Commodity price shocks and production networks[*]

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Abstract

This paper studies how different patterns in production networks affect the impact of commodity price shocks on real GDP. I construct a measure to capture the importance of a specific commodity sector on the economy that I call *dependency*. This variable captures the number of sectors who directly or indirectly interact with a particular commodity sector considering only high-intensity interactions. I document that for a specific commodity sector there are substantial differences among countries on the level of *dependency*. I show that these different patterns of *dependency* can significantly explain the impact of commodity price shocks on an economy. Using OECD data, I find that, even after controlling for the size of the commodity sector, the influence of the commodity sector as a customer within the network, *upstream dependency*, plays an important role in shaping the effect of commodity price shocks on real GDP. Then, I introduce these patterns in a novel network model from Silva et al. (2023) to evaluate counterfactual scenarios around commodity price shocks. I interchange the *dependency* pattern between the Chilean and Australian networks and show that, after imposing Chile's *dependency* pattern on Australia, commodity price shocks have a greater impact on Australia and vice versa. Theoretical results are consistent with empirical findings.

Keywords: Commodity price shocks, Production Networks, Diversification.

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1 Introduction

Commodity price shocks have been an important driver of the business cycle, for both developing and developed countries (Fernandez et al., 2017; Fernández et al., 2018). Different mechanisms have been studied to understand the propagation of these shocks. For example wealth and cost effects, financial conditions, fiscal or monetary policy (see Benguria et al., 2021; Drechsel and Tenreyro, 2018; Céspedes and Velasco, 2012, 2014). Recently, new studies have analized how the production structure may play an important role in the propagation of these shocks (Kohn et al., 2021; Silva et al., 2023).

Following this last strand in the literature, I study how different patterns associated to production networks can help to explain different impact of commodity price shocks between countries. Using OECD data, I start by documenting a new empirical pattern associated with commodity sectors that I call *dependency*. This pattern shows that, around a specific commodity sector, there are substantial differences among countries in the number of sectors who directly or indirectly interact with the commodity sector, once only high-intensity interactions are considered. Although it may be associated with general measures of networks such as density or sparcity, it is not the same. The main difference is that the *dependency* pattern is related to a specific commodity sector while density or sparcity are measures for the whole network.

I show that different patterns of *dependency* can significantly help to explain the impact of commodity price shocks. First of all, I introduce the concept of customer and supplier centrality, measures that conceptually captures the *dependency* pattern and use these measures to empirically study the impact of commodity price shocks on GDP. I found that the relevance of the commodity sector as a customer within the network plays an important role in the propagation of commodity price shocks. This is not the case for its relevance as an input supplier. After controlling by centrality measures of the commodity sectors, neither network density nor the size of the commodity sector play a role in the propagation. In second place, I introduce the *dependency* pattern in a novel network model from Silva et al. (2023) to evaluate counterfactual scenarios around commodity price shocks. I interchange the *dependency* pattern between the Chilean and Australian networks and show that, after imposing Chile's *dependency* pattern on Australia, commodity price shocks have a greater impact on Australia and vice versa. Theoretical results are consistent with empirical findings.

This paper contributes to two strands in the literature. On one hand, it extends the literature that investigates the impact of terms of trade or commodity price shocks on the business cycle (Mendoza, 1995; Kose, 2002; Schmitt-Grohé and Uribe, 2018; Fernandez et al., 2017; Fernández et al., 2018). Using measures of commodity sector centrality[1], I show its potential in the propagation of commodity price shocks. On the other hand, I contribute to the literature relating the production structure and the propagation of shocks (see Acemoglu et al., 2012, 2017, 2016; Barrot and Sauvagnat, 2016; Carvalho, 2008). Specifically, the contribution of this paper focuses on how the productive structure can contribute to the propagation of commodity price shocks and how systematic differences between countries can explain different magnitudes of this impact. Previous work has focused on general measures of production networks to undestand the propagation, whereas I use a specific measure associated with the commodities sector, which varies significantly from country to country.

The rest of the paper is organized as follows. Section 2 puts this work in context by presenting the related literature. Section 3 presents the main sources of data used throughout this paper. Section 4 introduces the *dependency* pattern, one of the main findings of this work. Section 5 introduces the centrality measure as a continuous measure that captures the idea of the *dependency* pattern and empirically studies the role of the centrality of commodity sectors in the propagation of shocks. Section 6 presents the model used to evaluate counterfactual scenarios, where the role of the *dependency* pattern in the propagation of commodity price shocks is examined. Finally, Section 8 concludes.

2 Related literature

The pioner work by Mendoza (1995) shown that terms-of-trade shocks account for sizeable fraction of GDP variability. It was the first paper that analyze the quantitative importance of terms of trade shocks in driving business cycles using a dynamic stochastic small open economy model. Kose (2002) extends Mendoza's work by developing a richer production structure that captures several empirically relevant features of developing economies. Kose (2002) found that world prices shocks — prices of primary, capital, intermediate goods, and world real interest rate — explain roughly 88% of aggregate output fluctuations. A recent work by Schmitt-Grohé and Uribe (2018) departed from calibrated business-cycle models to use empirical vector autoregression (SVAR) and found that terms-of-trade shocks play a modest role in generating aggregagte fluctuations in emerging and poor countries (less than 10%). As a conclusion, they mention that an improvement in the empirical model could be the use dissagregated commodity prices instead terms of trade aggregate indices. This work motivates articles as Fernandez et al. (2017) and Fernández et al. (2018) in the use of dissagregates price to measure commodity price shocks and their impact on aggregate activity.

[1]The definition of centrality measures will be discussed in Section 5

2.1 Empirical relevance of commodity price shocks

Commodity price shocks have shown to be an important driver of the business cycle, for both developing and developed countries. Based on a panel of 138 countries over 1960-2015 and using dissagregated world price measures, Fernandez et al. (2017) estimate an SVAR model to show that world shocks explain on average 33% of output fluctuation in individual economies. Moreover, for the post-2000 period, the estimate is doubled. However, the contribution of world shocks on output volatility is quite heterogeneus with estimated variance shares ranging from 14% to 54%.

Fernández et al. (2018) explore the hyphotesis that fluctuations in the price of commodities may be a key driver of business cycles in *small* emerging market economies (EMEs). On one hand, they document cyclical properties of commodity prices in EMEs: they are procyclical and lead the cycle of output, consumption and investment; and countercyclical to real exchange rates and measures of external risk premia. On the other hand, they estimate a structural model using data from Brasil, Chile, Colombia, and Peru to show that the median share of the forecast error variance in real output accounted by these shocks is 42% with considerable variability across countries, ranging from 27.5% in Brazil up to 77.1% in Chile. In their work, they focus on demand channel comming from income shocks triggered by commodity price fluctuations, leaving out supply channels that will be relevant in this article.

Similar results are found in Drechsel and Tenreyro (2018) for Argentina. Using post-1950 data, they estimate that the contribution of commodity price shocks to output growth represent nearly 38%, and around 62% of the variation in investment growth. Finally, Fernandez et al. (2020) study the rol of commodity price super-cycle in real activity for developed and developing countries. In line with their previous works, they found that shocks that drive commodity prices and world interest rate explain more than 50% of the variance of output growth on average, across countries. Moreover, most of the explanatory power comes from stationary shocks. They conclude that despite explaining part of aggregate activity at the country level, the contribution of commodity price super-cycles is smaller than that of stationary world shocks.

2.2 Propagation channels of commodity price shocks

Theoretical literature has explored different mechanisms for propagation of commodity price shocks. On the demand-side, the principal channel is wealth-effects. Increases in commodity prices raises the country income which allow increase consumption of non-commodity goods. One the supply-side, shocks to commodity prices modify firms labor demand, intermediate goods demand and affect firms' ability to borrow. A more detailed discussion of these channels can be found at Benguria et al. (2021) and Drechsel and Tenreyro (2018). Structural country characteristics as productive structure (Kohn et al., 2021; Caraiani, 2019; Silva et al., 2023) or financial market (Céspedes and Velasco, 2012; Shousha, 2016) also play a key rol on commodity shocks propagation. Finally, the way governments manage monetary and fiscal policies can stabilize output volatility (Céspedes and Velasco, 2012, 2014; Drechsel et al., 2019).

Benguria et al. (2021) use granular data for all sectors in the economy to explore two mechanisms for understanding how super-cycles affect economic activity: wealth and cost channels. In the wealth channel, higger commodity prices generate an increase in domestic demand that stimulates output, whereas the cost channel emerges because super-cycles induce wage increases. They build a multisector and multiregion model to quantity the transmission mechanisms and found that the labor market plays a crucial rol in the transmission of commodity super-cyles. A counterfactual economy, where commodity booms are purely endowment shocks experiences only 45% of the intersectoral labor reallocation between tradables and nontradables, and 40% of the labor reallocation between domestic and exported production within the tradable sector. Finally, they found that downward wage rigidity, prominent in an emerging-market context, reduces the welfare gains obtained from the super-cycle by more than 50% in comparison to a fully-flexible labor market (Benguria et al., 2021).

The mechanism of financial conditions and interest rates can be found in Drechsel and Tenreyro (2018), Shousha (2016), Fernández et al. (2018), and Céspedes and Velasco (2012). This mechanism stands that spreads seem to be lower during booms and higher during busts, affecting interest rates during commodity cycles. In the context of financial frictions, the lower interest rate reduces the costs related to working capital for firms and leads to a further boom in the commodity and nontradable sectors (Shousha, 2016). Relative to financial market development, Céspedes and Velasco (2012) provide evidence that the impact of commodity price shocks on output and investment tends to be larger for economies with less developed financial markets.

The productive structure also has been consider relevant for shocks propagation. In a recent work, Kohn et al. (2021) study the rol of differences in the patterns of production and international trade to explain business cycle volatility. Their work is based on systematic differences in production and trade patterns between developed and emerging countries, such as, share of tradable goods produced or commodities share of exports. Using a multi-sector small open economy where firms produce commodities or manufacture, they quantify the role of differences in production and trade patterns showing that these can account from 29% to 39% of the difference in real GDP volatility between emerging and developed countries. The main mechanism in this case comes from changes in capital and labor use, as well as productivity changes due to the reallocation of production between sectors induced by variations in international relative prices. Despite that this work use model of few sectors, the importance of sectoral interaction can be recognized. The following subsection presents a brief summary of the production network literature and some applications to the propagation of commodity price shocks.

