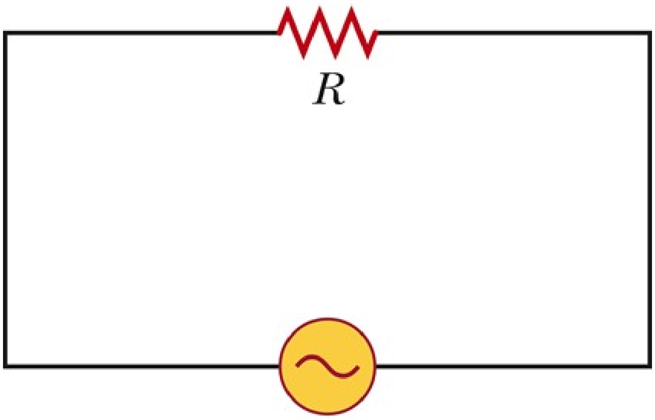
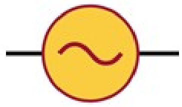
AC Circuit

AC Circuit Containing Resistance only



$$v = V_{max} \sin \omega t$$



Symbol of an ac source

Resistance is the opposition that an element offers to the flow of electric current. It is represented by the uppercase letter R. The standard unit of resistance is the ohm, sometimes written out as a word, and sometimes symbolized by the uppercase Greek letter omega: Ω. The behavior of a resistor is dictated by the relationship specified by Ohm's law.

Resistor is passive element. Passive element is an electrical component that does not generate power, but instead dissipates, stores, and/or releases it.

Consider a circuit consisting of an ac source and a resistor. The instantaneous voltage across the resistor

$v_R = v = V_{max} \sin \omega t$

From Ohm's law

$i = v / R = (V_{max} \sin \omega t) / R = I_{max} \sin \omega t$, **the instantaneous current flowing through the resistor.**

If V_{max} **and** I_{max} **be the maximum values of the voltage and current respectively, it follows that:** $I_{max} = V_{max} / R$**……… (1)**

But the r.m.s. value of a sine wave is 0.707 times the maximum value so that:

r.m.s. value of voltage = $V = 0.707\, V_{max}$

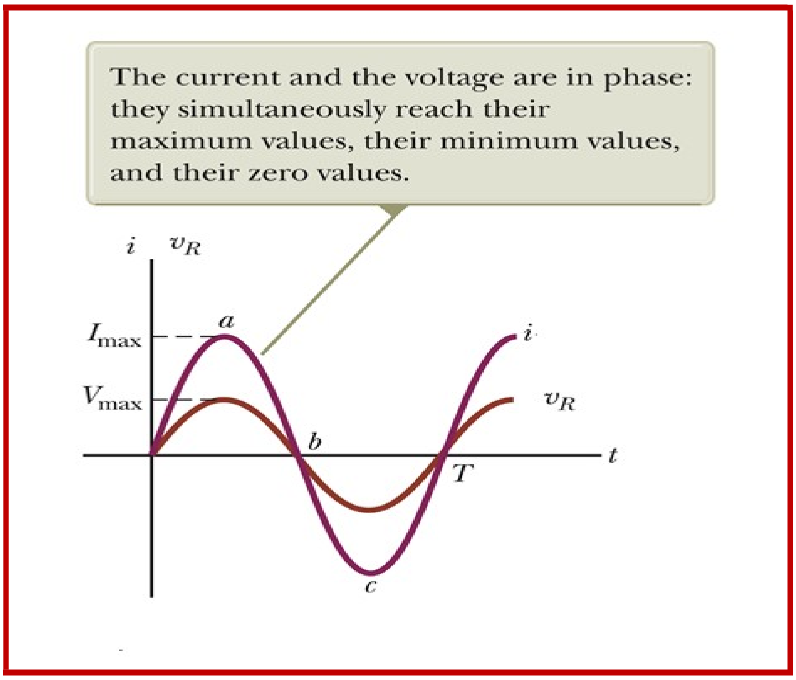
r.m.s. value of current = $I = 0.707\, I_{max}$

Substituting for V_{max} **and** I_{max} **in equation (1)**

$$I/0.707 = (V / 0.707) / R, \text{ and } I = V / R.$$

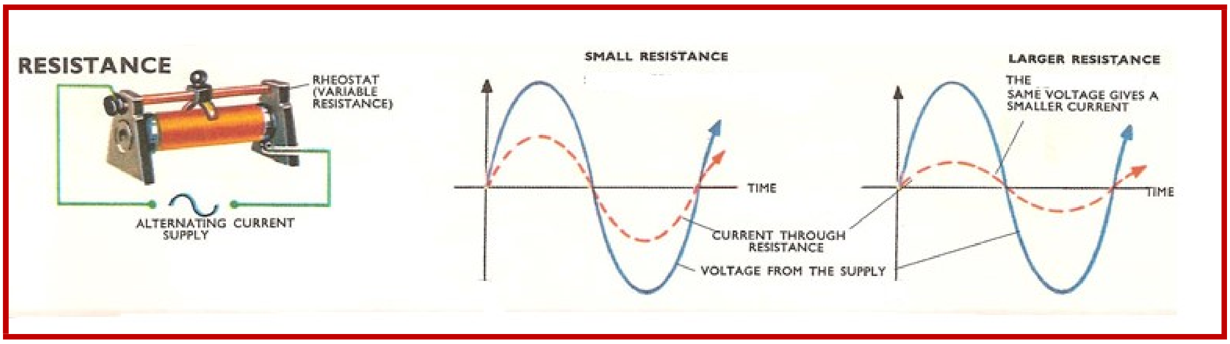
Hence Ohm's law can be applied without any modification to an a.c. circuit possessing resistance only.

