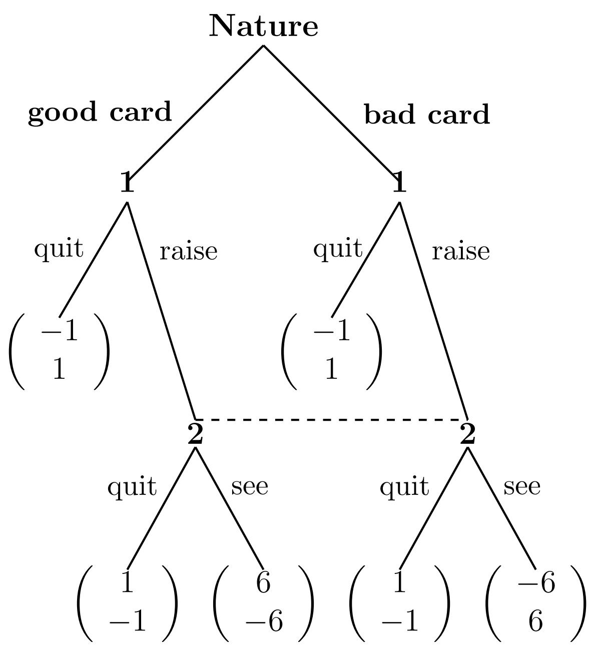
Problem set 2

Due date: Tuesday, December 3^{rd}, 1:15 pm.
Send your assignment to luiz.bissoto@epfl.ch

Exercise 1: The poker game (20 pts + 5 bonus)

Two players play a simplified poker game that works the following way: both players put 1 CHF in a pot. Then player 1 draws a card from a deck containing only two cards: a good card and a bad card. After seeing the card she drew player 1 decides whether to "quit" the game (in which case player 2 gets the money in the pot) or to "raise", which means putting another 5 CHF in the pot. Player 2 does not know player 1's card at this stage: this is a game of imperfect information. If player 1 "raises", player 2 must decide whether to "quit" (in which case player 1 gets the pot) or put an extra 5 CHF in the pot and "see". If player 2 sees, player 1's card is uncovered; if she had the good card she gets the pot, otherwise player 2 gets the pot.



1. What are the strategies available to player 1? And to player 2? (2 pts)
2. How many subgames does this game have? (1 pt)
3. Find the pure-strategy (Bayesian) Nash equilibria of this game (8 pts) (Hint: use the Hersanyi technique)
4. Find one mixed-strategy BNE. (9 points)
5. Bonus: Find a PBE, by giving the equilibrium strategy of the two players, and the probability that Player 2 assigns to the two nodes in the information set. (5 pts)

Exercise 2 (20 pts)

Consider a public good game similar to the one seen in class. There are now n players, each of which must decide whether to work or not work to produce a public good. The public good is produced if at least one player works. Work involves a constant cost $c < 1$, the same for everybody. The value of the good is v_i for each player i, and each player knows only his value. Each v_i is uniformly distributed in [0,1] (and each v_i is independent from the others). the total payoff to one player is $v_i - c$ if she works, v_i if she doesn't work and the good is produced by someone else, and 0 if the good is not produced.

1. Solve for a symmetric BNE in which each player works if v_i exceeds a threshold v^* (the same for everybody). (12 pts) (Hint: since you only have to find one mixed-strategy equilibrium, try a strategy in which player 1 mixes between **only two** strategies)

2. What is the probability that the good is produced, as a function of n? (8 pts)

Exercise 3 (20 pts)

There are five bags 10, 20, 40, 80, 160 gold coins, respectively. Two bags are selected randomly, with the constraint that one of the two bags contains twice as main coins as the other (otherwise said, the two bags are, with the same probability, the bags containing 10 and 20 coins, or those containing 20 and 40, or 40 and 80, or 80 and 160 coins). The two selected bags are then assigned to two players (each player gets one of the two bags with equal probability). After seeing the contents of her bag – but not the content of the other bag – each player is asked if she wants to switch bag with the other player. If both want to switch, the exchange occurs.

1. Find a BNE (13 pts) (Hint: would you be willing to switch if your bag contains 160 coins? and 80?)

2. Your friend tries to convince you that, if you have 20 or 40 coins, you should want to switch: if you have 20 coins, with the same probability the opponent has 10 or 40 coins, so upon switching you would get on average 25. Similarly, if you have 40, after switching you would get on average 50. Is the argument correct? (7 pts)

Exercise 4: the wallet game (20 pts)

Consider an auction with two bidders. The object for sale in the auction is a sum of money contained in two wallets: wallet 1 contains x_1 and wallet 2 contains x_2. Player 1 can see the content of wallet 1 and Player 2 can see the content of wallet 2. Each player knows that the content of the wallet she cannot see is uniformly distributed between 0 and \$50. The highest bidder, bidding b, gets a payout $x_1 + x_2 - b$, while the other player gets 0.

1. Is this a private-value or a common-value auction? (1 pts)

2. Is this a first-price or a second-price auction? (1 pts)

3. Take the point of view of Player 2 and assume that Player 1 uses a bidding strategy $b_1 = a^{(1)}x_1 + c^{(1)}$. If Player 2 bids b_2, what is the probability that she wins the auction?[1] (3 pts)

4. What is the expected surplus of Player 2, conditional on bidding b_2 and winning? (5 pts)

5. The expected payout of player 2, when bidding b_2, is equal to the probability of winning (that you found in point 3) times the expected surplus conditional on winning (that you found in point 4).
   - What is the optimal value of bid of Player 2 (i.e. the optimal b_2), as a function of x_2? (you will find an expression of the form $b_2 = a^{(2)}x_2 + c^{(2)}$) (5 pts)
   - Impose $a^{(1)} = a^{(2)}$ and $c^{(1)} = c^{(2)}$ to find the optimal symmetric strategy. You found the BNE! (5 pts)

Exercise 5: a second-price auction with entry fee (20 pts)

Consider a second-price private-value auction where you need to pay an entry fee E in order to participate. N people have already entered the room (you can assume that they did not need to pay the fee) and, if you decide to enter, nobody will enter after you. You know your private value v (such that $0 < v < 1$) and you know that the values of the other people in the room are i.i.d. and uniformly distributed in $[0, 1]$.

1. Find the condition, in terms of E and v, in which you would decide to pay the fee (12 pts). (Hint: compare your payout in case of not partcipating, which is 0, to the expected payout if you pay the fee and participate in the auction).

2. If you decide to pay the fee hence participate in the auction, what would your bidding strategy be? (8 pts)

[1] Note: taking the point of view of player 2 means that you know x_2 and you choose b_2. You do not know x_1, however you know the strategy of player 1, i.e. you know $a^{(1)}$ and $c^{(1)}$. Your answers from this point on can contain x_2, b_2, $a^{(1)}$ and $c^{(1)}$ but not x_1