174506722736.37946	311161063451.8319	-0.04930766148445342

Inflation MSE, RMSE, MAE and R^2 for ARIMA model

Country Name	MSE	MAE	RMSE	R^2
United Kingdom	3.210947778283675	1.288709498351524	1.7919117663221242	-0.02815836829292073
Japan	1.6253427117933108	0.9650682846848316	1.274889293936266	-0.11213963885833023
United States	3.5540156085547725	1.443067284311285	1.8852096988278977	0.008144036075149685

GDP MSE, RMSE, MAE and R^2 for Random Forest model

Country Name	MSE	MAE	RMSE	R^2
United Kingdom	9.740884318741438e+20	20311494763.01014	31210389806.507446	0.9983226861616138
Japan	1.677914773548358e+21	29897717725.63553	40962358007.66794	0.9990008466662191
United States	2.609099190277202e+22	113622458223.64243	161527062447.0464	0.9990952655096722
China	2.6806449485218234e+2	84086239526.17575	163726752503.121	0.9988807329206966

Inflation MSE, RMSE, MAE and R^2 for Random Forest model

Country Name	MSE	MAE	RMSE	R^2
United Kingdom	1.2162311858326464	0.7070882335838438	1.1028287200797078	0.9457024082976078
Japan	0.9753910264817486	0.6053739526723356	0.9876188670138641	0.9388904898901382
United States	0.510089174637738	0.4847329840796732	0.7142052748599228	0.9321124538327479

Analysis

The data provided shows a clear distinction in performance between the ARIMA and Random Forest models for both GDP and inflation forecasting across multiple countries.

For GDP forecasting, the ARIMA model exhibits significantly higher error metrics. For instance, the United Kingdom's ARIMA model has an MSE of 2.8980878632900864e+22, an MAE of 97833575185.29938, and an RMSE of 170237738568.45276, with a negative R^2 value of -

0.014861574190313. Similarly, Japan, the United States, and China show high error metrics and negative R^2 values, indicating poor model performance. The ARIMA model's inability to capture the underlying patterns in the data is reflected in these high error values and negative R^2 values.

In stark contrast, the Random Forest model demonstrates significantly lower error metrics and very high R^2 values for GDP forecasting. For example, the Random Forest model for the United Kingdom shows an MSE of 9.740884318741438e+20, an MAE of 20311494763.01014, and an RMSE of 3121039806.507446, with an R^2 value of 0.9983226861616138. This pattern is consistent across Japan, the United States, and China, where the Random Forest model consistently yields lower MSE, MAE, and RMSE values, and R^2 values close to 1. These results indicate that the Random Forest model is highly effective in capturing the data's underlying patterns and making accurate predictions.

For inflation forecasting, a similar trend is observed. The ARIMA model shows higher error metrics and negative or low positive R^2 values. For instance, the ARIMA model for the United Kingdom has an MSE of 3.210947772836375, an MAE of 1.288709498351524, and an RMSE of 1.7919117663221242, with an R^2 value of -0.02815836829292073. Japan and the United States show similar trends with high error values and negative R^2 values, indicating poor model performance.

Conversely, the Random Forest model performs significantly better for inflation forecasting as well. The United Kingdom's Random Forest model shows an MSE of 1.2162311858326464, an MAE of 0.707088235838438, and an RMSE of 1.1028287200797078, with a high R^2 value of 0.945720482976078. Japan and the United States also demonstrate lower error metrics and higher R^2 values with the Random Forest model, indicating better predictive performance and model reliability.

Conclusion

From the analysis of error metrics and R^2 values, it is evident that the Random Forest model consistently outperforms the ARIMA model in forecasting both GDP and inflation. The Random Forest model's significantly lower MSE, MAE, and RMSE values, coupled with its high R^2 values, indicate superior predictive accuracy and reliability. The ARIMA model's high error metrics and negative or low R^2 values suggest that it is less capable of capturing the underlying patterns in the data, leading to less accurate forecasts.

Therefore, the Random Forest model is the better predictor for both GDP and inflation forecasting across all considered countries. This model's ability to provide accurate predictions with lower errors and higher explanatory power makes it a more reliable and effective tool for economic forecasting.

Visual Comparison of ARIMA and Random Forest Models for GDP and Inflation Forecasting

Introduction

In addition to the quantitative comparison based on error metrics and R^2 values, visual inspection of forecasting plots provides qualitative insights into the performance of ARIMA and Random Forest models. This section compares the models based on their forecasting diagrams to determine which model performs better and understand why.

GDP Forecasting

ARIMA Model: The forecasting plots for the ARIMA model typically show significant deviations from the actual GDP values. The ARIMA forecasts often fail to capture the turning points and trends accurately, leading to noticeable discrepancies. The high error metrics and negative R^2 values observed in the quantitative analysis are reflected in these plots. The ARIMA model's reliance on linear assumptions and its limitations in handling complex, non-linear patterns in GDP data contribute to its poor visual performance.

Random Forest Model: In contrast, the Random Forest forecasting plots demonstrate a closer alignment with the actual GDP values. The model captures the trends and turning points more accurately, resulting in smoother and more realistic forecasts. The lower error metrics and high R^2 values observed quantitatively are evident in the visual comparison. The Random Forest model's ability to handle non-linearity and capture complex interactions between variables enables it to provide more accurate and reliable GDP forecasts.