2.3 Production networks and commodity price shocks propagation

In recent years, a growing body of literature has sought to understand the microfoundations of aggregate fluctuations and the production networks literature has played a relevant role (see Acemoglu et al., 2012, 2017, 2016; Barrot and Sauvagnat, 2016; Carvalho, 2008). A recent review of state of arts in the literature can be found in Carvalho and Tahbaz-Salehi (2019). One relevant result in this literature is that in an economy with intersectoral input-output linkages microeconomic idiosyncratic shocks may generate significant aggregate fluctuations. Moreover, if there exist significant asymmetry in the roles that sectors play as suppliers, idiosynctratic shocks can lead sizeable aggregate volatility (Acemoglu et al., 2012).

This literature has used aggregate network measures such as degree, density or sparcity to explain different aggregate outcomes. Degree is a measure of how many connections a specific sector has. It can be descomposed into in-degree, that in this context is the number of sectors a sector buys from, or out-degree, that is the number of sectors a sector sells from. Density describes the portion of the potential connections in a network that are actual connections so it takes the value 1 when all the sectors are connected or 0 if there is no relationship between them[2]. Finally, network sparsity measures the distribution of sectoral linkages. Sparsity measures the degree of input specialization in the economy and how crowded or dense these linkages are in the network. A network with high sparsity has fewer linkages, but these linkages are stronger and, on average, firms rely on fewer sources of input (Herskovic, 2018).

Measures as density, concentration or network sparcity has found relevant explaining different aggregate outcomes. For example, using OECD country data, Miranda-Pinto (2021) shown that GDP growth volatility declines with production network density. Herskovic (2018) demonstrate that concentration and sparsity are sufficient statistics for aggregate risk. Indeed, these latter two features are key for asset prices and determine the dynamics of aggregate output and consumption.

In addition to their role in explaining aggregate fluctuations from idiosyncratic shocks at firm

[2]Density of a network is constructed as follow:

$$\text{Density } = \frac{\sum_{i=1}^{N} \sum_{j}^{N} 1\left[\tilde{\omega}_{ij} > \underline{\omega}\right]}{N(N-1)}$$

where $1\left[\tilde{\omega}_{ij} > 0\right]$ is an indicator function that counts input-output connections that are greater than a small threshold $\underline{\omega} \in [0.001, 0.01]$

or sectoral level, production networks are also useful in explaining the propagation and amplification of macro-level shocks, as commodity price shocks. The specific literature on commodity price shocks and production networks is still under construction. Relative to commodity price shocks, Caraiani (2019) provides evidence that network features such as skewness in the in-degree or out-degree distribution, or density tend to amplifly the negative impact of oil shocks on GDP, using OECD data. Silva et al. (2023) study the role of domestic production networks in the transmission of commodity price shocks in small open economies. Using measures that they define as *customer* and *supplier* centrality, they found strong evidence for upstream propagation and muted downstream propagation on sectoral production. They also develop a static small open economy with production networks model to characterize the transmission channels of commodity price shocks through domestic production linkages. I utilize their novel model below in section 6 to introduce my measure of *dependency*.

In other contemporaneus work, Romero (2022) also studies the role of input-output linkages of commodity price fluctuations. It found that the positive correlation between commodity prices and GDP decreases in the *degree* of commodity sector. That is, when the commodity buy and sell a lesser fraction of their sales in intermediate market, the elasticity of GDP to commodity prices would be higher. Nevertheless, the network measures used do not capture indirect effects of networks or influence of commodity sector as this paper do.

Finally, Cao and Dong (2020) examine propagation trough input-output linkages for a commodity-exporting small open economy. They propose a theoretical model with diverse mechanism such as resource reallocation, exchange rate movements and monetary policy reaction to identify the importance of these transmission channels. When calibrating the model for the Canadian economy, they show that upstream and downstream input-linkages are relevant to explain the effect on aggregate output. Interestingly, and contrary to Silva et al. (2023), their results highlight the role of domestic downstream linkages, while upstream connections play a lesser role. In contrast to Cao and Dong (2020), I use a panel of 58 OECD and non-OECD countries to empirically show the role of upstream and downstream relationships.

The main contribution of this paper is to move away from aggregate measures of a production network to use commodity sector specific measures to study the propagation of commodity shock prices on aggregate outcomes. First, I present the *dependency* pattern in Section 4 to ilustrate striking differences in production networks between countries. Second, evidence is presented on how this pattern of intersectoral *dependency* associated with the commodity sector explains the differences in the impact on GDP. Third, I use the theoretical model developed by Silva et al. (2023) to evaluate counterfactual scenarios by modifying the pattern of *dependency* between countries.

In contrast to Caraiani (2019), that uses aggregate measures of the production network to explain the propagation of oil shocks, I use network characteristics associated specifically with the commodity sector. This paper is complementary to Silva et al. (2023) by presenting a new structural pattern in production networks that allows us to understand the differences in the propagation of commodity price shocks. I also focus on aggregate, not sectoral, outcomes.

3 Data

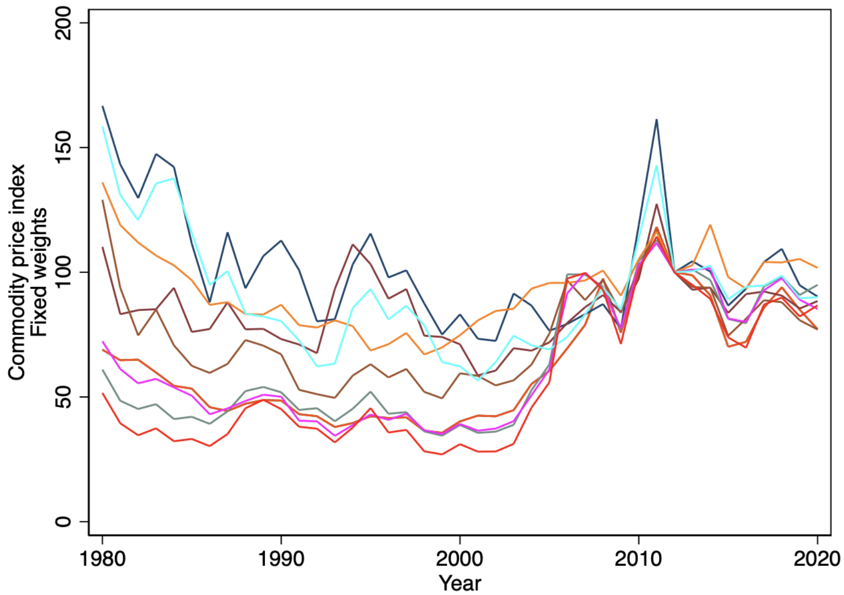
The principal database that I use in my analysis comes from OECD stats and includes 58 OECD and non-OECD countries. I use the 2018 edition of domestic Input-Output Tables (IOTs) that describe the sale and purchase relationships between sectors within an economy. These Input-Output tables are available from 2005 to 2015 and disaggregated into 36 productive sectors, as described in the Appendix. The values are in current US dollars for all countries.

I consider three sectors as commodity sectors: *Agriculture, forestry and fishing*, *Mining and extraction of energy producing products*, *Mining and quarrying of non-energy producing products*. Following the OECD notation, I will refer to these sectors as Sector 1, Sector 2 and Sector 3 respectively. Annual series for GDP at constant local currency and GDP per cápita at current or constant USD were obtained from World Bank databases for the 1980-2020 period.

Finally, I use the Commodity Export Price Index from International Moneraty Fund (IMF) as a proxy of exports commodity prices for each country. For each commodity, real prices are constructed as the commodity price in US dollars divided by the IMF's unit value index for manufactured exports (Gruss and Kebhaj, 2019). These specific prices for each commodity are weighted by their share relative to total commodity exports. I utilize the index with fixed weights to avoid changes due to variations in the composition of commodity exports [3]. Data are annual, based on 40 commodities and go back to 1962 for all countries present in the OECD databases. For further details about index construction see Gruss and Kebhaj (2019). Figure 1 present the IMF Commodity Price Index with fixed weights for the ten countries with the highest percentage of commodity exports.

[3]The commodity price index is then constructed as follows: $\Delta \log(\text{Index})_{i,t} = \sum_{j=1}^{J} \omega_{i,j,} \cdot \Delta \log(p_{j,t})$ where $p_{j,t}$ is the price of the commodity j in period t and $\omega_{i,j}$ is the fixed weight associated to commodity j in country i.

Figure 1: Commodity Export Price Index evolution



It can be seen that the price index is strongly correlated across countries. This comovement shown in Figure 1 has been highlighted using highly disaggregated commodity prices by Fernández et al. (2018) and Silva et al. (2023).

4 Empirical documentation

My principal empirical finding is the existence of remarkable differences among countries in the relevance of commodity sectors within the production network. I also show empirically how these differences affect the propagation of commodity price shocks. These differences are not due to size of the commodity sector, but to how they interact with other sectors within the production network and the influence they exert on other sectors. I show that in some countries, starting from the commodity sector, there is a consistently longer sequence of buyer (or seller) sectors when only high-intensity relationships are considered. These differences are substantive and cannot be detected when analyzing general measures of the production network, such as density.

From now on I use the notation following networks literature. Each sector in the economy it is called *node* and the flow between two nodes is called *edge*. There exist one edge for each transaccion between sectors in the economy. In order to highlight the clear differences in connections associated with the commodity sector between countries, I will use an *ego graph*. This kind of graph reveals all the nodes that interact directly or indirectly with a specific node, which in this case are the commodity sectors. For example, an *ego graph* for the commodity sector contain all the input providers for that sector, all the input providers for the providers of commodity sectors and so on. If there is a sector that is not directly or indirectly interconnected with the commodity sector it will not appear in the graph.