Phase Relationship between applied voltage and current in Resistor

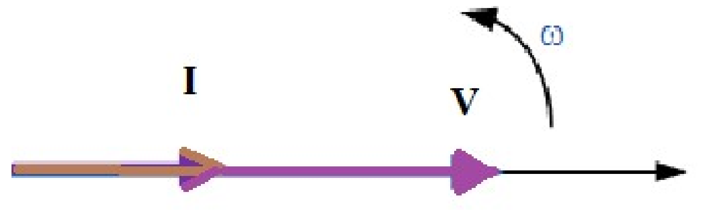


- **The graph shows the current through and the voltage across the resistor.**
- **The current and the voltage reach their maximum values at the same time.**

- The current and the voltage are said to be *in phase.*
- The direction of the current has no effect on the behavior of the resistor.
- Resistors behave essentially the same way in both DC and AC circuits.



This "in-phase" effect can also be represented by a phasor diagram. Therefore, as the voltage and current are both in-phase with each other, there will be no phase difference ($\theta = 0$) between them, so the vectors of each quantity are drawn super-imposed upon one another along the same reference axis.



Phasor Diagram for AC Resistance

Power in a resistive circuit

The instantaneous power in a resistive circuit is given by the product of instantaneous voltage and instantaneous current. The instantaneous power is given by

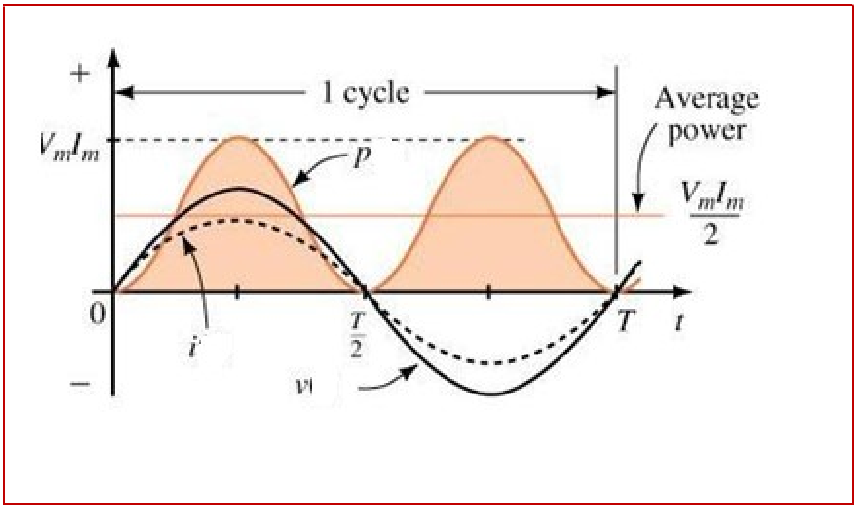
p = vi = $V_{max} \sin \omega t * I_{max} \sin \omega t$

Writing $V_{max} = V_m$ **and** $I_{max} = I_m$

$$p = (V_m \sin\omega t)(I_m \sin\omega t)$$

$$p = \frac{V_m I_m}{2}\, 2\sin^2\omega t = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} (1 - \cos 2\omega t)$$

$$p = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos 2\omega t$$



The average power consumed in the circuit over a complete cycle is given by

$$P_{average} = \frac{1}{2\pi}\int_0^{2\pi} p\, d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} \left\{\left(\frac{V_m I_m}{2}\right) - \left(\frac{V_m I_m \cos 2\omega t}{2}\right)\right\} d(\omega t)$$

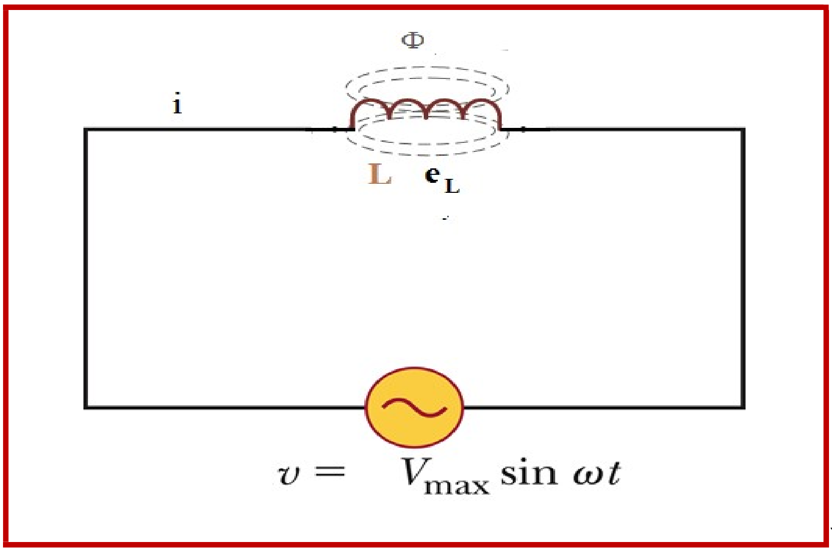
$$= \frac{V_m I_m}{2} = V_{r.m.s.} I_{r.m.s.} = VI$$

