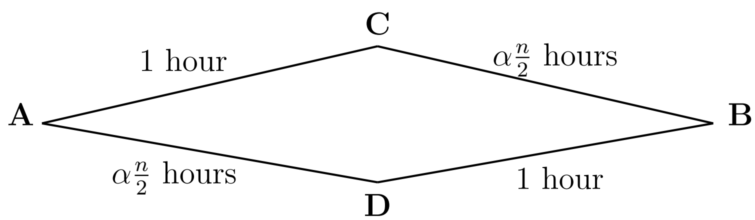
Problem set 1

Due date: Tuesday, October 29^{th}, 1:15 pm.

Exercise 1: The transportation paradox (14 points)

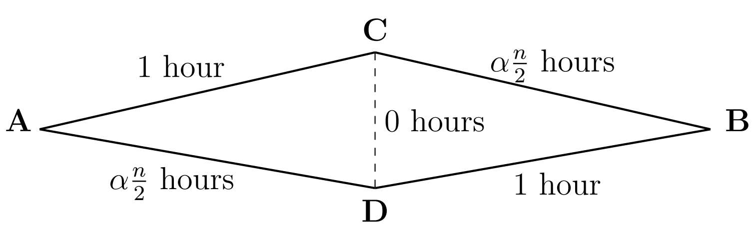
This is a simple version of the Braess paradox, which shows that it's possible that adding a new road to an existing transportation system (while not removing any of the existing roads) can increase transportation time for everybody. Student 1 and Student 2 live in location A and need to go to their school in location B every morning. There are two possible routes: one going through location C (call it route ACB) and onw going through location D (call it route ADB). The graph below shows the transportation time for every portion of the two routes.



While for the 2 stretches AC and DB the traveling time is always 1 hour, for the other two stretches the time depends on the number of people n on it. For example, if both students travel on CB, it takes them α hours to complete this portion, whereas if only one of them takes this route, it takes only $\frac{\alpha}{2}$ hours. **You know that** $\alpha < 1$

1. Find the pure-strategy equilibria for this game (The payout for choosing one route, e.g. the route ACB, is the negative of the total transportation time). (4 points)

   Imagine now that a new super-fast route is built between C and D, such that it takes 0 time to complete the stretch CD. Two new traveling possibilities are thus the routes ACDB and ADCB.



2. Show that the option of taking route ACDB is strictly dominated, and you can thus reduce the game to one with 3 choices for each student. (3 points)

3. Find the unique Nash equilibrium of this new game. For which values of α does this NE involve more time spent in transportation for both students, relative to the NEs of the game without the fast route? (7 points).

Exercise 2 (20 points)

Consider this game in matrix form with two players.

1 \ 2	L	R
U	(1,9)	(4,7)
M	(3,10)	(2,5)
D	(2,5)	(6,8)

1. This game has two pure-strategy Nash equilibria. What are they? (2 pts)
2. Suppose Player 2 plays a mixed strategy of the form $(p^2_L, 1 - p^2_L)$. What is the value of p^2_L such that player 1 is indifferent among actions M and D? (6 pts)
3. Suppose that Player 1 plays a mixed strategy of the form $(0, p^1_M, 1 - p^1_M)$. What is the value of p^1_M that makes player 2 indifferent between action L and action R? (6 pts)
4. Based on your responses above, describe a mixed-strategy Nash equilibrium of this game. Explain why this is in fact a Nash equilibrium. (6 pts)

Exercise 3 (16 points)

Consider the following game

1 \ 2	L	R
T	(4,2)	(3,2)
M	(0,0)	(1,1)
B	(1,1)	(0,0)

1. Are there any strictly dominated strategies for Player 1? For Player 2? (2 pts)
2. Are there any weakly dominated strategies for Player 1? For Player 2? (2 pts)
3. Reduce this game as much as you can by eliminating *only strictly dominated strategies.* Which Nash equilibrium or equilibria do you find? (4 pts)
4. Now reduce this game by eliminating strictly and weakly dominated strategies. Can you find two ways to do this, so that you end up with a different Nash equilibrium? (8 pts)

Exercise 4: The travelers' paradox (16 points)

Two travelers come back from a trip to Peru with two identical artifacts. The artifacts are destroyed during the plane trip, and the manager of the airline company accepts to reimburse the travelers. He devises the following plan to induce the travelers to declare the real value of the artifacts: the players are put in two different rooms and have to declare the value of the artifact. The maximum value they can declare is \$100 and the min is \$3, and the value has to be an integer number. If they declare the same value $v_1 = v_2$, they both get reimbursed for that amount. If one player declares a lower value, it is assumed that that is the true value of the artifact, and the player who declared the lower value (v_L) gets $v_L + 2$, and the other player gets $v_L - 2$.

1. Find the unique Nash equilibrium of this game. (8pts)
2. Do you think this Nash equilibrium is realistic, or you find something unsatisfactory about it? (1pts)
3. Consider now an alternative plan in which the player who declared the lower value (v_L) gets $v_L + 1$, and the other player gets $v_L - 1$. Find the Nash equilibria of this game. (7pts)

Exercise 5: Splitting the dollars (16 points + 4 bonus)

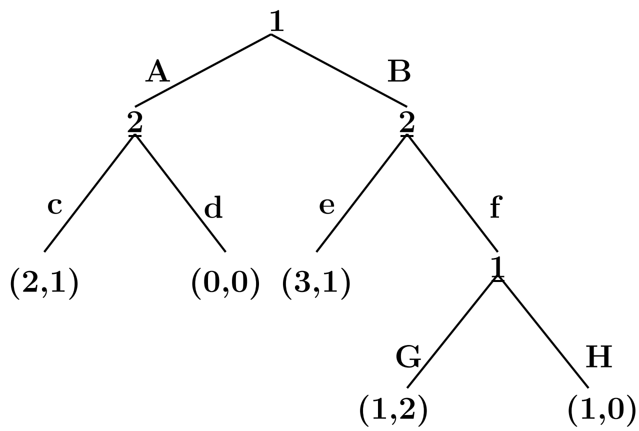
Players 1 and 2 are bargaining over how to split \$ 10. Each player i demands an integer amount s_i between \$1 and \$10 for herself. The choices are made simultaneously. We will consider this game under different sets of rules. In both cases, if $s_1+s_2 \leq 10$, both players get the amount they demanded (and the remainder, if any, is destroyed). Find the pure-strategy Nash equilibria in the following two cases:

1. In the first case, if $s_1 + s_2 > 10$ both players get 0.
2. In the second case, if $s_1 + s_2 > 10$, and $s_1 \neq s_2$, the player who demanded the smaller amount gets the amount she demanded and the other player gets the remaining money (e.g. if $s_1 = 6$ and $s_2 = 7$, then Player 1 gets 6 and player 2 gets 4). If $s_1 + s_2 > 10$ and $s_1 = s_2$, then both players get 5.

Bonus (4 points): Now suppose instead that s_i can be any real number $0 \leq s_i \leq 10$, with the same rules as in part 2. (e.g. if $s_1 = \$4.6721$ and $s_2 = \$7.4523$ then player 1 gets s_1 and player 2 gets $1 - s_1$. What are the Nash equilibria of this game?

Exercise 6 (18 points)

Consider the foilowing extensive-form game with two players (Player 1 and player 2)



1. How many pure strategies has Player 1? and Player 2? (3 pts)
2. Write the game in normal form and find all the pure-strategy Nash equilibria. (5 pts)
3. Using backward induction, find all the pure-strategy subgame-perfect equilibria. (6 pts)
4. Identify one NE which is not an SPE and show that it involves a non-credible threat. (4 pts)