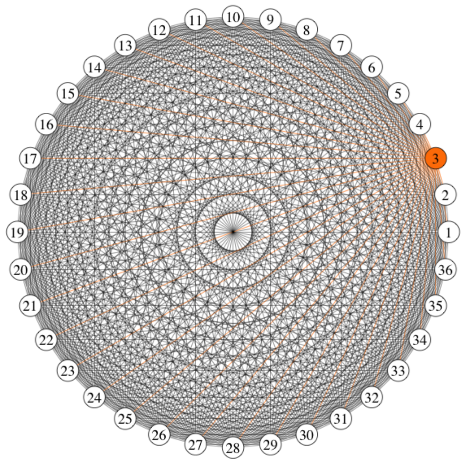
To construct the *ego graph* I apply filters to keep only the more relevant transactions. There exist two points of view for this relevance: for the selling or buying sectors. On one hand, I keep an edge from sector i to j if this sale represents more than 10% *i's* intermediate sales. I define this relevance as *upstream relevance* because the focus is on the sealling sector. On the other hand, I keep an edge from i to j if this purchase represents more than 10% *j's* intermediate purchases. This filter is defined as *downstream relevance* because the focus is on the purchasing sector. After apply one of these two filters its posible see how many sectors in the economy are associate directly or indirectly with one specific commodity sector.

To fix ideas, Figure 2 compare the complete production network for Australia and Chile for 2014. Each node represent one sector in the economy and each edge between nodes represent a sale from sector i to sector j[4]. Orange node is a commodity sector (*mining and quarrying of non-energy producing products*) and orange edges are direct purchases or sales by this sector. At this point it is impossible to see differences between networks because in the data almost all sectors sell a non-zero value to others. Density of production network at this level is the same for both economies. Considering a threshold of 0.1%, the mean density for period 2005-2015 in Australia is 10% bigger than in Chile. Is this difference relevant to the propagation of commodity price shocks? The answer is not clear.

[4]The names of each sector are described in the Appendix.

Figure 2

(a) Australia: Complete network



(b) Chile: Complete network

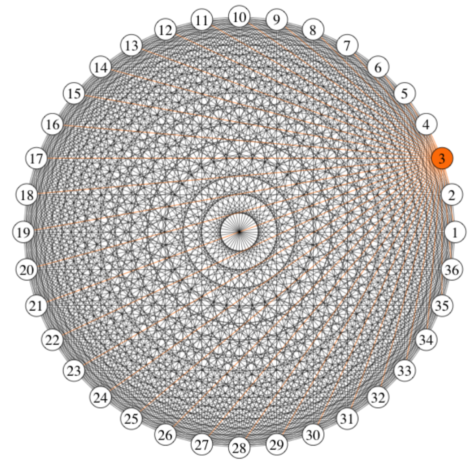
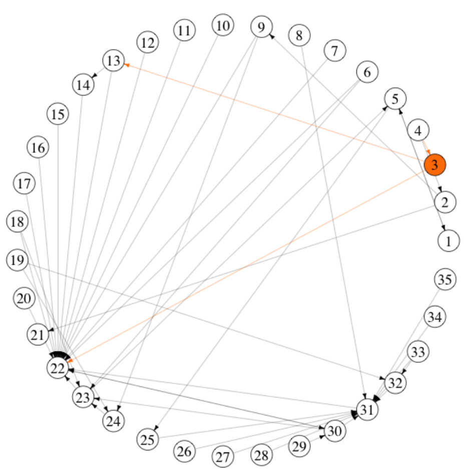


Figure 3

(a) Australia: filtered network



(b) Chile: filtered network

Figure 4

(a) Australia: upstream ego graph

(b) Chile: upstream ego graph

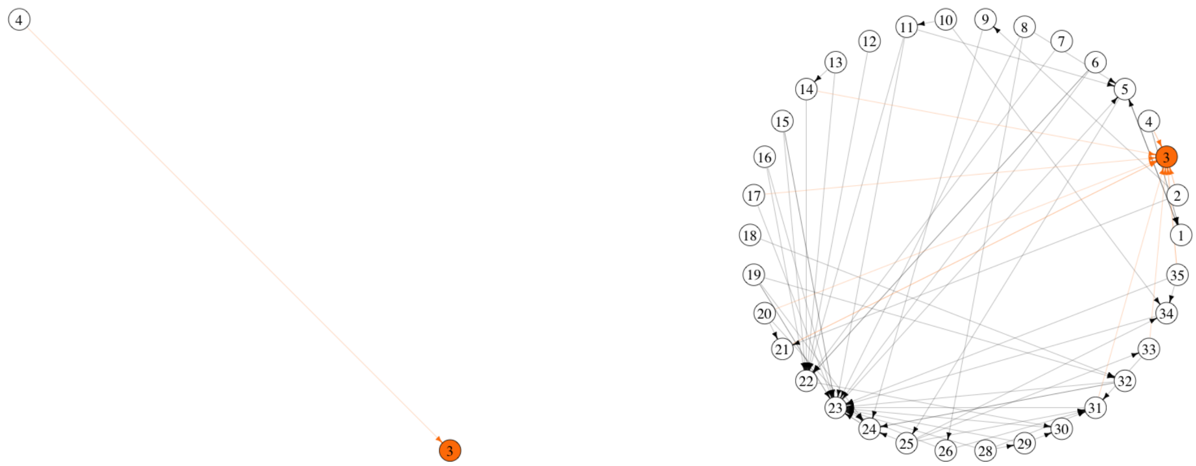


Figure 3 shows the networks after applying a threshold: all sales representing less than 10% of total intermediate sales for the selling sector are dropped (*upstream relevance*). It can be seen that in Chile, the sector 3 is a relevant buyer for at least 9 sectors whereas the same node in Australia only is a relevant buyer for 1 sector. In Chile, commodity sector 3 sells more than 10% of their intermediate sales to three sectors (Chemicals and pharmaceutical products, Basic metals, and Construction) whereas in Australia only does for Basic metals and Construction sectors. This can be seen by following the arrows in the graph: the arrow points to the sector that buys inputs from others. Finally, it is possible to see that, beyond the commodity sector, sectoral sales are more concentrated. For example, sectors from 22 to 32 emit (or receive) more edges in Chile than in Australia. Figure 3 helps us to visualize more clearly the most intense sector relationships, as only sales representing more than 10% for the selling sector are shown.

Finally, Figure 4 shows the *upstream ego graph*. This figure contains all sectors that sells above than 10% of their intermediate sales to the commodity sector (node 3), sectors that sells above than 10% of their intemediate sales to commodity sector input providers and so on. For the case of Australia, only one sector sell a relevant share of their intermediate sales to sector 3. In Chile, 9 sectors sell a relevant share of their intermediate sales to sector 3. At the same time, there exists a surprising number of sectors which in turn sell a significant percentage of their intermediate sales to sector 3 suppliers and so on.

Following the above procedure, Table 1 shows the average number of sectors between 2005-

2015 associated with commodity sector 3 in the *ego graph* for different countries. That is, the number of sectors that directly or indirectly sell a relevant share of their total intermediate sales to the commodity sector. A significant difference can be observed in the number of sectors maintained by Chile and Peru in relation to other exporting non-energy mining products countries. The first column keep sales that represent more than 10% of intermediate sales of upstream sectors while the second column keep sales that represent more than 5% of total production of upstream sectors for robustness. For countries like United Kingdom, Canada or Iceland, no sector sell more than 10% of their intermediate sales to this commodity sector whereas in countries like Chile and Peru there exist more direct and indirect sales that represent important shares for providers. Table 2 shows the same pattern for exporting mining of energy producing products (sector 2) countries. Again, the sub-graph for some countries is notably larger in some countries than in others.

Table 1: Mean of nodes from upstream ego graph for sector 3

	Filter		Filter
Country	Edge / TIS > 0.1	Country	Edge / Total production > 0.05
Peru	28	Chile	10
Chile	19	Peru	8
Australia	2	Australia	3
Brazil	2	Brazil	2
United States	2	United States	2
Iceland	1	Canada	2
Canada	1	United Kingdom	1
United Kingdom	1	Iceland	1

Note. TIS: Total intermediate sales. The mean was calculated as the average of the resulting sectors for each *ego graph* between 2005 and 2015.

Table 2: Mean of nodes from upstream ego graph for sector 2

	Filter		Filter
Country	Edge / TIS > 0.1	Country	Edge / Total production > 0.05
Colombia	22	Colombia	6
Saudi Arabia	8	Saudi Arabia	4
Tunisia	3	Indonesia	2
Norway	3	Australia	2
Australia	2	Mexico	2
Indonesia	2	Norway	2
Mexico	2	Tunisia	2
United Kingdom	2	United Kingdom	2
United States	2	United States	2

Note. TIS: Total intermediate sales. The mean was calculated as the average of the resulting sectors for each *ego graph* between 2005 and 2015.

The number of sector in the *ego graph* presented in the previous tables shows a kind of dependency between sectors in the network. I refer to this finding as *dependency pattern*, that is, the fact that in some countries sectors directly or indirectly related to the commodity sector concentrate their sales in few buyers. The *dependency* pattern is closely related to sparcity measures used in Herskovic (2018). A network with high sparsity has fewer linkages, but these linkages are stronger and, on average, firms (or sectors) rely on fewer sources of input. This fact can be associated with diversification patterns in the production networks and as I will show later, can have sizeable effects in the propagation of shocks. However, measures such as sparcity or density are measures for the entire production network. The pattern I call *dependency* is a measure of inter-sectoral dependence associated specifically with the commodity sector. Recall that *ego graph* only considers sectors that interact directly or indirectly with a given commodity sector.