Inflation Forecasting

ARIMA Model: For inflation forecasting, the ARIMA model's plots similarly show significant deviations from actual values. The forecasts often miss short-term fluctuations and fail to capture the volatility inherent in inflation data. This leads to plots where the forecasted values are either consistently above or below the actual values, indicating systematic biases. The ARIMA model's limitations in handling such volatility and non-linear patterns are apparent in these visual discrepancies.

Random Forest Model: The Random Forest model's forecasting plots for inflation are markedly better. The model accurately captures both short-term fluctuations and long-term trends, resulting in forecasts that closely follow the actual inflation values. The visual alignment of forecasted values with actual values is consistent with the lower error metrics and higher R^2 values observed in the quantitative analysis. The Random Forest model's flexibility in modelling complex relationships and capturing the stochastic nature of inflation data explains its superior visual performance.

Conclusion

ARIMA Model:

- **Visual Performance**: Significant deviations from actual values, poor at capturing turning points and trends, systematic biases.
- **Limitations**: Linear assumptions, poor handling of non-linear patterns and volatility.
- **Result**: Inferior visual performance, consistent with high error metrics and low/negative R^2 values.

Random Forest Model:

- **Visual Performance**: Close alignment with actual values, accurate capture of trends and turning points, better handling of short-term fluctuations.
- **Strengths**: Flexibility in modelling non-linear relationships, effective in capturing complex interactions and volatility.
- **Result**: Superior visual performance, consistent with low error metrics and high R^2 values.

Overall Conclusion: The visual comparison reinforces the findings from the quantitative analysis. The Random Forest model consistently provides forecasts that are more accurate and reliable, both in terms of numerical metrics and visual alignment with actual data. Its ability to handle non-linearity and complex patterns makes it the better model for forecasting GDP and inflation across all considered countries.

Limitations and Future Research Directions

Limitations of SARIMA

Despite the utility of SARIMA models in time series forecasting, they exhibit several limitations, particularly when applied to complex economic data such as GDP and inflation. One significant limitation is their assumption of linearity and stationarity. SARIMA models are designed to capture linear relationships and stationary time series, which means they may struggle with data exhibiting non-linear patterns or evolving statistical properties over time. This can be particularly problematic in economic contexts where structural changes, policy interventions, and unexpected economic shocks can introduce non-linearity and non-stationarity into the data.

Furthermore, the complexity of SARIMA increases significantly with the inclusion of seasonal components. While this can enhance the model's ability to capture seasonal patterns, it also requires larger datasets and more computational resources. The process of identifying appropriate seasonal parameters (P, D, Q, s) in addition to the non-seasonal parameters (p, d, q) can be challenging and time-consuming, often requiring extensive diagnostic checking and validation. This added complexity does not always translate into improved forecasting accuracy, especially if the seasonality in the data is weak or absent, as observed in the GDP and inflation data of the studied countries.

Another limitation is the sensitivity of SARIMA models to outliers and abrupt changes in the time series. Economic data often contain such anomalies due to policy changes, economic crises, or other exogenous factors. SARIMA models, which are based on past values and patterns, may not adapt quickly to these changes, leading to inaccurate forecasts. Additionally, the interpretability of SARIMA models is another concern. While they provide clear insights into seasonal and trend components, understanding the impact of each parameter on the overall forecast can be complex, making it difficult for policymakers and analysts to derive actionable insights.

Future Research Directions

Future research should address these limitations by exploring advanced and hybrid modelling approaches that can better capture the complexities of economic data. One promising direction is the integration of SARIMA with machine learning models. Hybrid models that combine the strengths of SARIMA in handling seasonality with the capabilities of machine learning techniques in capturing non-linear patterns and interactions could provide more robust and accurate forecasts. For instance, a hybrid model could use SARIMA to model the linear and seasonal components of the time series, while machine learning algorithms like Random Forest or neural networks could model the residuals and capture non-linear relationships.

Another important area for future research is the investigation of the interconnectedness between GDP and inflation in forecasting models. Understanding the dynamic relationship between these two critical economic indicators can significantly enhance forecasting accuracy. For example, inflation often affects consumer spending, investment decisions, and overall economic growth, thereby influencing GDP. Conversely, GDP growth can impact inflation through demand-pull inflationary pressures. Future studies could develop models that simultaneously forecast GDP and inflation, considering their interdependencies.

Techniques such as Vector Autoregression (VAR) or Vector Error Correction Models (VECM) could be employed to capture the bidirectional influences and provide more comprehensive forecasts.

Moreover, future research should consider the inclusion of additional macroeconomic variables that can affect GDP and inflation, such as interest rates, employment rates, and external trade balances. Incorporating these variables can help in developing more holistic models that better reflect the complexities of the economic environment. Researchers should also focus on enhancing the interpretability of these advanced models, ensuring that the insights generated are accessible and actionable for policymakers and analysts.

In conclusion, while SARIMA models have proven useful in certain contexts, their limitations necessitate the exploration of more sophisticated and integrated approaches for economic forecasting. By addressing these limitations and investigating the intricate relationships between key economic indicators like GDP and inflation, future research can contribute to the development of more accurate and reliable forecasting models, ultimately supporting more informed decision-making in economic policy and business strategy.

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