Due date: Friday, December 20^{th}, 23:59 pm.
Send your assignment to luiz.bissoto@epfl.ch

Exercise 1 (17 pts): Application of the Revenue-Equivalence Theorem

Consider a fourth-price private-value auction. Each of the $N >> 4$ bidders has a private valuation of the object for sale, and each valuation is independent and uniformly distributed in $[0, 1]$. The highest bidder gets the object and pays a price equal to the bid of the fourth-highest bidder.

1. Knowing that, according to the revenue-equivalence theorem, the expected payment from each bidder in fourth-price auctions is the same as in first- or second-price auctions, how much is the expected payment for a bidder i whose private value is v_i? (5 pts)

2. Guessing that the optimal bidding strategy is of the form $b(v) = av$ with constant a, and using the formula for the $k-th$ statistic seen in class (i.e. the expected value of the k-th of N draws uniformly distributed in $[0, v]$) find the value of a, i.e. the optimal bidding strategy $b(v)$ in this type of auction. (12 pts)

Exercise 2 (16 pts): Twice-Repeated Game

The following stage game is repeated twice

1 \ 2	L	M	R
L	(1,1)	(5,0)	(1,0)
M	(0,5)	(4,4)	(0,0)
R	(0,1)	(0,0)	(3,3)

1. What are the pure-strategy Nash Equilibria of the stage game? (4 pts)

2. Find three Subgame Perfect Equilibria of the twice-repeated game, one of which needs to involve both players playing M in the first stage. The discount factor is 1. (12 pts)

Exercise 3: Price Competition in Duopoly (17 pts)

Two firms are playing an infinitely-repeated pricing game of the following form

1 \ 2	Low	High
Low	(5,5)	(20,0)
High	(0,20)	(10,10)

If a firm choose Low prices, it acts competitively, trying to take market share from the opponent. The outcome (High, High) can be interpreted as a collusive behavior to share the market and keep high profits. The firms simultaneously set prices at regular intervals.

1. What are the NE of the stage game? (2 pts)
2. In the infinitely-repeated game, describe a SPE in which in equilibrium both firms set High prices in every period. Specify what value of the discount factor you need for your strategy profile to be a SPE. (15 pts)