From Tables 1 and 2 two things are worth mentioning. First, despite the position in the ranking is maintained, the number of sectors decreases sharply when total production is considered instead of total intermediate purchases. This is not caused by the size of commodity sector because the *upstream ego graph* use sales shares for providers (i.e, sales for sector j to commodity sector over j's total production). Instead, is the size of insume providers that reduce the sales share. Second, the number of sectors in the *ego graph* is a discontinue measure for the relevance or linkages asociated at commodity sector. Although it does shed light on the interactions behind it, an arbitrary threshold of 10% eliminates sales representing, for example, a share of 7%. Lower thresholds would successively increase the number of sectors in the ego graph for all countries in a non-linear way. For this reason, in the following section I will present a measure that captures in a continuous and formal way the relevance of the commodity sector within the production network.

Finally, the pattern of *dependency* is also found when studying the downstream relationships for the commodity sector. The *downstream ego graph* is constructed in a similar way but using the relevance for the buyer (i.e. sales from sector i to j over j's total intermediate purchases). Tables 3 and 4 show that some countries maintain many more intensively related sectors, making up long chains of successive purchases.

Table 3: Mean of nodes from downstream ego graph for sector 3

	Filter		Filter
Country	Edge / TIP > 0.1	Country	Edge / Total Inputs > 0.1
Peru	17	Peru	13
Chile	17	Chile	11
Brazil	9	Australia	6
United States	7	Brazil	5
Australia	7	Canada	4
Canada	5	United States	3
Iceland	2	Iceland	1
United Kingdom	1	United Kingdom	1

Note. TIP: Total intermediate purchases. The mean was calculated as the average of the resulting sectors for each *ego graph* between 2005 and 2015.

Table 4: Mean of nodes from downstream ego graph for sector 2

	Filter		Filter
Country	Edge / TIP > 0.1	Country	Edge / Total Inputs > 0.1
Indonesia	32	Indonesia	29
Colombia	26	Colombia	23
Saudi Arabia	26	Mexico	20
Mexico	25	Saudi Arabia	13
Tunisia	25	United States	8
United States	13	Australia	6
Australia	9	Norway	5
Norway	6	Tunisia	4
United Kingdom	5	United Kingdom	3

Note. TIP: Total intermediate purchases. The mean was calculated as the average of the resulting sectors for each *ego graph* between 2005 and 2015.

The differences in production structures associated with the commodity sector can lead to a better understanding of the impact commodity price shocks have on output or GDP. In this sense, the empirical results using a panel of 58 countries presented in Section 5, along with the simulation exercises in Section 7, demonstrate that empirical and theoretical results are in harmony. Therefore, showing that the stronger the pattern of *dependency*, the greater the aggregate effect of a commodity price shock.

5 Centrality measures and commodity shocks propagation

Section 4 documents the existence of very different patterns of connections associated to commodity sectors between countries. As discussed previously, the measures associated with the *ego graph*, which give rise to the *dependency pattern*, are discontinuous and information about relationships below the threshold used is lost. For this reason, in this section I introduce a formal measure of the centrality (or influence) of a specific sector within the production network. This measure conceptually capture the *dependency pattern* and is not dependent on the arbitrariness of the threshold set. In consequence, I use the centrality measures to understand the effect of network structure on the propagation of commodity price shocks.

There are diverse centrality measures in networks. Bloch et al. (2021) provide a taxonomy of centrality measures and synthesizes their differences. Sargent and Stachurski (2022) also discusses about different centrality measures and their applications in economics. One of these measures (Katz centrality) conceptually captures the idea of cross-sectoral dependence observed in the *ego graph*. Katz centrality is a measure of influence. It relies on the idea that the influence of node i is related to the influence of her neighbors. High centrality is conferred to a node when it is linked to by nodes that themselves have high centrality.

Formally, Katz centrality is computed by $\kappa_i = \beta \sum_j a_{ij}\kappa_j + 1$[5], where β takes values between (0, 1/rank(A)), and a_{ij} is an element of the adjacency matrix corresponding to the weighted digraph[6]. The elements of the adjacency matrix, which act like weights for the centrality measure, are key to giving an interpretation to these values. I have to ways to use weights in the adjacency matrix. On one hand, if I compute the centrality of sector i using as a weight elements $c_{ij} = P_j M_{ij}/P_j Q_j$ (sales of j to i over j output), I am measuring how important is sector i as buyer for j both directly and indirectly, so I can interpret the centrality as *customer centrality*. On the other hand, if the weight in the adjacency matrix are $s_{ij} = P_j M_{ij}/P_i Q_i$ (sales of j to i over i output), I am measuring how important is j as input supplier for the sector i. Then, this last measure will be the *supplier centrality*.

These weights also allows to associate the centrality measure with the *dependency pattern*. In the *ego graph*, the relevance of the commodity sector is observed in that its input suppliers are heavily dependent on purchases from the commodity sector. In turn, these suppliers have high intensity relationships with their own suppliers and so on. When calculating the *customer centrality*, the influence of the commodity sector will depend on the influence of the sectors with

[5] or $\kappa = (I - \beta A)^{-1}\mathbf{1}$ in matricial form.

[6] The adjacency matrix of a weighted digraph (V, E, w) with vertices $(v_1, ..., v_n)$ is the matrix $A = (a_{ij})_{1\leqslant i,j\leqslant n}$ with $a_{ij} = \begin{cases} w(v_i, v_j) & \text{if } (v_i, v_j) \in E \\ 0 & \text{otherwise} \end{cases}$

which it interacts most, and I know that these sectors are in turn highly relevant to their suppliers.

Figures 5 and 6 show the relationship between GDP per capita and centrality measures[7]. In the same spirit as tables 2 and 1, figure 5 shows that countries where the customer centrality of the commodity sector is greater tend to have lower levels of development. However, this relationship is not linear and there are some notable exceptions. Figure 6 instead relates to tables 4 and 3. There is a clear negative relationship. Countries where the supplier centrality of the commodity sector is greater tend to be countries with lower GDP per capita.

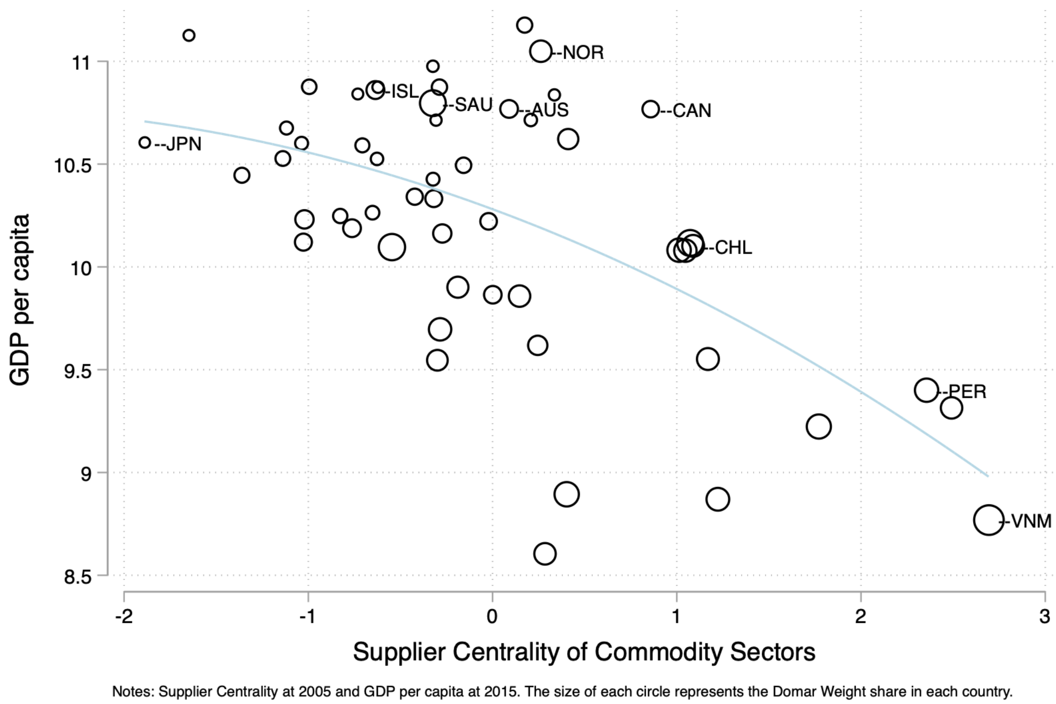
Finally, I would like to highlight the relationship between the centrality measures and the size of the commodity sectors. In both figures, the size of each marker represents the Domar weight of the commodity sectors. There is a lot of heterogeneity in the relationship between Domar weight and centrality measures. Although the commodity sector tends to be large in countries with high centrality, there are several reverse cases where the sector is large and the centrality measure is low.

Figure 5: GDP and Customer Centrality

Notes: Customer Centrality at 2005 and GDP per capita at 2015. The size of each circle represents the Domar Weight share in each country.

[7]Both centrality measures have been normalized. Thus, a value of one means 1 standard deviation from the mean of our panel.

Figure 6: GDP and Supplier Centrality



5.1 Impulse responses

5.2 Impulse responses

To measure the impact of commodity price shocks on GDP and the role that play linkages as amplification factor I use Local Proyections (LPs). Plagborg-Møller and Wolf (2021) prove that LPs and Vector Autoregressions (VARs) estimate the same impulse responses. They show, among other things, that (i) LP and VAR are simply dimension reduction techniques with common estimands but different finite sample properties; (ii) VAR-based structural identification can be performed using LPs, and vice versa; (iii) linear VARs are as robust to nonlinearities as linear LPs.