In summary Power to a Resistive Load

- **p is always positive. From the above equation it is clear that whatever may be the value of ωt the value of cos2ωt cannot be greater than 1 hence the value of p cannot be negative. The value of p is always positive irrespective of the instantaneous direction of voltage v and current i.**

- **Power flows only from source to load.**
  - **All of the power delivered by the source is absorbed by the load.**
- **This power is known as active power. Power to a pure resistance consists of active power only.**
- **Average value of power is halfway between zero and peak value of $V_m I_m$.**
- $P = V_m I_m/2$
- **If V and I are in RMS values**
  - **Then $P = VI$**
- **Also, $P = I^2 R$ and $P = V^2/R$**
- **Active power relationships for resistive circuits are the same for ac as for dc.**

AC Circuit Containing Inductance only



Inductance is the property of an electrical conductor by which a change in current through it induces an electromotive force in the conductor. It consists of a conductor such as a wire, usually wound into a coil.

An "ideal inductor" has inductance, but no resistance.

When the current flowing through an inductor changes, the time-varying magnetic field induces an "e.m.f." (e_L) in the coil, according

to Faraday’s law of electromagnetic induction. According to Lenz's law the direction of induced "e.m.f." is always such that it opposes the change in current that created it. As a result, inductors always oppose a change in current. The instantaneous value of the induced e.m.f. is given by

$$e_L \alpha - \frac{d\phi}{dt} = -L\frac{di}{dt}$$

Since the resistance of the coil is assumed to be negligible, the whole of the applied voltage is absorbed in neutralizing the induced e.m.f.,

$$\therefore v = applied\ voltage = -e_L = L\frac{di}{dt}$$

$$di = \frac{1}{L}\ vdt,\ \int di = \frac{1}{L}\int V_{max}\sin\omega t\ dt$$

$$\text{i} = -\frac{V_{max}}{\omega L}\cos\omega t = \frac{V_{max}}{\omega L}\sin(\omega t - \frac{\pi}{2})$$

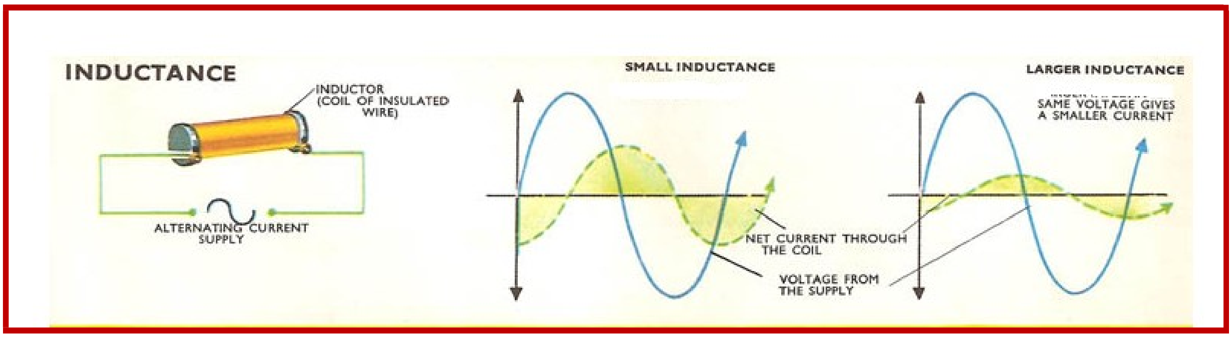
$$= I_{max}\sin\left(\omega t - \frac{\pi}{2}\right)$$

where $I_{max}= V_{max}/\omega L$.

Thus, the current in an inductor lags the applied voltage by an angle $\pi/2$ or 90°. Also, from the expression it follows that maximum value of the current is $V_{max}/\omega L$, i.e., $I_{max}= V_{max}/\omega L$, so that $V_{max}/ I_{max} = \omega L= 2\pi fL$.

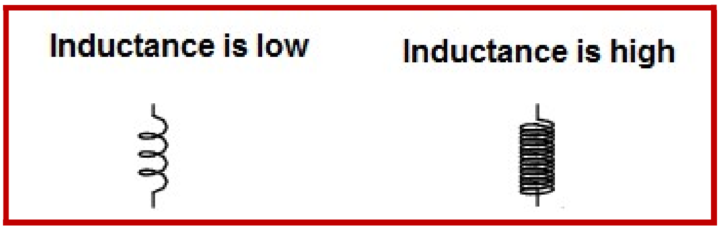
If V and I are the r.m.s. values, then $V/I = 0.707V_{max}/0.707I_{max} = 2\pi fL$ = *inductive reactance*. The term inductive reactance is denoted by the symbol X_L. Hence, $I = V/2\pi fL = V/ X_L = V/ \omega L$. This is similar to $I = V / R$.

The *inductive reactance* is the opposition that an inductor (or coil) offers to the alternating current. Therefore, ωL plays the same role as that of a resistor. The inductor impedes the flow of alternating current in the circuit. Unit of X_L is also ohm.