An advantage of LPs is the ease with which interactions or non-linearities can be added. I consider the following interactions variables for commodity price shocks: customer centrality, supplier centrality, density of the network and commodity sector size (Domar weight). For centrality, Domar weights, and density measures, I use measures from 1995 and hold their values constant (first year of the sample). Because there are five commodity sectors in the data, I generate the customer and supplier centrality measures as the sum of the individual centrality for each sector. Equation (1) summarize the main regression of this section. The LHS, $y_{i,t+h} - y_{i,t-1}$, is the h-period change for the variable of interest in country i. In this case, I focus on natural logarithm of real GDP at local currency. Δcp is the change of natural logarithm of commodity price. Given this specification β_0^h represent the cumulative impact of an innovation in commodity price on real GDP. $\Delta\text{cp}_{i,t} \times \text{Int}_{i,t}^k$ is the term that incorporate interaction variables mentioned above. The term μ_i^h are country fixed effects; ν_t^h are time fixed effects and $\epsilon_{i,t+h}$ a random disturbance. For all estimations I also included three lags of Δcp and $\Delta y_{i,t}$. Real GDP and commodity prices are in logarithm while the interaction terms are all in levels and normalized. For this reason, γ_k^h parameter measure the impact of an increase of a standard deviation above the mean in the interaction term.

$$\begin{aligned} y_{i,t+h} - y_{i,t-1} = \alpha^h + \beta_0^h \Delta\text{cp}_{i,t} + \sum_{k \in I} \gamma_k^h \Delta\text{cp}_{i,t} \times \text{Int}_{i,t}^k \\ + \sum_{j=1}^{J} \beta_j^h \Delta\text{cp}_{i,t-j} + \sum_{j=1}^{J} \rho_j^h \Delta y_{i,t-j} \\ + \mu_i^h + \nu_t^h + \epsilon_{i,t+h} \end{aligned} \tag{1}$$

Table 5 presents the LP using customer centrality as interaction term. A commodity shock price Δcp generates an increase of GDP. This cumulative effect is increasing, as the impact on contemporaneous GDP is not different from zero, but increases over time. Four periods after a 1% shock on the commodity price index, GDP increase 0.169 percent. The second row shows that the customer centrality of commodity sectors amplifies the impact of commodity shocks. One period after the shock, in countries with a standard deviation above the mean of the customer centrality measure, the effect is amplified by 1/3 relative to a pure commodity price shock. This means that in countries where the pattern of *dependency* is stronger, GDP will be more volatile after commodity price shocks.

Table 5: Interaction with Commodity Customer Centrality

	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Δ cp	0.015	0.054**	0.095***	0.136***	0.169***
	(0.014)	(0.026)	(0.029)	(0.038)	(0.043)
Δ cp $\times$ Customer Centrality	0.008	0.019**	0.026***	0.027**	0.025*
	(0.006)	(0.007)	(0.010)	(0.013)	(0.013)
Observations	1109	1059	1008	957	906
R^2	0.634	0.621	0.630	0.653	0.694

Notes: Each regression controls for country and year fixed effect. Standard erros in parentheses. Key: *** significant at 1%; ** 5%; * 10%.

Table 6 shows the amplification factor of the size of commodity sectors, measured by Domar weight. The results are quite similar to those using customer centrality as an interaction variable, although the coefficients are less precise. One possibility is that customer centrality is largely explained by the size of the commodity sector. In such a case, Domar weight would remain the key statistic for understanding the propagation of shocks. Table 7 uses as the interaction term the residual of customer centrality on Domar weight, i.e. the part of customer centrality that is not explained by the size of the commodity sector. Despite the loss of precision, it is observed that the effect of commodity price shocks is amplified in countries where the customer centrality of the commodity sector is higher.

Finally, Table 8 presents results using the full set of interactions. A commodity price shock Δcp generates an increase in GDP. On the one hand, I observe that customer centrality remains significant as an amplifying factor. On the other hand, the effect of supplier centrality is muted. These results are similar and consistent with the results of Silva et al. (2023), who study sectoral output. The results obtained so far highlight the importance of the commodity sector as a buyer—directly and indirectly—of inputs rather than as a seller. Regarding the size of the commodity sector and the density of the production network[8], the results indicate that, after controlling for measures of customer and supplier centrality, neither of these measures plays a significant role in the propagation of commodity shocks.

Table 6: Interaction with Commodity Dommar Weight

	$h=0$	$h=1$	$h=2$	$h=3$	$h=4$
Δ cp	0.014	0.049*	0.090***	0.131***	0.163***
	(0.014)	(0.026)	(0.028)	(0.037)	(0.042)
Δ cp $\times$ Domar Weight	0.003	0.015*	0.018*	0.020*	0.020*
	(0.006)	(0.008)	(0.010)	(0.011)	(0.011)
Observations	1109	1059	1008	957	906
R^2	0.633	0.620	0.628	0.652	0.694

Notes: Each regression controls for country and year fixed effect. Standard erros in parentheses. Key: *** significant at 1%; ** 5%; * 10%.

[8] I use a threshold equal to 0.1% following Miranda-Pinto (2021)

Table 7: Interaction with residual Customer Centrality

	$h=0$	$h=1$	$h=2$	$h=3$	$h=4$
Δ cp	0.016	0.055**	0.098***	0.138***	0.171***
	(0.014)	(0.027)	(0.030)	(0.039)	(0.043)
Δ cp × Residual Customer Centrality	0.008	0.015*	0.021**	0.021	0.019
	(0.006)	(0.007)	(0.010)	(0.013)	(0.013)
Observations	1109	1059	1008	957	906
R^2	0.634	0.620	0.629	0.652	0.694

Notes: Residual customer centrality is the residual of the regression of Customer centrality on Dommar Weight. Each regression controls for country and year fixed effect. Standard erros in parentheses. Key: *** significant at 1%; ** 5%; * 10%.

Table 8: All Interactions

	$h=0$	$h=1$	$h=2$	$h=3$	$h=4$
Δ cp	0.014	0.050*	0.092***	0.132***	0.163***
	(0.014)	(0.027)	(0.030)	(0.038)	(0.043)
Δ cp × Customer Centrality	0.011	0.019**	0.026**	0.024	0.021
	(0.007)	(0.008)	(0.011)	(0.015)	(0.015)
Δ cp × Supplier Centrality	0.001	-0.002	-0.008	-0.003	0.009
	(0.008)	(0.008)	(0.008)	(0.013)	(0.014)
Δ cp × Domar Weight	-0.005	0.009	0.015	0.014	0.005
	(0.007)	(0.010)	(0.012)	(0.016)	(0.017)
Δ cp × Network Density	-0.009	-0.006	-0.002	-0.002	-0.008
	(0.006)	(0.009)	(0.012)	(0.015)	(0.016)
Observations	1109	1059	1008	957	906
R^2	0.635	0.622	0.630	0.653	0.695

Notes: Each regression controls for country and year fixed effect. Standard erros in parentheses. Key: *** significant at 1%; ** 5%; * 10%.

To complement the empirical analysis, in the next section I present a novel production network model that explicitly incorporates features of commodity sectors. The model allows counterfactual exercises to evaluate how the *dependency pattern* affects the propagation of shocks in the economy.

6 Model

The model of this section is an static version of production networks economy for an small open economy where the commodity price is exogenous and depend of international demand. This section is based on the model proposed by Silva et al. (2023) and its novelty is how it incorporates external demand and prices for commodity sector in a production network framework. To the best of my knowledge, this work along Cao and Dong (2020) and Romero (2022) are the first that explicitly incorpores particular features of commodity sector within production networks. Another relevant contribution to the literature provided by Silva et al. (2023) is a clear decomposition of the channels through which a commodity price shock propagates in the economy.

They consider a economy with $N+1$ sectors/firms. The first N sectors produce non-tradable goods, while sector $N+1$ produces a commodity. The price of commodity good is exogenous and taken as given for the sector $N+1$. Finally, the demand side is modeled as simple as possible to emphasize the role of linkages in the propagation of commodity price shocks. Each block of this economy is described below.

6.1 Firms

While the non-tradable sectors feature constant returns to scale, the commodity sector features decreasing returns to scale in production. This feature for the commodity sector allows that its marginal cost curve will be increasing in the quantity produced. This *scale effect* is absent in the non-tradable sector where the marginal cost is independent of the scale of production due to the constant returns to scale production function. The problem of each type of firm is outlined below.

6.1.1 Non-Tradable Sectors: $i = 1, 2, ..., N$

There is a representative firm in each i sector producing using a CES production function that combines labor and intermediate goods from other sectors. In particular, sector i operates using the following production function

$$Q_i = Z_i \left(a_i^{1-\rho_Q} L_i^{\rho_Q} + (1-a_i)^{1-\rho_Q} M_i^{\rho_Q} \right)^{\frac{1}{\rho_Q}} \tag{2}$$

where $\rho_Q \equiv \frac{\sigma_Q - 1}{\sigma_Q}$. Q_i is gross output, Z_i is physical productivity, L_i is labor and M_i is a composite of intermediate goods specified below. The parameter σ_Q account for the elasticity of substitution between labor and the intermediate input bundle.

The CES aggregator of intermediate inputs from all sectors for firm i is defined as:

$$M_i = \left(\sum_{i=1}^{N+1} \omega_{ij}^{1-\rho_I} M_{ij}^{\rho_I} \right)^{\frac{1}{\rho_I}} \tag{3}$$

where $\rho_I \equiv \frac{\sigma_I - 1}{\sigma_I}$. M_{ij} is the demand of firm i for inputs of firm j, σ^I is the elasticity of substitution across intermediates and ω_{ij} is a parameter that reflects the importance of sector j as an input supplier to sector i. As usual, cost minimization of the intermediate input bundle defines its price index

$$P_i^I = \left(\sum_{j=1}^{N+1} \omega_{ij} P_j^{1-\sigma_i^I} \right)^{\frac{1}{1-\sigma_i^I}}.$$

6.1.2 Commodity Sector: $i = N + 1$

The commodity sector features decreasing returns so the production function is given by

$$Q_{N+1} = Z_{N+1} \left(a_{N+1}^{1-\rho_Q} L_{N+1}^{\rho_Q} + (1 - a_{N+1})^{1-\rho_Q} M_{N+1}^{\rho_Q} \right)^{\frac{1}{\rho_Q} \delta_{N+1}} \tag{4}$$

where decreasing returns to scale are governed by $0 < \delta_{N+1} < 1$. The commodity sector aggregates the intermediate input bundle in the same way as the other sectors.