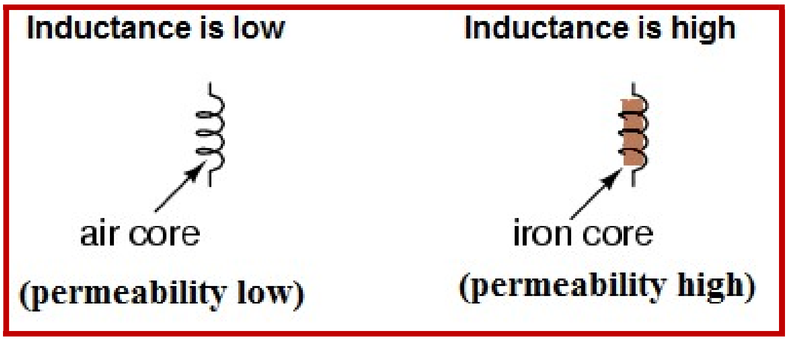


To have a large reactance the coil

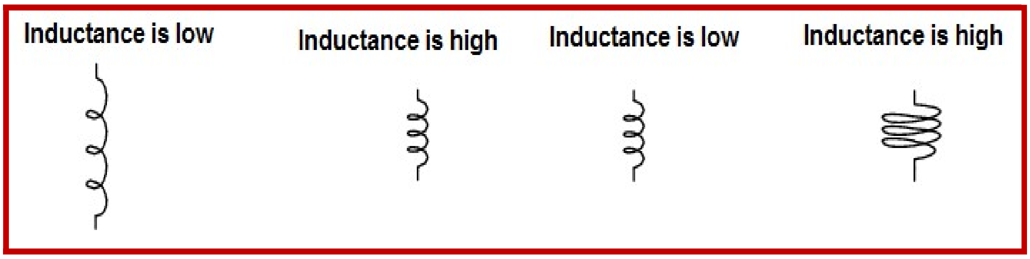
(i) Should have many turns as L ∝ N.



(ii) Should have an iron-core as L ∝ $\mu_r\mu_o$.

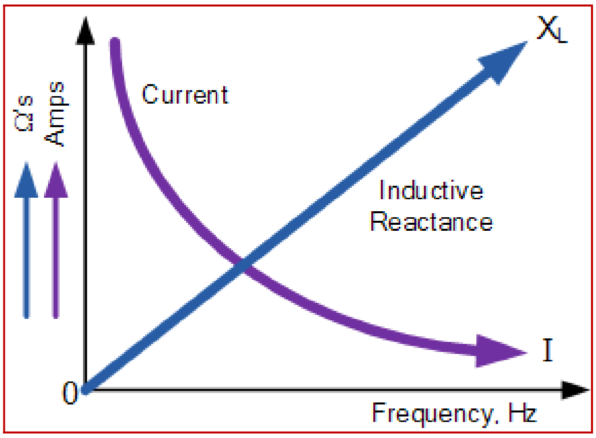


(iii) Length and area of the coil as L ∝ area / length.



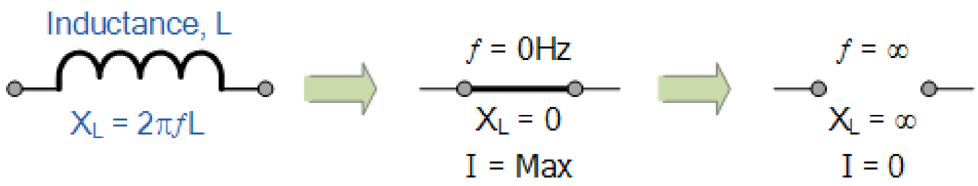
(iv) **Also, the frequency of a.c should be high.**

iv) X_L in case of DC (direct current), is zero.



The inductive reactance of an inductor increases as the frequency increases. Also, as the frequency increases the current flowing through the inductor also reduces in value.

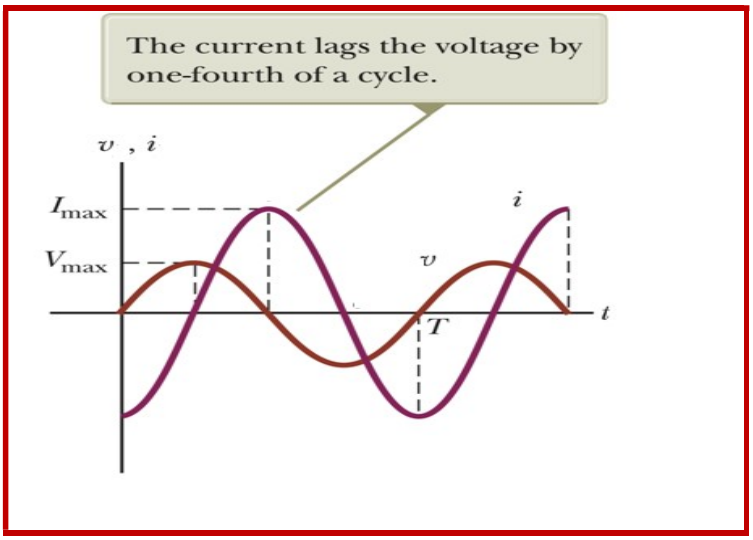
The effect of very low and very high frequencies on the reactance of a pure AC Inductance as follows:



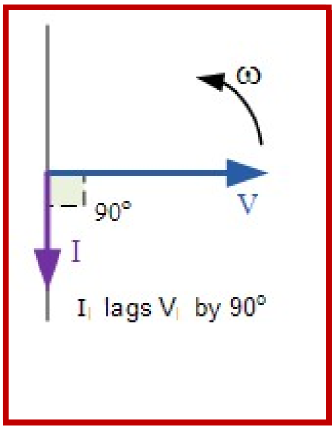
In an AC circuit containing pure inductance the following formula applies:

$$\text{Current, I} = \frac{\text{Voltage}}{\text{Opposition to current flow}} = \frac{V}{X_L}$$

Phase Relationship between applied voltage and current in Inductor



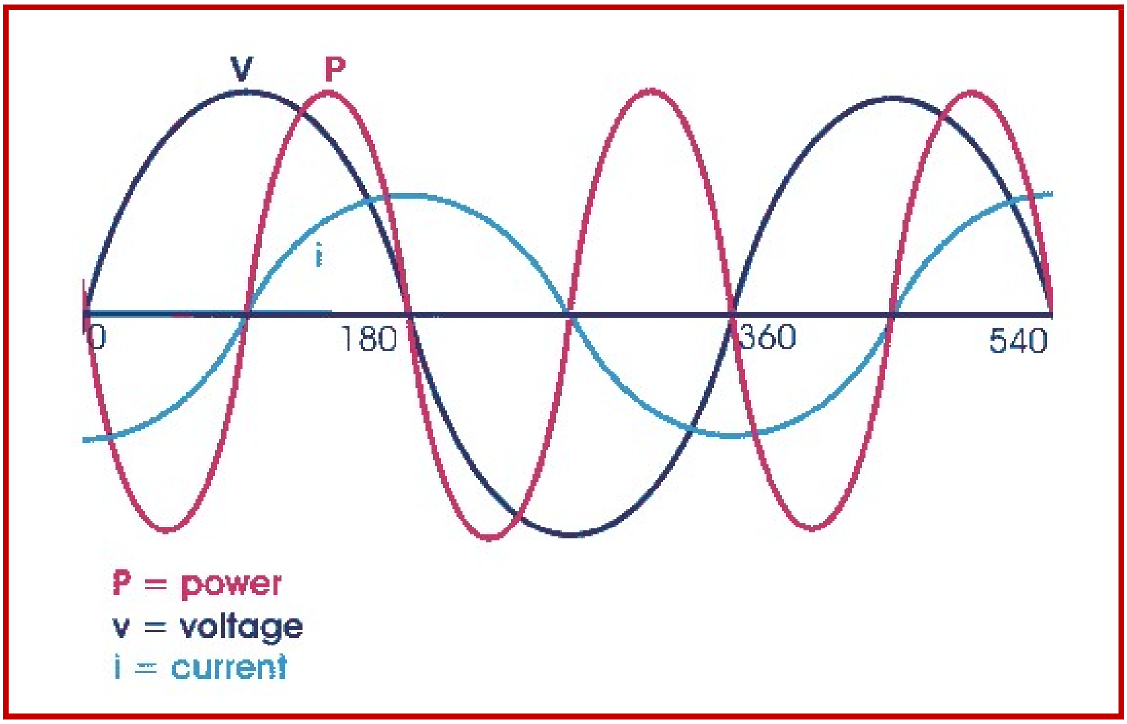
Phasor Diagram for an Inductor



Power in a Inductive circuit

The instantaneous power delivered to the purely inductive circuit is obtained by

$$
\begin{aligned}
p &= vi \\
&= (V_m \sin \omega t)(I_m \sin(\omega t - \pi/2)) \\
&= (V_m \sin \omega t)\ (-\ I_m \cos \omega t) \\
&= -(2V_m I_m \sin \omega t \cos \omega t)\ /2 \\
&= -(V_m I_m/2) \sin 2\ \omega t
\end{aligned}
$$



In the above expression, it is found that the power is flowing in alternative directions. From 0º to 90º it will have negative half cycle, from 90º to 180º it will have positive half cycle, from 180ºto 270º it will have again negative half cycle and from 270º to 360º it will have again positive half cycle. Therefore, this power is alternating in nature with a frequency, double of supply frequency. As the power is flowing in alternating direction i.e., from source to load in one quarter cycle and from load to source in next half cycle the average value of this power is zero. The

implication is that the inductive element receives energy from the source during one-quarter of a cycle of the applied voltage and returns exactly the same amount of energy to the driving source during the next quarter of a cycle. Therefore, this power does not do any useful work.

The power associated with an inductance is *reactive power*.

Energy Stored in an Inductor

If the circuit is purely inductive, energy will be stored in the magnetic field during quarter of a cycle and is obtained by integrating power wave p between limits of t = T/4 and t = T/2,

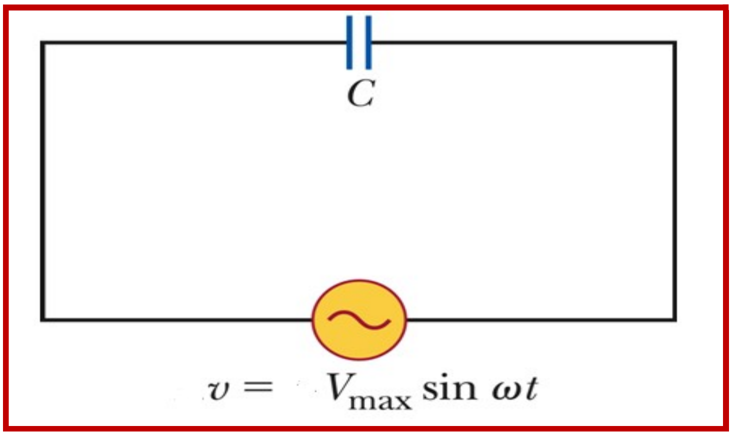
$$W_L = \int_{T/4}^{T/2} -\frac{V_m I_m}{2} \sin(2\omega t) dt$$

$$= \frac{V_m I_m}{2\left(\frac{4\pi}{T}\right)} \left[\cos\left(\frac{4\pi}{T} t\right)\right]_{T/4}^{T/2}$$

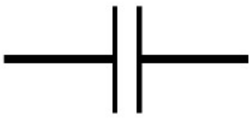
$$= \frac{V_m I_m}{2\omega} = \frac{(\omega L\ I_m) I_m}{2\omega} = \frac{L I_m^2}{2}$$

If L is in henrys and I_m is in amperes respectively, W_L is given in joule.

AC Circuit Containing Capacitance only



A capacitor (originally known as a condenser) is a passive two-terminal electrical component used to store electrical energy temporarily in an electric field. The forms of practical capacitors vary widely, but all contain at least two electrical conductors (plates) separated by a dielectric (i.e. an insulator that can store energy).



An ideal capacitor is wholly characterized by a constant capacitance *C*, defined as the ratio of charge *Q* on each conductor to the voltage *V* between them.

The circuit contains a capacitor and an AC source. An inductor opposes a change in current. A capacitor does the opposite. It opposes a change in voltage. Pure capacitor has zero resistance. When an alternating voltage applied across the capacitor, the capacitor first charged in one direction and then in another direction. The charge q is given by

$$q = Cv = CV_{max}\sin\omega t$$

The flow of electrons "through" a capacitor (i.e., the charging current) is directly proportional to the *rate of change of voltage* across the capacitor.

Expressed mathematically, the relationship between the current "through" the capacitor and rate of voltage change across the capacitor is as such:

$$i = \frac{dq}{dt} = C\frac{dv}{dt} = C\frac{d}{dt}V_{max}\sin(\omega t)$$

$$= \omega C V_{max}\cos(\omega t)$$

$$= \left[V_{max}\left(\frac{1}{\omega C}\right)\right]\sin\left(\omega t + \frac{\pi}{2}\right) = I_{max}\sin\left(\omega t + \frac{\pi}{2}\right)$$

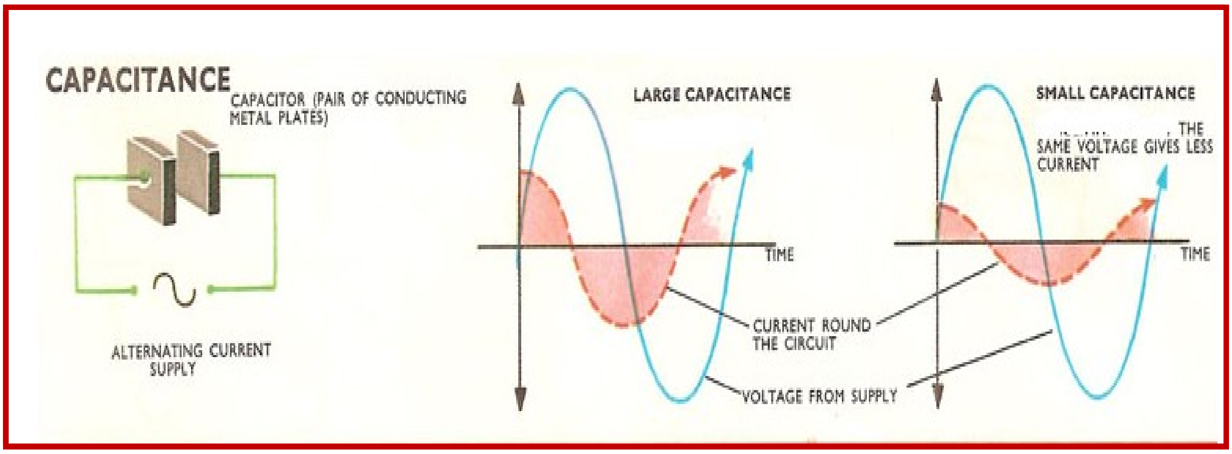
Thus, the current in a pure capacitor leads the applied voltage by π/2 radian or 90°. From the above expression it follows that the maximum value of the current is $\omega C V_{max}$ or $2\pi f C V_{max}$.

∴ $V_{max} / I_{max} = 1 / 2\pi f C$. If V and I are the r.m.s. values of voltage and current, then

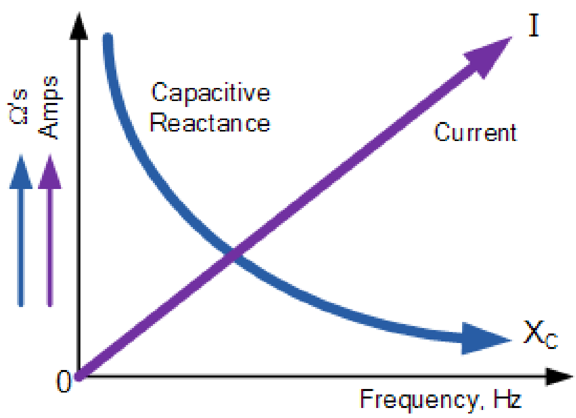
$\frac{V}{I} = \frac{0.707V_{max}}{0.707I_{max}} = \frac{1}{2\pi f C}$ **= *capacitive reactance*. The impeding effect of a capacitor on the current in an AC circuit is called the *capacitive reactance*. The capacitive reactance is expressed in ohms and is represented by X_C.**

- ★ **Capacitive reactance *decreases* with increasing frequency. In other words, the higher the frequency, the less it opposes (the** more it “conducts”) the AC flow of electrons and the current increases.