6.2 Consumer's Preferences

The demand side is modeled as simple as possible to emphasize the role of linkages in the propagation of commodity price shocks. They assume that the representative consumer at home has Cobb-Douglas preferences over all $N + 1$ goods:

$$U(\{C_i\}_{i=1}^{N+1}) = \prod_{i=1}^{N+1} C_i^{\beta_i}, \qquad \sum_{i=1}^{N+1} \beta_i = 1 \tag{5}$$

The consumer receives income from labor and from firms' profits. Its budget constraint thus reads

$$\sum_{i=1}^{N+1} P_i C_i = W\bar{L} + \sum_{i=1}^{N+1} \Pi_i + rB, \tag{6}$$

where P_i denotes the price of good i, W is the wage, which will be set as the numeraire, $\bar{L}$ is labor supply, Π_i is firm i's profits and rB are assets at the steady state.

The foreign demand for the commodity sector and its price is modeled as function of the rest of the world income, Y^*:

$$P_{N+1} = (Y^*)^{\frac{1}{\sigma_S^*+\sigma_D^*}} (P^*)^{(\sigma_D^*-1)\frac{\sigma_S^*}{\sigma_D^*+\sigma_S^*}} = \eta_{N+1}(Y^*)^{\frac{1}{\sigma_S^*+\sigma_D^*}} \tag{7}$$

$$C_{N+1}^* = \left(\frac{P_{N+1}}{P^*}\right)^{-\sigma_D^*} \frac{Y^*}{P^*} = \eta_C (Y^*)^{\frac{\sigma_S^*}{\sigma_S^*+\sigma_D^*}}, \tag{8}$$

where P^* is the price index of the rest of the world, σ_S^* and σ_D^* are the elasticity of world supply and demand of commodity sector. The constants $\eta_C = \eta_{N+1}^{-\sigma_D^*}(P^*)^{\sigma_D^*-1}$ and $\eta_{N+1} = (P^*)^{(\sigma_D^*-1)\frac{\sigma_S^*}{\sigma_D^*+\sigma_S^*}}$ depend on the price index of the rest of the world.

6.3 Equilibrium

A competitive equilibrium in this economy is defined in the usual way. Taking prices as given consumers and firms maximize utility and profits and all markets clear. The resource constraint for each sector is

$$Y_i = C_i + C_i^* + \sum_{j=1}^{N+1} M_{ji}, \tag{9}$$

where C_i is domestic consumption demand, M_{ji} is demand from firm j, and C_i^* is the external demand. Since the commodity sector is the only one that exports in this economy, $C_i^* = 0$ for all $i \neq N+1$.

Finally, the labor market clearing condition reads

$$\sum_{i=1}^{N+1} L_i = \bar{L}. \tag{10}$$

6.4 Commodity Price Shocks Propagation

Silva et al. (2023) show how changes in the commodity price P_{N+1} propagates throughout the economy and affects sectoral output, providing a first order approximation for changes in sectoral output. Since the focus in this paper is on changes in aggregates, I generate a weighted sum of sectoral output changes to calculate the variation on aggregate output. Then, up to a first-order, the effect of a differential change in the commodity price $\hat{P}_{N+1}$ on aggregate output, $\hat{Q}$, satisfies:

$$\hat{Q} = \sum_{i=1}^{N+1} \theta_i \hat{Q}_i \tag{11}$$

where

$$\begin{aligned}
&\theta_i = P_i Q_i / \sum_{k=1}^{N+1} P_k Q_k, \\
&\hat{Q}_i = (\zeta_i^{N+1} + \zeta_i^{Q} \phi_{P_{N+1}}^{N+1}) \hat{P}_{N+1}, \qquad \text{for } i = 1, \ldots, N \\
&\hat{Q}_{N+1} = \phi_{P_{N+1}}^{N+1} \hat{P}_{N+1}
\end{aligned}$$

The expressions for $\hat{Q}_i$ and $\hat{Q}_{N+1}$ come from propositions 2 and 3 in Silva et al. (2023). Variables ζ_i^{N+1}, ζ_i^{Q}, and $\phi_{P_{N+1}}^{N+1}$ are constant that depend on model parameters and network structure. See Appendix B for details.

The term ζ_i^{N+1} capture three different channels through commodity prices shocks impact the output in non-commodity sectors (see Appendix B). The first one is the well known *wealth effect*, that represents how the increase in domestic income due to higher commodity prices allows consumers to increase their domestic consumption. The second one is the *buyers' substitution effect*, that captures how intermediate inputs' buyers try adjust their demand to other inputs to avoid the increase in the price of the commodity sector. Finally, the third component is the *pure downstream effect*, that captures the increase in marginal cost experienced by sector i after a change in the commodity price.

As shown in expression (11), many factors determine the total effect of one shock to the commodity price. Silva et al. (2023) discuss how different elasticites or structural parameters affect shocks propagation. Since the *dependency pattern* is associate with differences on allocation matrix (M) and expenditure matrix (Ω), I use these differences to explain the different responses of aggregate output across countries.

The mapping from *dependency pattern* to the model elements is as follows. The *dependency pattern* is generated by the shares of purchases and sales between sectors. The more concentrated these shares are, the more dependent one sector is on another. In the model, the elements of allocation matrix (M), are defined as $m_{ij} = \frac{M_{ij}}{Q_j}$, i.e., how much of good j is allocated to sector i. Then, higher m_{ij} implies that sector j is more dependent as seller from sector i (upstream relevance). The matrix (Ω), whose elements are $\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$, defines the expenditure of goods from j by sector i (downstream relevance). These two matrices govern the shares of purchases and sales between sectors, and therefore, the *dependency pattern*.

7 Simulations

In this section I use the model presented above to quantify differences in propagation of a commodity price shock modifying the production structure and, therefore, the pattern of *dependency* shown in Section 4. For these counterfactual exercises I use OECD data from Australia, Chile and Peru in 2015.

Calibration. Parameters were set as follows. Throughout all exercises I set $\delta_{N+1} = 0.9$. For the three countries, I assume equal elasticities for the production function and test the robustness of the results using different combinations of elasticities, where $\sigma_I \in (0.8, 1.2, 2)$ and $\sigma_{VA} \in (0.8, 1.2, 2)$. The parameter b_{N+1}, that represent the ratio of intermediate purchases over total cost was calibrated from data at 0.8 for Chile, 0.77 for Australia and 0.72 for Peru. Finally, α_{N+1} and α_B are set to 0.05 and -0.6, respectively. All the other variables are constructed using OECD data at 36 sectors level for 2015. For all exercises it is assumed that the shock was to commodity sector 3 (*Mining and quarrying of non-energy producing products*).

The aggregate effect on output depends on upstream and downstream effects. On one hand, for a given allocation and expenditure matrix, higher σ_{VA} reduce the positive upstream effect because sectors substitute intermediate inputs toward labor. On the other hand, higher σ_I increase the upstream effect by fostering substitution between intermediates. The same is true for the negative effect of pure downstream propagation through prices. Lastly, α_B (ratio of debt to GDP) determines the sign and strength of the wealth effect. Negative values produce a positive wealth effect and vice versa. This, in turn, increases the upstream propagation defined in equation (14).

7.1 Simulations results

Using expression (11), I evaluate changes in aggregate output by computing changes in sectoral output and then aggregating these percentage changes by adjusting for the size of each sector. Starting from a symmetric equilibrium I generate T = 200 perturbations to commodity price and compute the aggregate output change.

The counterfactual exercise consists of taking the dependency pattern from one country to another. For example, imputing the structure observed in Figure 4 (b) to Australia and the structure in Figure 4 (a) to Chile. Details about this transformation are presented in the Appendix. A clarification is important to make at this point. To impute the upstream dependency structure from one country to another, it is also necessary to modify the downstream dependency structure, since the change is made at the allocation matrix (M) level. This second change generates a new downstream dependency pattern that is not the original one nor the one of the country in comparison, but it is the only possible structure after matching the upstream dependency pattern. This implies that the following results are not only a consequence of the change in the upstream dependency pattern, but a combination of both. Regardless of the aforementioned, the exercise is useful as a first approximation to understand how different production structures affect the propagation of shocks. Section 5 will show how each of these dependency patterns (upstream and downstream) affect the propagation of shocks separately.

Table 9: Counterfactual exercises: Chile vs Australia

	Chile	Australia
Panel A: Mean output change after an $U[0, 0.1]$ positive shock sequence		
Original	0.073	0.048
Counterfactual	0.058	0.065
Ratio C/O	-20%	+35%
Panel B: Volatility of output change after an $U[-0.1, 0.1]$ shock sequence		
Original	0.089	0.058
Counterfactual	0.071	0.079
Ratio C/O	-20%	+36%

Note: Counterfactual means that the production structure of the country with which it is compared has been imputed. Panel A shows the mean impact on aggregate output after the commodity price shock sequence. Panel B compute the standard deviation of these changes. Simulations were made using $(\sigma_I, \sigma_Q) = (2, 1.2)$ but results remains almost identical using $(\sigma_I, \sigma_Q) = (1.2, 2)$ instead.

Table 9 shows the results for simulations. *Original* rows compute mean impact or volatility using the original input-output structure for the country whereas *Counterfactual* rows shows values for the counterfactual economy. Panel A of Table 9 shows that mean impact on output after commodity price shocks is bigger in Chile than Australia. At the same time, the volatility of these changes is bigger in Chile than Australia. Counterfactual results reveal that when the Australian pattern of upstream dependency is imputed to Chile, the mean impact and volatility drops 20%. At the same time, imputing Chilean upstream dependecy pattern to Australian economy generates an increase in the effect that commodity price shocks have on total output for Australia. Table 10 shows the same exercise for Peru and Australia. The results are qualitatively similar.

The results on Tables 9 and 10 were tested under different parameterizations. Although there may be significant changes in the average impact of the shock or the volatility generated, the main result remains the same. That is, imputing the Australian dependence pattern to Chile — or Peru — reduces their impact on aggregate output and the opposite for Australia. More precise and country-specific calibration would be needed to obtain more accurate measures that fit the data in terms of output impact or volatility.