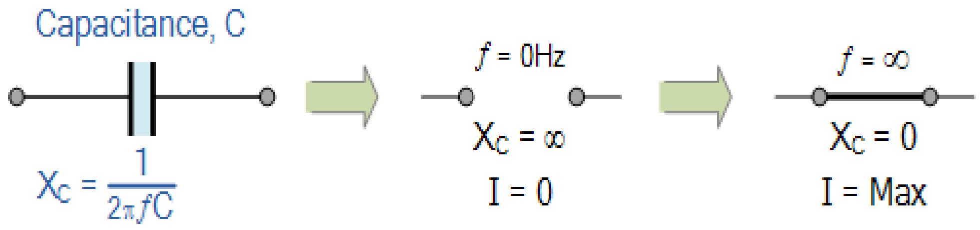
★ As the frequency approaches zero, X_C approaches infinity and the current approaches zero.



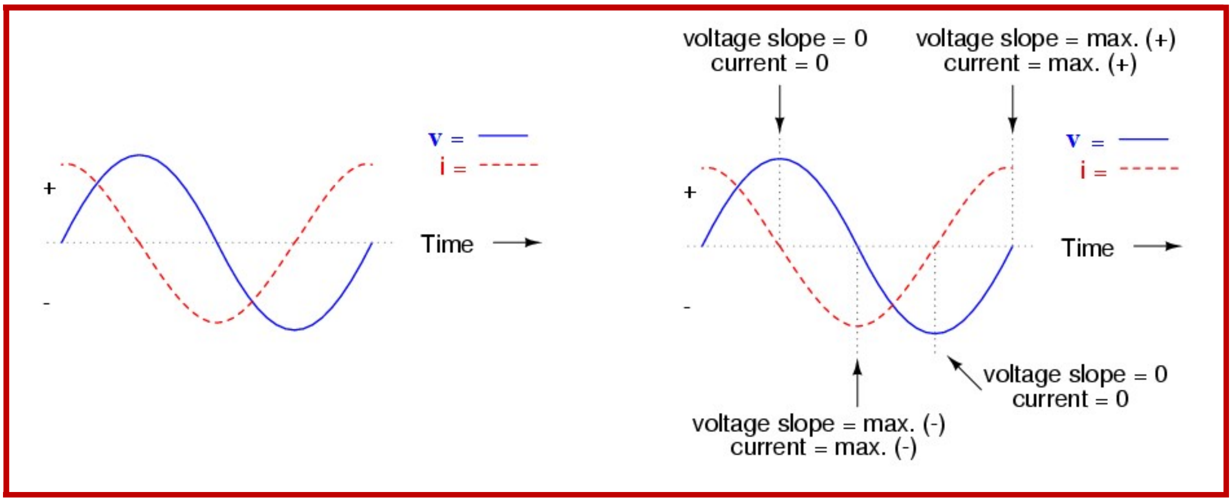
Capacitive Reactance against Frequency



The effect of very low and very high frequencies on the reactance of a pure AC Capacitance as follows:

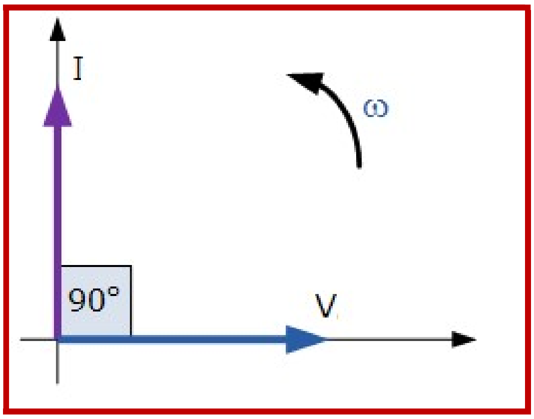


Phase Relationship of applied voltage and current in Capacitor



Pure capacitive circuit waveforms

Phasor Diagram for AC Capacitance

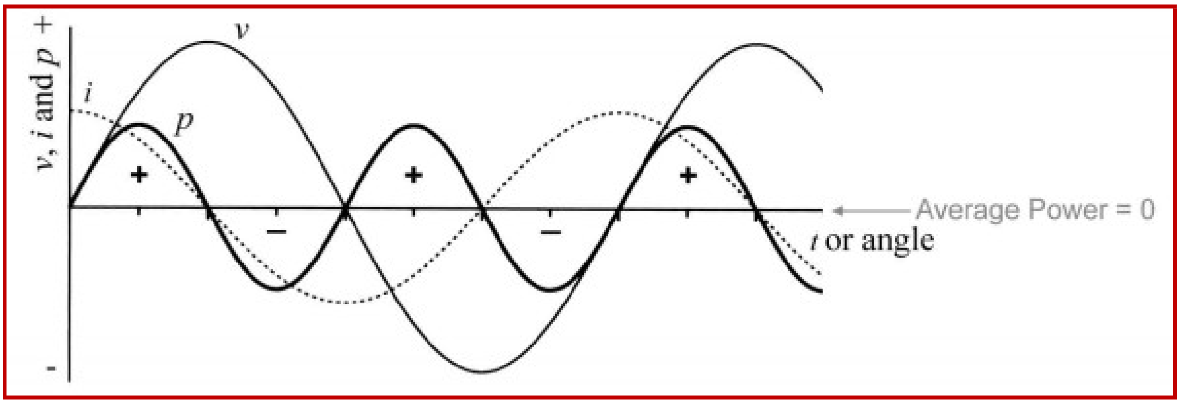


Power in a purely Capacitive circuit

$$\mathbf{p} = (V_m \sin\omega t)(I_m \sin(\omega t + {}^{\pi}/_{2})$$

$$= V_m I_m \sin\omega t \cos\omega t$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\,\omega t$$



In the first-quarter cycle both v and i are positive, therefore the power is also positive (since $p = vi$, at any instant). In the second quarter-cycle v stays positive while i have gone negative, therefore p is negative. In the third-quadrant both i and v are negative and so p is positive. Finally, in the fourth-quadrant i is positive and v is still negative resulting in p being negative. The power wave is thus a series of identical positive and negative pulses whose average value over a half-cycle of voltage is zero, also note that its frequency is twice the frequency of the voltage.

During the first and third quarter-cycles the power is positive means that power is supplied by the circuit to charge the capacitor. In the second and fourth quarter-cycles the capacitor is discharging and thus supplies the energy stored in it back to the circuit, thus p has a negative value. The minus or plus signs simply indicate the direction in which the power is flowing. Since this interchange of energy dissipates no average power no heating will occur and no power is lost.

The capacitive power does not do any useful work. This power is also a reactive power.

Energy Stored in a Capacitor

The amount of energy received by the capacitor during quarter of a cycle and is obtained by integrating power wave p between limits of t = 0 and t = T/4,

$$W_c = \int_0^{T/4} \frac{V_m I_m}{2} \sin 2\omega t \, dt$$

$$= \frac{V_m I_m}{2(\frac{4\pi}{T})} [-\cos\frac{4\pi}{T}t]_0^{T/4} = \frac{V_m I_m}{\frac{4\pi}{T}} = \frac{V_m I_m}{2\omega}$$

Since $I_m = \omega C V_m$, $W_C = \frac{V_m^2 C}{2}$

Summary

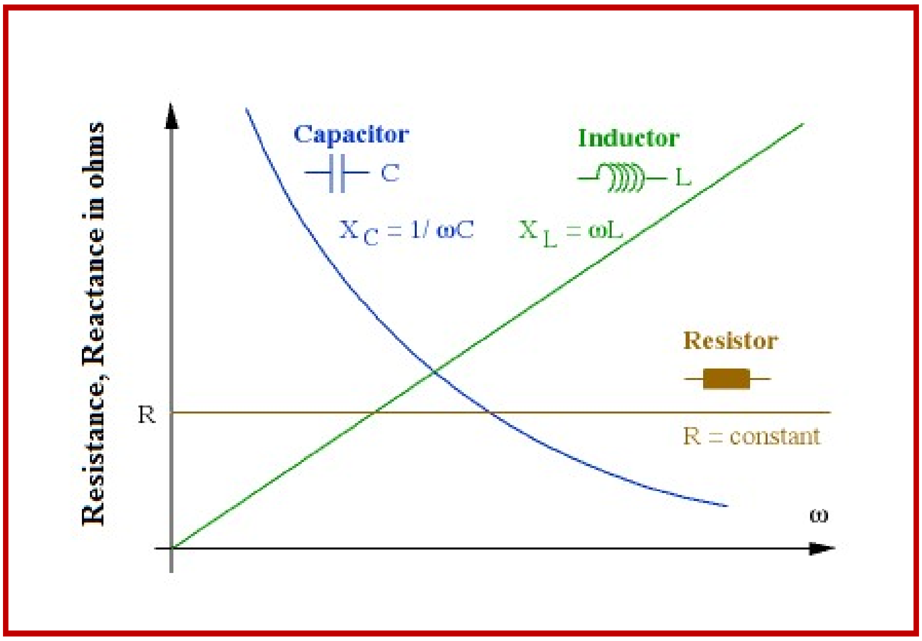
Resistance, Reactance

The following is a summary of the relationship between voltage and current in circuits:

★ **Resistance is the special case when φ = 0.**

★ **Reactance the special case when φ = ± 90°.**

Component	**Resistor** R	**Inductor** L	**Capacitor** C
Difference of Phase between Voltage and Current	**Voltage and Current is in phase**	**Current lags behind Voltage by π/2**	**Voltage lags behind Current by π/2**
Ohm's Law	$R = V / I$	$X_L = V / I = \omega L$	$X_c = V / I = 1 / \omega C$



Memory Aid for Passive Elements in AC

An old, but very effective, way to remember the phase differences for inductors and capacitors is:

"E L I" and "I C E"

Emf E is before current I in inductors L;

Emf E is after current I in capacitors C.