Table 10: Counterfactual exercises: Peru vs Australia

	Peru	Australia
Panel A: Mean output change after an $U[0, 0.1]$ positive shock sequence		
Original	0.051	0.048
Counterfactual	0.045	0.057
Ratio C/O	-12%	+19%
Panel B: Volatility of output change after an $U[-0.1, 0.1]$ shock sequence		
Original	0.064	0.060
Counterfactual	0.056	0.072
Ratio C/O	-12%	+19%

Note: Counterfactual means that the production structure of the country with which it is compared has been imputed. Panel A shows the mean impact on aggregate output after the commodity price shock sequence. Panel B compute the standard deviation of these changes. Simulations were made using $(\sigma_I, \sigma_Q) = (2, 1.2)$ but results remains almost identical using $(\sigma_I, \sigma_Q) = (1.2, 2)$ instead.

In contrast to the empirical section, the simulations provide results on aggregate output rather than GDP. This is because in the model the effect on GDP is zero. The reason is because in the model labor is an inelastic factor and there are no frictions or distortions. Considering that the empirical evidence in Section 5 shows that commodity shocks have important effect on GDP, through the production structure, I consider exploring a version of the model where there are effects on GDP as an important topic for future research.

8 Conclusion

This paper studies how different patterns in production networks can help to explain the impact of commodity price shocks on aggregate outcomes. Using OECD data, I present a new empirical pattern associated with commodity sectors that I call *dependency*. This pattern shows that, around a specific commodity sector, there are substantial differences between countries in the number of sectors who directly or indirectly interact with the commodity sector, once only high-intensity interactions are considered. This pattern can be associated with general measures of networks such as density or sparcity, but is not the same. The main difference is that the *dependency* pattern is associated to a specific commodity sector while density or sparcity are measures for the whole network.

I show that different patterns of *dependency* can significantly help to explain the impact of commodity price shocks. In order to empirically assess the role of the production structure in the propagation of commodity price shocks, I introduce the concept of *eigenvector centrality*, a measure captures the idea behind the *dependency* pattern. I found that the influence of the commodity sector as a buyer within the network plays an important role in the propagation of commodity price shocks. Once controlling for network measures associated specifically with the commodity sector, general network measures such as density or sparcity do not play a role in propagation. In a second step, I introduce the *dependency* pattern in a novel network model from Silva et al. (2023) to evaluate counterfactual scenarios around commodity price shocks. I interchange the *dependency* pattern between the Chilean and Australian networks and show that, after imposing Chile's *dependency* pattern on Australia, commodity price shocks have a greater impact on Australia and vice versa. Theoretical results are consistent with empirical findings.

The main contribution of this paper is to move away from aggregate measures of a production network to use commodity sector specific network measures to study the propagation of commodity shock prices on aggregate outcomes. This paper contributes to two strands in the literature. On one hand, it extends the literature that investigates the impact of terms of trade or commodity price shocks on the business cycle. On the other hand, contributes to the literature relating the production structure and the propagation of shocks. Finally, the null effect of commodity price shocks on GDP in the model and the significant effects found in the empirical evidence opens an avenue for future research.

References

Acemoglu, D., U. Akcigit, and W. Kerr (2016): “Networks and the macroeconomy: An empirical exploration,” *Nber macroeconomics annual*, 30, 273–335.

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012): “The network origins of aggregate fluctuations,” *Econometrica*, 80, 1977–2016.

Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi (2017): “Microeconomic origins of macroeconomic tail risks,” *American Economic Review*, 107, 54–108.

Barrot, J.-N., and J. Sauvagnat (2016): “Input specificity and the propagation of idiosyncratic shocks in production networks,” *The Quarterly Journal of Economics*, 131, 1543–1592.

Benguria, F., F. Saffie, and S. Urzúa (2021): “The transmission of commodity price super-cycles,”Technical report, National Bureau of Economic Research.

Bloch, F., M. O. Jackson, and P. Tebaldi (2021): “Centrality measures in networks,” *Available at SSRN 2749124*.

Cao, S., and W. Dong (2020): “Production Networks and the Propagation of Commodity Price Shocks,”Technical report, Bank of Canada.

Caraiani, P. (2019): “Oil shocks and production network structure: Evidence from the OECD,” *Energy Economics*, 84, 104559.

Carvalho, V. M. (2008): *Aggregate fluctuations and the network structure of intersectoral trade*: The University of Chicago.

Carvalho, V. M., and A. Tahbaz-Salehi (2019): “Production networks: A primer,” *Annual Review of Economics*, 11, 635–663.

Céspedes, L. F., and A. Velasco (2012): “Macroeconomic performance during commodity price booms and busts,” *IMF Economic Review*, 60, 570–599.

——— (2014): “Was this time different?: Fiscal policy in commodity republics,” *Journal of Development Economics*, 106, 92–106.

Drechsel, T., M. McLeay, and S. Tenreyro (2019): “Monetary policy for commodity booms and busts.”

Drechsel, T., and S. Tenreyro (2018): “Commodity booms and busts in emerging economies,” *Journal of International Economics*, 112, 200–218.

Fernandez, A., S. Schmitt-Grohé, and M. Uribe (2017): “World shocks, world prices, and business cycles: An empirical investigation,” *Journal of International Economics*, 108, S2–S14.

——— (2020): “Does the Commodity Super Cycle Matter?,” *National Bureau of Economic Research*, (No. w27589).

Fernández, A., A. González, and D. Rodríguez (2018): “Sharing a ride on the commodities roller coaster: Common factors in business cycles of emerging economies,” *Journal of International Economics*, 111, 99–121.

Gruss, B., and S. Kebhaj (2019): *Commodity terms of trade: A new database*: International

Monetary Fund.

Herskovic, B. (2018): “Networks in production: Asset pricing implications,” *The Journal of Finance*, 73, 1785–1818.

Kohn, D., F. Leibovici, and H. Tretvoll (2021): “Trade in commodities and business cycle volatility,” *American Economic Journal: Macroeconomics*, 13, 173–208.

Kose, M. A. (2002): “Explaining business cycles in small open economies:‘How much do world prices matter?’,” *Journal of International Economics*, 56, 299–327.

Mendoza, E. G. (1995): “The terms of trade, the real exchange rate, and economic fluctuations,” *International Economic Review*, 101–137.

Miranda-Pinto, J. (2021): “Production network structure, service share, and aggregate volatility,” *Review of Economic Dynamics*, 39, 146–173.

Plagborg-Møller, M., and C. K. Wolf (2021): “Local projections and VARs estimate the same impulse responses,” *Econometrica*, 89, 955–980.

Romero, D. (2022): “Domestic Linkages and the Transmission of Commodity Price Shocks,” *Working Papers of the Central Bank of Chile.*

Sargent, T. J., and J. Stachurski (2022): “Economic Networks: Theory and Computation,” *arXiv preprint arXiv:2203.11972.*

Schmitt-Grohé, S., and M. Uribe (2018): “How Important are Terms-Of-Trade Shocks?” *International Economic Review*, 59, 85–111.

Shousha, S. (2016): “Macroeconomic effects of commodity booms and busts: The role of financial frictions,” *Unpublished manuscript.*

Silva, A., J. Miranda-Pinto, J. Olaya-Agudelo, and P. Caraiani (2023): “Commodity Shocks and Production Networks in Small Open Economies,” *Central Bank of Chile Working Paper*, 977.

Appendix A

1. OECD sectors

Table 11: OECD sectors description

Sector number	Sector name	OECD code
1	Agriculture, forestry and fishing	01T03
2	Mining and extraction of energy producing products	05T06
3	Mining and quarrying of non-energy producing products	07T08
4	Mining support service activities	09
5	Food products, beverages and tobacco	10T12
6	Textiles, wearing apparel, leather and related products	13T15
7	Wood and products of wood and cork	16
8	Paper products and printing	17T18
9	Coke and refined petroleum products	19
10	Chemicals and pharmaceutical products	20T21
11	Rubber and plastic products	22
12	Other non-metallic mineral products	23
13	Basic metals	24
14	Fabricated metal products	25
15	Computer, electronic and optical products	26
16	Electrical equipment	27
17	Machinery and equipment, nec	28
18	Motor vehicles, trailers and semi-trailers	29
19	Other transport equipment	30
20	Other manufacturing; repair and installation of machinery and equipment	31T33
21	Electricity, gas, water supply, sewerage, waste and remediation services	35T39
22	Construction	41T43
23	Wholesale and retail trade; repair of motor vehicles	45T47
24	Transportation and storage	49T53
25	Accomodation and food services	55T56
26	Publishing, audiovisual and broadcasting activities	58T60
27	Telecommunications	61
28	IT and other information services	62T63
29	Financial and insurance activities	64T66
30	Real estate activities	68
31	Other business sector services	69T82
32	Public admin. and defence; compulsory social security	84
33	Education	85
34	Human health and social work	86T88
35	Arts, entertainment, recreation and other service activities	90T96
36	Private households with employed persons	97T98

2. Counterfactual transformation

This subsection explains the counterfactual transformation for simulations exercises. The main idea is to change the diversification pattern of IO tables keeping total intermediate sales and gross output constant. The procedure consists of 6 steps detailed as follows. Table 12 present IO tables for two countries (Left and Right). In both countries there exists three sectors. Column I.S. means intemediate sales and column G.O. is gross output. These two countries exhibit different IO tables and gross output.

Table 12: **Step 1.** Original IO table

	1	2	3	I.S.	G.O.
sector 1	55	60	10	125	155
sector 2	12	10	20	42	50
sector 3	5	23	60	88	128

	1	2	3	I.S.	G.O.
sector 1	30	30	30	90	130
sector 2	25	70	33	128	200
sector 3	39	40	81	160	170

Table 13 compute intermediate sales shares for each sector and the total intermediate sales share (i.e., total intermediate sales over gross output). An element i, j represent sales from sector i to j over gross output of sector i. In the *upstream ego graph* presented in Section 4, shares below 0.1 are dropped. Column I.S. share is the row sum, that is, the share of intermediate sales over gross output.

Table 13: **Step 2.** Intermediate shares (Sales / G.O.)

	1	2	3	I.S. share	G.O.
sector 1	0.35	0.39	0.06	0.81	155
sector 2	0.24	0.20	0.40	0.84	50
sector 3	0.04	0.18	0.47	0.69	128

	1	2	3	I.S. share	G.O.
sector 1	0.23	0.23	0.23	0.69	130
sector 2	0.13	0.35	0.17	0.64	200
sector 3	0.23	0.24	0.48	0.94	170

After calculating intermediate sales share, Table 14 compute the ratio between each sector intermediate sales share and the total intermediate share. For example, the element $(3, 2)$ in the left panel is computed as 0.18/0.69. This means that for all intermediates sales from sector 3, 26% go to sector 2. These are the target ratios that are intended to be exchanged between countries. A counterfactual economy of country Left is expected to have the shares in the right panel in Table 14, and vice versa.

Table 14: **Step 3.** Sector share / Total I.S. share

	1	2	3
sector 1	0.44	0.48	0.08
sector 2	0.29	0.24	0.48
sector 3	0.06	0.26	0.68

	1	2	3
sector 1	0.33	0.33	0.33
sector 2	0.20	0.55	0.26
sector 3	0.24	0.25	0.51

To permute these shares between countries, I compute new shares as presented in Table 15. Note, for example, that in Table 13 sector 1 of Right country distributes its sales evenly. Now, in Table 15, we have that sector 1 on Left country distributes its sales evenly.

Table 15: **Step 4.** Modified Intermediate shares (Sales / Gross output)

	1	2	3	I.S. share
sector 1	0.27	0.27	0.27	0.81
sector 2	0.16	0.46	0.22	0.84
sector 3	0.17	0.17	0.35	0.69

	1	2	3	I.S. share
sector 1	0.30	0.33	0.06	0.69
sector 2	0.18	0.15	0.30	0.64
sector 3	0.05	0.25	0.64	0.94

Table 16 check that, for all sectors, the ratio between the sales share and the total intermediate sales share (I.S. share) is identical to that of the other country presented in Table 14. This means that all sectors distribute their sales as the comparison country keeping its total intermediate sales share constant (compare column 4 in Table 13 and 15).

Table 16: **Step 5.** Check modified shares (Sector share / I.S. share)

	1	2	3
sector 1	0.33	0.33	0.33
sector 2	0.20	0.55	0.26
sector 3	0.24	0.25	0.51

	1	2	3
sector 1	0.44	0.48	0.08
sector 2	0.29	0.24	0.48
sector 3	0.06	0.26	0.68

Finally, Table 17 computes the counterfactual IO matrix for each country. These new tables meet two conditions. First, gross output and intermediate sales remains constant in relation to the original values (see Table 12). Second, the ratios of sectoral intermediate share over total intermadiate share is identical to that of the economy in comparison (see Tables 14 and 16). This means that the *upstream dependency pattern* is relocated from one country to another.

Table 17: **Step 6.** Counterfactual IO tables

	1	2	3	I.S.	G.O.
sector 1	42	42	42	125	155
sector 2	8	23	11	42	50
sector 3	21	22	45	88	128

	1	2	3	I.S.	G.O.
sector 1	40	43	7	90	130
sector 2	37	30	61	128	200
sector 3	9	42	109	160	170

Appendix B

Before introduce Proposition 1 to 3 from **?**, I present the notation that they use.

Notation. They use **bold** letters to refer to vectors and matrices and use a hat notation to denote changes relative to a given equilibrium i.e. $\hat{X} = d\log X = \log X - \log X^*$. The notation for matrices and vectors are in the following table.

Table 18: Notation

Notation	Typical Element	Comment
Matrices ($N \times N$, domestic sectors only)		
Ω	$\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$	Expenditure of goods from j by sector i
$\Psi = (I - \Omega)^{-1}$	Ψ_{ij}	Importance of j as a direct and indirect supplier to i
M	$m_{ij} = \frac{M_{ij}}{Q_j}$	How much of good j is allocated to sector i
$\Psi^U = (I - M')$	Ψ^U_{ij}	Importance of j as a direct and indirect buyer to i
Vectors ($N \times 1$)		
Ω_0	$\Omega_{0i} = \frac{P_i C_i}{GDP}$	
$\Omega_{N+1}(b)$	$\Omega_{N+1,j} = \frac{P_j M_{N+1,j}}{P_{N+1} Q_{N+1}}$	Importance of good j as a direct supplier to sector $N+1$
$\Omega_{N+1}(s)$	$\Omega_{i,N+1} = \frac{P_{N+1} M_{i,N+1}}{P_i Q_i}$	Importance of sector $N+1$ as a direct supplier to sector i
$\lambda = \Psi'\Omega_0 + \Psi'(\Omega_{N+1}(b))\lambda_{N+1}$	$\lambda_i = \frac{P_i Q_i}{GDP}$	Domar Weight of Sector i

Source: **?**.

After introduce their notation I present their main propositions.

Proposition 1. *Consider a differential change in the commodity price, $\hat{P}_{N+1}$. Up to a first-order, the change in domestic prices satisfy*

$$\hat{P}_i = \left(\sum_{k=1}^{N} \Psi_{ik} \Omega_{k,N+1} \right) \hat{P}_{N+1}. \tag{12}$$

Proposition 2. *Consider a differential change in the commodity price, $\hat{P}_{N+1}$. Up to a first-order,*

$\hat{Q}_{N+1}$ *satisfies*

$$\hat{Q}_{N+1} = \phi^{N+1}_{P_{N+1}} \hat{P}_{N+1}, \tag{13}$$

where

$$\begin{aligned}
\phi^{N+1}_{P_{N+1}} &= \frac{\delta_{N+1}}{1-\delta_{N+1}} \left(1 - b_{N+1}\tilde{\omega}_{N+1,N+1}\right) - \frac{\delta_{N+1}}{1-\delta_{N+1}} b_{N+1} \left(\sum_{i=1}^{N} \tilde{\omega}_{N+1,i} \left(\sum_{k=1}^{N} \Psi_{ik}\Omega_{k,N+1} \right) \right), \\
b_{N+1} &= \frac{P^I_{N+1} M^I_{N+1}}{P^B_{N+1} B_{N+1}}, \\
\tilde{\omega}_{ij} &= \frac{P_j M_{ij}}{P^I_i M_i}.
\end{aligned}$$

Proposition 3. Consider a differential change in the commodity price, $\hat{P}_{N+1}$. Up to a first-order, $\hat{Q}_i$, for $i \neq N+1$, satisfies

$$\hat{Q}_i = (\zeta^{N+1}_i + \zeta^Q_i \phi^{N+1}_{P_{N+1}}) \hat{P}_{N+1}, \tag{14}$$

where

$$\begin{aligned}
\zeta^{N+1}_i = & \underbrace{-\alpha_{N+1}\alpha_B \sum_{k=1}^{N} \Psi^U_{ik} \frac{\Omega_{0k}}{\lambda_k}}_{\text{Wealth Effect}} + \underbrace{\left(\alpha_{N+1} + \left(\sum_{k=1}^{N} \Psi^U_{ik} \left(\xi^{N+1}_{ki} m_{ki} + m_{N+1,k} \left[\xi^{N+1}_{N+1,k} + (1-\alpha_{N+1}) \right] \right) \right) \right)}_{\text{Buyers' Substitution}} \\
& - \underbrace{\sum_{k=1}^{N} \Psi_{ik}\Omega_{k,N+1}}_{\text{Pure Downstream Effect}}
\end{aligned}$$

$$\begin{aligned}
\zeta^Q_i &= -\alpha_{N+1}\alpha_B \sum_{k=1}^{N} \Psi^U_{ik} \frac{\Omega_{0k}}{\lambda_k} + \left(\alpha_{N+1} + \left(1 - \alpha_{N+1} - \xi^Q_{N+1}\right) \left(\sum_{k=1}^{N} \Psi^U_{ik} m_{N+1,k} \right) \right) \\
\phi^{N+1}_{P_{N+1}} &= \frac{\delta_{N+1}}{1-\delta_{N+1}} \left(1 - b_{N+1}\tilde{\omega}_{N+1,N+1}\right) - \frac{\delta_{N+1}}{1-\delta_{N+1}} b_{N+1} \left(\sum_{i=1}^{N} \tilde{\omega}_{N+1,i} \left(\sum_{k=1}^{N} \Psi_{ik}\Omega_{k,N+1} \right) \right) \\
b_{N+1} &= \frac{P^I_{N+1} M^I_{N+1}}{P^B_{N+1} B_{N+1}} \\
\tilde{\omega}_{ij} &= \frac{P_j M_{ij}}{P^I_i M_i} \\
\alpha_{N+1} &= \frac{\Pi_{N+1}}{GDP} \\
\alpha_B &= \frac{rB}{\bar{L} + \Pi_{N+1} + rB}
\end{aligned}$$

$$
\begin{aligned}
\xi_{ji}^{N+1} &= (1-\sigma_j^I)\sum_{k=1}^{N}\Psi_{ik}\Omega_{k,N+1} + (\sigma_j^I - \sigma_j^{VA})\left(\sum_{i=1}^{N}\tilde{\omega}_{ji}\left(\sum_{k=1}^{N}\Psi_{ik}\Omega_{k,N+1}\right) + \tilde{\omega}_{j,N+1}\right) \\
&\quad + (\sigma_j^{VA}-1)\sum_{k=1}^{N}\Psi_{jk}\Omega_{k,N+1} \qquad \text{for all } j = 1, ..., N \\
\xi_{N+1}^{Q} &= \frac{(1-\delta_{N+1})(\sigma_{N+1}^{VA}-1)}{\delta_{N+1}} \\
\xi_{N+1,i}^{N+1} &= (1-\sigma_{N+1}^I)\sum_{k=1}^{N}\Psi_{ik}\Omega_{k,N+1} + (\sigma_{N+1}^I - \sigma_{N+1}^{VA})\left(\sum_{i=1}^{N}\tilde{\omega}_{N+1,i}\left(\sum_{k=1}^{N}\Psi_{ik}\Omega_{k,N+1}\right) + \tilde{\omega}_{N+1,N+1}\right) \\
&\quad + (\sigma_{N+1}^{VA}-1).
\end{aligned}
$$